

Infinity and Continuity

Marinus Dirk Stafleu

Infinity and continuity are two recurring themes in Danie Strauss's philosophy of mathematics based on Herman Dooyeweerd's Philosophy of the Cosmonomic Idea. Stressing the relevance of the distinction between law and subject, this paper provides some marginal comments on Strauss's and Dooyeweerd's views on mathematics, which may be influenced by intuitionism, constructivism and essentialism.

1. Introduction

The first time I met Danie Strauss (in the seventies), he asked my opinion on the reality of actual infinity. I don't remember my answer, but it was probably the classical one: infinity is never actual. Much later, Strauss observed that "... due to his leaning towards Cantorean set theory Stafleu developed an alternative approach 'uprooting' continuity, cancelling the qualifying role of the meaning nucleus of the numerical aspect with regard to its disclosed structure, and implicitly used the actual infinite which he wanted to reject otherwise."¹ I welcome the opportunity to reply to this admonition.

Danie Strauss was several times concerned with the problem of potential and actual infinity, or 'successive' and 'at-once' infinity, as he calls it, at least as early as 1970 and more recently in 1996.² Strauss appears to start from a moderate intuitionist and maybe essentialist worldview that I do not share. Therefore I shall present an alternative view that I believe to be no less consonant with Dooyeweerd's systematic philosophy than Strauss's. Alongside, I shall make some marginal comments on his views. Basically, there are two related problems: the meaning of the so-called transfinite numbers and the meaning of the set of real numbers, which I call 'continuous', in line with most mathematicians but against Strauss's objections.

1 Strauss 1995, 132. Strauss 1973, 185-186. Strauss refers to Stafleu 1972.

2 Strauss 1970-71. Strauss 1973, 183-186; Strauss 1996.

2. Potential and actual infinity

Until the first half of the 19th century, philosophers and mathematicians accepted Aristotle's distinction of 'potential' and 'actual' infinity. An endless sequence of numbers may have a 'potential' limit, but this limit is never 'actually' reached. For example, in the sequence of rational numbers $1/1, 1/2, 1/3, \dots 1/n, \dots$ where n is an increasing natural number, $1/n$ approaches but never reaches zero. The limit is a potential number, not an actual one. This is in particular relevant if the limit is not a rational number but an irrational one (like the square root of 2), for in this context 'rational' means 'reasonable' besides 'a ratio of two integral numbers'. Of old, this view of an infinite sequence presented quite a few problems, many of which were solved by 19th-century mathematicians like A.L. Cauchy (1789-1857), B. Bolzano (1781-1848) and K.W.T. Weierstrass (1815-1897). They avoided rather than solved the mostly philosophical problems of infinity, in particular by defining the limit of an infinite 'Cauchy-sequence' of rational numbers without assuming that it could be reached in an actual procedure. They demonstrated that any rational or irrational real number is the limit of a Cauchy-sequence of rational numbers. Later on, I shall discuss whether one may consider these limits to be 'numbers'.

3. The cardinality of infinite sets

Since 1870, G.F.L.P. Cantor (1845-1918) presented his theory of infinite sets, freely talking of 'transfinite numbers'. I believe that a lot of confusion could have been prevented if he had used a different word than 'transfinite number', and I want to argue that these are not numbers at all. A much better and often used expression is 'cardinality' or 'power'. The cardinal number of a *finite* set is its number of elements. Cantor observed that in many sets the elements have a one-to-one correspondence to the members of the infinite set of natural numbers. This applies, *e.g.*, to the set of all even numbers, to the set of all triples and to the set of all rational numbers. It also applies to the set of all square roots of (positive or negative) integral numbers. This set contains natural numbers (like 4), irrational numbers (like $\sqrt{2}$) and even complex numbers (like $-2i$). All these sets are infinite, but because of this property mathematicians call them 'discrete' or 'denumerable' or 'having the cardinality of the natural numbers'. By analogy, J.W.R. Dedekind (1831-1916) defined an infinite denumerable set as a set having a one-to-one correspondence to at least one of its proper sub-sets.

Cantor proposed to designate this property by the symbol \aleph_0 , aleph-nought, after the first letter of the Hebrew alphabet. Cantor assumed this

‘transfinite number’ to be the first in a sequence, $\aleph_0, \aleph_1, \aleph_2, \dots$. In 1892, Cantor proved by his famous diagonal method that the set of *real* numbers is not denumerable.³ Cantor indicated the cardinality of the set of real numbers by C . He posed the problem of whether C equals \aleph_1 . At the end of the 20th century, this problem was still unsolved. Maybe it is not solvable.⁴

This means that the set of real numbers cannot be reduced to a denumerable set, not even if it is dense. A *dense set* like the set of rational numbers has the property that between any two elements of the set (in which ‘betweenness’ is defined) there is at least one other element of the same set. For instance, $1/2(a+b)$ lies between the rational numbers a and b in the arithmetical order of smaller and larger.⁵ As a consequence, in a dense set there are infinitely many elements between a and b . Nevertheless, a dense set (like that of the rational numbers) may be denumerable, though in a different order than in that of increasing magnitude. An infinite Cauchy-sequence of rational numbers has a limit that may be rational or irrational. If one could complete the set of rational numbers with the set of all irrational limits of Cauchy-sequences, one would arrive at the set of real numbers. However, because the set of Cauchy-sequences is not denumerable, there does not exist a denumerable set of operations to achieve this goal. One would commit the fallacy of *petitio principii* by *defining* the set of real numbers (being non-denumerable) by adding all limits of Cauchy-sequences to the set of rational numbers, for this would require a non-denumerable set of operations (of calculating these limits).

It turns out that the only possible way of defining a continuous set (including that of the real numbers) is by reference in one way or another

3 Strauss, 1996, 157-158.

4 Cantor showed that the set of all sub-sets of an infinite discrete set is not denumerable. It does not have the cardinality \aleph_0 of a denumerable set but a ‘higher’ one, \aleph_1 . The set of all sub-sets of a set of cardinality \aleph_1 has the cardinality \aleph_2 , and so on. In a similar way, he showed that the set of all sub-sets of a continuous set has a cardinality D different from C . Consequently, besides the series $\aleph_0, \aleph_1, \aleph_2, \dots$ one has a similar series C, D, E, \dots . Now the continuum hypothesis states that these two series coincide in one way or another. K. Gödel (1906-1978) and P.J. Cohen demonstrated that the continuum hypothesis cannot be derived from and does not contradict the other axioms of set theory. This means that the identification of C with \aleph_1 may be treated as an independent axiom in the theory of sets.

5 If $a < b$, $a < 1/2(a+b) < b$. Observe that the cardinality of an infinite set is independent of the order of its elements, like the number of elements in a finite set is independent of their order. When speaking of denseness, one assumes that the set of rational numbers is arithmetically ordered, according to the order of smaller and larger. Only because the set of rational numbers is dense, Cauchy-sequences exist.

to the spatial relation frame, for instance applying the axiom that the set of real numbers can be projected on a spatial line. The assumption that any real number can be considered a Cauchy-sequence of rational numbers implies that the *dense* set of rational numbers constitutes a metric for the *continuous* set of real numbers. Therefore, the completion of the denumerable set of rational numbers to the continuous set of real numbers (only possible because the set of rational numbers in its arithmetical order is dense) does not imply the reduction of the latter to the former, *if it is performed via the spatial relation frame*.

As a consequence, we cannot determine the cardinality (in terms of the \aleph 's) of a continuous set like that of all points within a square. Instead one defines a *spatial* magnitude for such a set, like the area of a square. Usually this measure is defined such that it is subject to arithmetical laws like those of addition and subtraction, meaning that this spatial magnitude is projected on a numerical scale. This requires a metric, *i.e.*, a law for a scale, a unit and at least one measurement method.⁶ It is a remarkable fact that if one wants to measure the length of *e.g.* the diagonal in a unit square in an empirical way, for instance by using a ruler, one finds inevitably a rational number, like 1.41. In contrast, if one would calculate its length using Pythagoras's theorem, one finds an irrational number, as the Pythagoreans discovered to their dismay.⁷ On the one side the projection of spatial relations on quantitative ones (requiring the continuous set of real numbers) is *retrociptatory*. On the other hand, the definition of a continuous set by the projection of real numbers on a spatial line is *anticipatory*, requiring the continuity of spatial figures. In Strauss's words, the former requires the *concept* of a measure, the latter the *idea* of a continuous set.

4. Law and subject

Dooyeweerd's philosophy emphasizes the relation of laws and their subjects: laws make no sense without subjects, whereas subjects being *lawful* do not exist without laws.⁸ In this paper I am more concerned with modal laws than with the specific laws determining characters like those of groups, of vectors or of spatial figures.⁹ Modal laws are taken together in modal aspects or in relation frames, as I prefer to call them, for these

6 Stafleu, 1972; 1980, chapter 3.

7 Strauss, 1996, 145-146.

8 Strauss, 1996, 167-168.

9 Stafleu, 2002, chapter 2-3.

laws concern subject-object and subject-subject relations. In the quantitative relation frame, being the first one, we do not find objects. The only quantitative relations are those between quantitative subjects like numbers. Natural numbers satisfy a set of laws, usually called axioms, like those formulated by G. Peano (1858-1932). From these one derives the relation of numerical difference and the law of addition and subtraction, leading to the introduction of negative integers. Next, one discovers the law of multiplication and division, introducing the rational numbers. This shows the interplay of the law-side and the subject-side of created reality: one discovers some arithmetic laws for intuitively known subjects (*i.e.*, the natural numbers), next one finds that other subjects respond to the same laws, one discovers more laws, and so on. It is a never-ending story. Even the investigation of the set of natural numbers yields new results occasionally.

This makes clear what we should consider as a number: it is something that is only subject to numerical laws.¹⁰ For this reason, besides natural numbers one recognizes negative numbers, fractions, real and complex numbers, because all of these are subject to numerical laws.¹¹

There are some exceptions to be mentioned. The most important is that (in the arithmetical order of smaller and larger) each natural number has a unique successor (3 succeeds 2, for instance). This also applies to the integral numbers, but not to the dense set of rational numbers or the continuous set of real numbers, in which no member has a unique successor. In the sets of natural numbers, integrals, rational numbers and real numbers (though not the complex numbers), for each two different numbers a and b either $a < b$ or $b < a$. If these sets are projected on a spatial line, the relative position of the corresponding points can be uniquely determined by a real number. In fact, one should not consider the order of the points on a line to be successive, but simultaneous.

To make this clear, consider a point like the centre of gravity of a physical body moving along a line. On the one hand, because the set of spatial points is continuous, no spatial point has a unique successor. On the other hand, we assume that the moving point passes all points on its path

10 It is a bit more complicated, for one has to abstract from other relation frames, like the spatial, the kinetic or the physical ones. For instance, sugar is subject to numerical laws, as is clear when you want to calculate the price of 3 kg sugar, knowing the price per kg. Nevertheless, one does not consider sugar a number.

11 Complex numbers are numerical vectors, having a specific character, and therefore a specific kind of addition *etc.* derived from arithmetical relations.

successively. This is not a contradiction or an antinomy. Rather, it means that motion cannot be reduced to space or number: the kinetic relation frame is irreducible to the quantitative and spatial ones. The kinetic temporal order of ‘uniform succession’ is fundamentally different from the spatial order of simultaneity or the numerical order that had better be called ‘serial’. This insight should preclude the interpretation of a Cauchy-sequence as actually ‘approaching’ its limit, subject to ‘arithmetical progression’.¹²

5. The transfinite numbers are not numbers

I do not consider Cantor’s transfinite numbers to be numbers, for these do not confirm to arithmetical laws for addition *etc.*¹³ Because they are called transfinite numbers, one often attempts to treat them as such. For instance, one writes that $\aleph_0 + \aleph_0 = \aleph_0$, or $\aleph_0 + 7 = \aleph_0$. This means that if one takes two denumerable infinite sets together, or if one adds seven elements to a denumerable infinite set, one still has a denumerable infinite set, which is hardly surprising. It has nothing to do with the laws for addition of *numbers*. Rather, these examples concern the addition of *sets*, an operation that is subject to rules differing from those for the addition of numbers. For instance, if one adds 4 to 4, one gets 8, but if one adds a set to itself, one gets nothing but the same set. It means that one cannot consider sets to be subjects to arithmetic laws only (like numbers are). It appears that sets cannot be understood without taking into account spatial laws besides quantitative ones.¹⁴ I conclude that Cantor’s transfinite numbers are not numbers. They indicate the ‘cardinality’ or ‘power’ of an infinite set, nothing more or less than a property distinguishing various classes of sets from each other. There is nothing wrong with that.

12 Strauss, 1996, 167-173, quoting Dooyeweerd, 1953-1958, II, 92; I, 98-99.

13 Beth, 1944, 180-181.

14 Stafleu, 2002, chapter 2. Like Strauss, I believe that the relation of a whole and its parts is spatial rather than numerical. A set having sub-sets cannot be understood completely numerically. The relation of a set to its elements is numerical, whereas its relation to its sub-sets is spatial. Therefore, an element should be carefully distinguished from a sub-set containing only one element, Quine, 1963, 30-32 notwithstanding. The elements of a denumerably infinite set can be serially ordered, but its sub-sets cannot. (That is why the set of all such sub-sets has a different cardinality). However, these are simultaneously (‘at once’ according to Strauss) present in the set. If one considers a line as a set of points (to which Strauss objects, see below), then line-segments are sub-sets, which measure is their length. A sub-set containing merely one element (or zero elements) has length zero. Only a sub-set having at least two different points has a length different from zero, at least equal to the distance between these two points. So length turns out to be a property of a sub-set of a line. A point (being not a sub-set but an element of the set) has no length, not even zero; see Stafleu, 2002, 75.

Similarly, the cardinality C of a continuous set is not a number. To his surprise, Cantor discovered that the set of points on (*e.g.*) a side of a spatial square has the same cardinality as the set of the points within the square. It would be confusing to conclude that there are as many points within the whole square as there are on one of its sides. When Galileo (1564-1642) discovered that the set of natural numbers (1, 2, 3, ...) has a one-to-one correspondence to the set of (numerical) squares of natural numbers (1, 4, 9, ...), he was prudent enough to conclude that "... the attributes 'equal', 'greater', and 'less' are not applicable to infinite, but only to finite quantities."¹⁵

6. Intuitionism, constructivism and essentialism

Above, I deliberately used the word 'intuitively'. Like all mathematical intuitionists, Strauss appears to be a constructivist.¹⁶ Constructivists believe that something can only be considered a number if it can be constructed from the natural numbers in a *finite* number of steps. Consequently, Cantor's transfinite numbers cannot be numbers, and by an infinite Cauchy-sequence one cannot construct a number.

I agree with the first conclusion (and shall return to the second one presently), but not on constructivist arguments. I believe that intuition is an important starting point in philosophical analysis. In particular, the choice of the relation frames is first of all based on intuition, and this also applies to the initial insight into the laws and relations in each frame. The relation frames are conditions for the existence of anything that is created, in particular for human existence and for human knowledge. This means that *starting* the analysis of a relation frame requires intuition. But as far as science and scientific philosophy are concerned, the empirical and theoretical investigation soon overtakes this intuition, by finding new relations, new laws and new subjects, as illustrated above. Scientific analysis 'opens up' reality, inclusive of mathematical reality.

Therefore, though I believe intuition to play an important initial part, I reject its hypostatization in intuitionism. I have no objection at all to

¹⁵ Galileo Galilei, 1954, 32-33. Beth, 1944, 173-174.

¹⁶ Strauss, 1996, 158-159. An intuitionist constructivist should not be confused with a *social-constructivist*, somebody who believes that all scientific concepts are constructs made by people in a social context, see Howell, Bradley, (Eds.) 2001, chapter 12. Strauss, 1996, 167-168 observes that Dooyeweerd was influenced by intuitionism. In fact, Strauss is at most a moderate intuitionist, for he is quite critical of the real mathematical intuitionists. In particular he shares their constructivism as long as it is based on a finite number of steps. (This is also called *finitism*.) Sometimes intuitionists accept proof based on a denumerable infinite series of steps, which Strauss criticizes.

procedures that involve infinite series, like the proof of complete induction,¹⁷ rejected by the Intuitionist L.E.J. Brouwer (1881-1966).

For this reason and contrary to Strauss,¹⁸ I have no objection against considering the limit of a Cauchy-sequence of rational numbers to be a number itself, for it can be proved that such limits are subject to the same arithmetical laws as rational numbers do. For instance, if one adds the number 1 to all elements of a Cauchy-sequence, its limit increases by 1. Or if one doubles each element of this series, the limit is doubled too. If one adds the elements of two series one-by-one, the limits are added as well. And so on. Consequently, one does not need the idea of actual infinity to argue that the limit of a Cauchy-sequence is subject to arithmetical laws.

Such a limit may be a fraction itself, or it is an irrational number, like $\sqrt{2}$ or π . In this way it is possible to define or calculate a denumerable set of irrational numbers (for instance, the set of all square roots of natural numbers), but not the whole set of all real numbers, which is not denumerable, as argued above. This set can only be defined by reference to a *spatial* set, like the set of all spatial points on a straight line. I agree with Strauss that the set of real numbers anticipates the spatial aspect. However, this does not apply to each irrational number apart. The square roots of natural numbers can be defined by reference to arithmetical laws only. Yet, the meaning of numbers like $\sqrt{2}$ or π is most easily illustrated by pointing out that these refer to the quantitative ratio of the diagonal and the side of a square, respectively the circumference and the diameter of a circle.

Strauss considers the view that the limit of a Cauchy-sequence could define a number to be a *petitio principii*, for it would presuppose that the limit is a number. This objection would be quite right if one would have the intention to grasp the *essence* of numbers. But I am not an essentialist. For me it is sufficient that the limits of Cauchy-sequences are subject to the same arithmetical laws as do fractional numbers.

Strauss strongly objects to the conception of a spatial line as a continuous set of points.¹⁹ From a constructivist point of view this objection seems to

17 If $P(n)$ is a proposition defined for each natural number $n < a$, and $P(a)$ is true, and $P(n+1)$ is true if $P(n)$ is true, then $P(n)$ is true for any $n < a$.

18 Strauss, 1996, 151, 153.

19 Strauss, 1970-71, 173-175. We have already seen that a line-stretch as a sub-set should be distinguished from points being the *elements* of the set. Apparently, Strauss confuses the continuity of a line and its sub-sets with the continuity of a single point, or a discrete set of points, which indeed makes no sense. Only finite or infinite sets and their sub-sets have cardinality (in this case, continuity), not their elements.

be valid, for it can hardly be doubted that a line cannot be *constructed* from a set of points. A more empirical view, however, does not attempt to construct a line, but simply establishes as a fact that on a line one has ‘at once’ an infinity of points constituting a set. This set can be shown to be non-denumerable, and one calls it continuous. Assuming that the set of real numbers can be projected on such a line (in a one-to-one correspondence), one states that the set of real numbers has the same cardinality C as the set of spatial points on a line. For this reason, one calls the set of real numbers ‘continuous’.

For Strauss this appears to be *anathema*, an *antinomy*. Like Dooyeweerd and Vollenhoven he maintains that the meaning nucleus of the spatial aspect is continuous extension. To call the set of real *numbers* ‘continuous’ would eradicate the irreducible difference between the spatial and the numerical aspect. I consider this an essentialist fallacy. Like Strauss I believe that the set of real numbers can only be defined by anticipating the spatial aspect, as follows from the axiom that it can be projected on a spatial line. It is the only way to define the *complete set* of real numbers, although it is possible to define various denumerable infinite subsets of real numbers in a purely numerical way, like the set of all square roots of natural numbers. One needs both the numerical and the spatial relation frame to define the complete set of real numbers. Calling the set of real numbers ‘continuous’ in the sense of ‘having the same cardinality as the set of spatial points on a line’ implies that the continuity of the set of real numbers anticipates the spatial relation frame. This does not make any difference for the mutual irreducibility of the numerical and the spatial relation frames.

This might be considered a semantic problem, as if Strauss and I merely apply different meanings to the *word* continuous, but there is more at stake. Interpreting the *meaning* of a relation frame rigorously by a single word (concept or idea) implies that one believes this meaning nucleus to characterize the modal aspect *essentially*. The numerical aspect would be essentially ‘discrete quantity’, the spatial one essentially ‘continuous extension’.²⁰ In the Philosophy of the Cosmomic Idea *meaning* refers to

20 Strauss, 1996, 171 states that it belongs to the nature of a spatial continuum that it allows of infinite successive division of all its parts. However, this property is shared by a dense set like the set of rational numbers, as Strauss himself observes. Only the applicability of Dedekind’s cut (Strauss, 1996, 156) is a criterion of continuity. Until the end of the 19th century, even mathematicians did not always distinguish between dense and continuous sets, see Grünbaum 1973, 10-11. Weierstrass’s definition of a ‘continuous’ variable quoted by Strauss, 1996, 152 applies to a dense set as well.

the Origin of the creation, in particular (but not exclusively) to the Origin of the laws for the creation. The question of the Origin of laws (including mathematical laws) can only be answered in a religious way. The meaning of any relation frame is to be found in its laws, fallibly expressed in human-made law statements like mathematical axioms. Dooyeweerd's meaning nucleus and the related anti- and retrocipations of any relation frame may be helpful to express this meaning in a nutshell, but it should never be absolutized in an essentialist way.

7. Transcendence

Strauss's distinction of (retrociatory) concept and (anticipatory) idea gives the impression that he considers the latter 'transcendental', though it is not entirely clear to me whether Strauss would draw that conclusion, for he uses the term 'transcendental idea' only in a critical sense. Therefore, my final remark may be more directed to Dooyeweerd than to Strauss.

Dooyeweerd calls the anticipating direction in the serial order of the modal aspects *transcendental*. Anticipation in a relation frame points from that frame to a succeeding one, like retrocipation refers to an earlier one. The final frame has no anticipations, but the aspect of faith being the final one would point out of temporal reality to eternity in an eschatological sense.²¹ Apparently, Dooyeweerd applies two quite different meanings to the concept of transcendence. Normally he understands by transcendence: to transcend temporal reality. That is something entirely different from anticipating a later modal aspect, which occurs *within* created reality. In the case of the aspect of faith, he identifies these two meanings. This equivocation is at least confusing and in my view it is wrong. The mutual projections of the relation frames are temporal, never transcending reality. With respect to the first modal aspect, Dooyeweerd only observes that it has no retrocipations. For the final aspect he assumes that it anticipates eternity. It appears to be more correct to state that the final aspect has no anticipations like the first one lacks retrocipations. Everything created refers via the law to the Creator, who as the lawgiver transcends the law. This vertical transcending relation does not uniquely apply to the relation frame of faith, for it applies to all relation frames and all character types, conceived as sets of general, respectively specific natural laws and normative principles. Moreover, a human being concentrates through all

21 Dooyeweerd, 1953-58, I, 33. *Ibid.* II, 302 and Dooyeweerd, 1959, 88 calls the aspect of faith 'the window to eternity'. In my view this applies to any relation frame. Dengerink 1989 assumes that the modal aspect of eternity succeeds that of faith.

relation frames toward the religious source of her existence. For Christians, this is Jesus Christ, through whom they know God and have eternal life.²² Each person performs this concentration with all his/her heart, with all his/her soul, with all his/her mind,²³ with his/her religion, starting from each relation frame, not only that of faith.²⁴

8. Conclusion

My critical remarks in the margin of Strauss's discussion of the philosophy of mathematics do not diminish my admiration for and appreciation of Danie Strauss's many and valuable contributions to the development of the Philosophy of the Cosmomic Idea. I shall never forget his invaluable help in publishing my *Time and Again* (1980).

Bibliography

- BETH, E.W. 1944. *De wijsbegeerte der wiskunde van Parmenides tot Bolzano*. Antwerpen: Standaard Boekhandel.
- DENGERINK, J.D. 1989. Mens, kosmos, tijdelijkheid, eeuwigheid. *Philosophia Reformata*, 54: 83-102.
- DOOYEWEERD, H. 1953-58. *A new critique of theoretical thought*, four volumes (Revised translation by D.H. Freeman, W.S. Young and H. de Jongste of *De wijsbegeerte der wetsidee*, 1935-36). Amsterdam: Paris.
- DOOYEWEERD, H. 1959. *Vernieuwing en bezinning*. Zutphen: Van den Brink.
- GALILEO GALILEI. 1954. *Dialogues concerning two new sciences* (Translation 1914 by H. Crew, A. de Salvio of *Discorsi e dimostrazioni matematiche intorno à due nuove scienze*. Leyden 1638: Elsevier). New York: Dover.
- GRÜNBAUM, A. 1973. *Philosophical problems of space and time*. Dordrecht: Reidel.
- HOWELL, R.W., BRADLEY, W.J. (Eds.) 2001. *Mathematics in a postmodern age, A Christian perspective*. Grand Rapids: Eerdmans.
- QUINE, W.V.O. 1963. *Set theory and its logic*. Cambridge Mass. 1971: Harvard University Press.
- STAFLEU, M.D. 1972. Metric and measurement in physics. *Philosophia Reformata*, 37: 42-57.
- STAFLEU, M.D. 1980. *Time and again, A systematic analysis of the foundations of physics*. Toronto: Wedge; Bloemfontein: Sacum.
- STAFLEU, M.D. 2002. *Een wereld vol relaties, Karakter en zin van natuurlijke dingen en processen*. Amsterdam: Buijten en Schipperheijn.
- STRAUSS, D.F.M. 1970-71. Number-concept and number-idea. *Philosophia Reformata*, 35: 156-177; 36: 13-42.
- STRAUSS, D.F.M. 1973. *Begrip en idee*. Assen: Van Gorcum.
- STRAUSS, D.F.M. 1995. The significance of Dooyeweerd's philosophy for the modern natural sciences. In: Griffioen, S., Balk, B.M. (Eds.), *Christian philosophy at the close of the twentieth century*. Kampen: Kok.

22 John 17, 3.

23 Matthew 22, 37; Mark. 12, 30; Luke 10, 27.

24 Dooyeweerd, 1953-58, II, 298; Troost, 2004, 41.

STRAUSS, D.F.M. 1996. Filosofie van de wiskunde. In: Van Woudenberg, R. (Red.), *Kennis en werkelijkheid*. Amsterdam: Buijten en Schipperheijn.

TROOST, A. 2004. *Vakfilosofie van de geloofswetenschap, Prolegomena van de theologie*. Budel: Damon.