Philosophy in the Context of our Time

I

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Samevatting

Hierdie artikel bevat die eerste sistematiese reaksie op die Festschrift *wat einde 2006 aan die outeur (D.F.M. Strauss) opgedra is. Dat die geskiedenis van die filosofie deurlopend in die greep was van uiteenlopende sienings van die eenheid en verskeidenheid in die skepping, het veral neerslag gevind in die ontwerpe wat verskeie filosowe daargestel het ten opsigte van die eienskappe wat in die werklikheid aangetref word. Die negatiewe kant van hierdie erfenis is daarin geleë dat dergelike pogings telkens gestrand het in een of ander vorm van reduksionisme – 'n benadering wat die teenoorgestelde uiterste van die posisie-keuse van die reformatories-wysgerige tradisie verteenwoordig. In laasgenoemde tradisie word tewens daarna gestreef om die verskeidenheid in die werklikheid ernstig op te neem – sonder om een of ander aspek te kies in terme waarvan alle ander verklaar kan word. Hierdie ideaal van 'n nie-reduksionistiese ontologie vind allereers gestalte in die teorie van die modale aspekte van die werklikheid, nader toegespits op die verskillende samehangsmomente tussen die aspekte van die werklikheid. Met vermelding van verskeie bydraes word veral aandag geskenk aan hierdie probleem van inter-modale samehange, met besondere verwysing na die bydrae van Stafleu (oor oneindigheid en kontinuïteit) – hoewel ander outeurs se bydraes ook ter sprake kom. In die besonder word gelet op die samehang tussen getal en die rol van 'n kwalifiserende sin-kern en die plasing van kontinuïteit in hierdie inter-modale konteks.*

1. Orientation

I am truly thankful for the *Festschrift* dedicated and presented to me in November 2006. I appreciate all the contributions – I know how much

valuable time and effort enter into an end-product like this. One of the contributors, Johan Visagie, suggested that I should respond to the work in a number of articles. Since I can identify myself with much that has been said in all the diverse articles, the obvious course of action would be to focus on some of the most important issues that deserve further reflection and interaction. In this first response I shall therefore merely "open the floor" in order to get the conversation rolling – with the explicit understanding that any contributor may eventually decide to send a response or even rebuttal to the *Journal for Christian scholarship*.

2. Contemporary philosophizing cannot be divorced from the history of philosophy

The history of philosophy provides to the interested scholar an immensely rich and varied picture of the most intriguing kaleidoscope of intellectual articulations and traditions. The remarkable fact about this legacy is that no philosopher ever seems to be fully "out-dated," for even the earliest Greek philosophers still exert an influence on present-day philosophical thinking. With a view to the contribution of Stafleu – on *Infinity and continuity* – it is worth mentioning a recent excellent example of the many elements of continuity with the history of philosophy as found in a Ph.D. that was submitted to the Free University of Brussels in 2006 by Karin Verelst. She advanced a penetrating and radically new challenge to the traditional interpretations of Zeno's paradoxes, involving a number of mathematical sub-disciplines. She starts with a well-articulated and lucid description of known paradoxes, followed by an equally significant explanation of Cantor's transfinite number theory, including an account of transfinite ordinal and cardinal numbers (the specific theme of her Appendix II), Cantor's diagonal proof of the non-denumerability of the real numbers and a statement of his continuum hypothesis. Reflections on truth and reality pave the way for looking at paradoxes in terms of their origin, focusing on the Parmenidean and Heraclitian legacies. In the context where she accounts for Plato's reaction to Heraclitus' *flux* theory we get closer to the discussion of the relation between Parmenides and Heraclitus. The *now-being* serves to highlight the affinity between these two thinkers – Heraclitus merely describes a *succession* of *now-moments*. All of this intimately coheres with the problem of *being* and *not-being*.

It is certainly one of the outstanding merits of the reformational philosophical tradition that it opened up an insight into the underlying problems of uniqueness and coherence operative within the philosophical legacy of the West. The idea of being and non-being is related to *identity* and *difference*, to 'succession' and 'now', even to universality and what is individual. Descriptions of properties assumed to be applicable (or attached) to the things and processes we experience are present in the

thought schemes of virtually all prominent philosophers – from Plato and Aristotle up to Thomas Aquinas, René Descartes, Immanuel Kant and Nicolai Hartman. However, what is absent in all these instances is a truly systematic analysis of this dimension of the universe. In its theory of modal aspects reformational philosophy advanced such a systematic analysis – one that aims at accepting the diversity within reality without attempting to reduce it merely to one or a few of its aspects. In other words, reformational philosophy aims at developing a non-reductionist ontology.

It is therefore not surprising that in a number of the contributions to the *Festschrift* the theory concerning the inter-aspectual (inter-modal) connections between the various irreducible facets of reality does play an important role. We find it for example in the contribution of Botha on metaphor (see Botha, 2006:24 ff.), in the discussion of Jandl (2006:78 ff.), Stafleu (2006:163-173), as well in those of Visagie, Weideman and Wessels (see in particular Weideman, 2006). One can say that the entire article of Martin Rice is directed at exposing the untenability of a *reductionist ontology* – particularly focused on the impasse of *materialism*.

3. The issue of infinity and continuity

The uniquely new way in which Verelst approaches the paradoxes of Zeno intimately coheres with the introduction of a procedure through which the totality of *Z* could be presented in a way capable of generating two different kinds of infinity at once – as *divisions* and as *parts*. The connection with basic ideas of Cantor is accomplished by accepting the idea that Zeno advanced a view in which a simultaneous division *through-and-through* is intended.[1]

The crux of these arguments is centered in the fundamental difference between the two kinds of infinity found throughout the history of philosophy and mathematics, traditionally known as the potential and the actual infinite. Particularly the latter idea was always accompanied by implicit notions of continuity. It is therefore quite appropriate that Stafleu has decided to choose this relationship (namely between infinity and continuity) as theme for his contribution.

There is in fact a remarkable coincidence in this regard, because without knowing anything about his forthcoming *Festschrift* contribution, I

1 Important parts of modern mathematics are fitted into the development of this argument – also explored in order to show that there are intimate links between Zeno's paradoxes of plurality and motion.

presented at the Free University of Brussels a guest lecture on October 12, 2006 investigating the same problem: *infinity and continuity*.[2]

3.1 A conversation started in the seventies of the 20th century

In reaction to my articles on *Number-concept and Number-idea* that appeared in *Philosophia reformata* (1970 & 1971), Stafleu at the time sent two letters to me (March 10, 1971 & March 29, 1971), accompanied by a draft article on "The discrete and the continuous". The thrust of this article is summarized in his letter of March 29, 1971. The element relevant to our current discussion is his remark that a disclosure of the meaning of an aspect *relativizes* its closed meaning and with this also the meaning of the meaning-nucleus of an aspect. The first formulation given by Stafleu mentions that the *meaning* of an *aspect* changes through the opening up of anticipations and his second statement claims that the meaning-nucleus *itself* is changed.[3] This ambiguity reflects an issue on a more fundamental level: what is the idea of an aspect? Is it the aspect or the meaning-nucleus that contains the analogies referring to other aspects? Of course there are four options: (a) aspects refer to aspects; (b) the meaning-nucleus of an aspect refers to other aspects; (c) aspects refer to the meaning-nuclei of other aspects; or (d) meaning-nuclei refer to meaning-nuclei.

Before we try to find out which one of these options are best suited to account for the coherence between the diverse aspects of reality we have to consider some of the crucial elements found in the development of the discipline of mathematics by the end of the 19th century.

The way in which Dedekind, Weierstrass and in particular Cantor, advanced new mathematical foundations for the (irrational) real numbers during the last couple of decades of the 19th century, once again gave prominence to the relationship between the potential infinite and the actual infinite (sometimes also designated as the *uncompleted* and *completed infinity*). Cantor explicitly defines[4] irrational numbers in terms of actually infinite sets of rational numbers. From the perspective of his new view on

2 This fairly extensive paper has been prepared as a little booklet – with its own *Table of Contents*, *Index of Subjects* and *Index of Persons* (size: A5 format). In PDF format it is available on the WEB at the following address: http://www.freewebs.com/dfmstrauss/articles.htm.

3 "Anderzijds, wanneer wij het betreffende aspect in zijn ontsloten vorm gaan beschouwen, dus met zijn anticipaties, dan verandert met iedere anticipatie de zin van het modale aspect. Dooyeweerd spreekt van een *verdieping* van de zin, of een *ontsluiting*; ik heb in mijn artikel (Analysis) aangetoond, dat door het openingsproces de *gesloten* zin *gerelativeerd* wordt. Wanneer men dit ontkent, en stelt dat ook in geopende toestand de zinkern dezelfde is als in gesloten toestand, dan maakt men zich schuldig aan een verabsolutering van de betreffende zin (dit heeft b.v. betrekking op Newton's 'absolute' ruimte en tijd)."

the actual infinite Cantor also defines continuity in arithmetical terms, namely as a point-continuum, where *point* could be replaced by *real number* such that the non-denumerable, actual infinite set of real numbers may be designated as *continuous*: "Thus I had no other choice but to develop what is possibly the most general purely arithmetical concept of a continuum of points" (Cantor, 1962:192).

Subsequently it is customary in modern Cantorian set theory to distinguish between the so-called *discrete* set of integers, the *dense* set of rational numbers and the *continuous* (perfectly-coherent)[5] set of real numbers. In this way Cantor laid the foundation for the dominant tendency in modern mathematical set theory that continues to identify the original spatial meaning of continuity with the properties of the set of all real numbers. On the basis of this arithmeticistic basic orientation it is subsequently attempted to arrive at a consistent conception of the extended linear continuum as an aggregate of non-extended elements.[6] Against this brief sketch we can now return to the views of Stafleu, for, during the next year (1972), he published an article in which he articulated his understanding of the core meaning of number and its disclosure in line with the remarks made in his letter of 1971. According to this article *continuity* always refers to a set and, as a result, it must be seen as "an analogical concept" (Stafleu, 1972:48). However, an *analogy* always refers to a different aspect in which the analogical moment concerned appears in its original (non-analogical) sense as its meaning-nucleus. By contrast Stafleu states: "only discrete sets have an original meaning, continuous sets have an analogical meaning" (Stafleu, 1972:48). According to him *continuity* in its original sense neither belongs to the arithmetical aspect nor to the spatial aspect. The disclosure of the original meaning of the numerical aspect even implies "the loss of the *discrete* character of number" (Stafleu, 1972:48). The problem is that the idea of a modal aspect entails that every analogical moment is always qualified by the meaning-nucleus of the aspect in which the core meaning of a different aspect appears *analogically*. This represents option (c) of the four options mentioned above[7] – and it corresponds to the original and authentic idea of a modal aspect as developed by Dooyeweerd himself (see Dooyeweerd, 1997-II:75, 77).

4 Compare Hasse and Scholz (1928:12).

5 Cantor criticizes the one-sidedness in the views of Bolzano and Dedekind regarding the continuum and then remarks: "I believe to have acknowledged in both these predicates 'perfect' and 'coherent' the necessary and sufficient hall-marks of a continuum of points and therefore define a point continuum within Gn as a perfect-coherent set" (Cantor, 1962:194). Note that for Cantor Gn represents a "n-dimensional arithmetical space."

6 Cf. Grünbaum, A. 1952. A Consistent Conception of the Extended Linear Continuum as an Aggregate of Unextended Elements. In: *Philosophy of Science*, Vol.19, nr.2, April (pp.288-306).

The original ('authentic') theory of a modal aspect therefore conjectures that (c) represents the correct view. However, in some instances, an expression may be employed having the 'face' of (d), but the intention of (c). Expressions such as "love life" and "retributive balance" appear to be instantiations of (d), but in fact speaks of nothing but "moral life" and "jural balance" (belonging to (c)).[8]

3.2 Stafleu's later views

The article contained in the 2006 *Festschrift* proceeds from the important systematic work published by Stafleu in 2002 – on a "world full of relations". To a large degree this 2002 work presents a fresh and in some instances innovative systematic account of reality. Stafleu introduced the term "relation frame" ('relatiekader') as designation of an aspect (Stafleu, 2002:16 ff.). The second element of this phrase, the Dutch term 'kader', may mean 'framework', 'cadre' 'skeleton' and it alludes to the whole-parts relation.[9] The first element, namely the term 'relation', is derived from the spatial meaning of connectedness, echoing a basic idea of his 1980 work on the foundations of physics where he pointed out that the modal aspects are not merely modes by being, since they are also modes of explanation and "universal modes of temporal relations" (Stafleu, 1980:15). The last phrase acquired a prominent place in his more recent work on relations. As a substitute for his former employment of the phrase "typical laws" (or the term 'typicality' – see Stafleu, 1980:6, 11), Stafleu now (in 2002) introduces the term 'character' as a general designation of a "cluster of laws of similar things, events or their relations" (Stafleu, 2002:9). The only connotation that may help to strengthen the intuitive appeal of the term character is given in its apparent implicit reference to the 'nature' of things, allowing for classifying certain groups of things, distinct from other groups or classes of things.[10]

Nonetheless, Stafleu does not intend to introduce the term *character* as "the essence or nature of things" or processes, for his intention is to emphasize that a cluster of laws determine the mutual relations between things and processes (Stafleu, 2002:9). It seems as if Stafleu, in his fear for what he calls 'essentialism', underplays the *thingness* of things by focusing on *relations*. This emphasis comes dangerously close to

7 Aspects refer to the meaning-nuclei of other aspects.
8 This is simply an effect of the fact that the semantic domains of terms like 'love' and 'retributive' include the meaning nuances 'moral' and 'jural'.
9 "Fit in with the whole scheme of things."
10 Stafleu does point out, however, that scientific classification is something different from typifying characters on the basis of universal "relation cadres" (modal aspects) (Stafleu, 2002:31).

functionalism – the 'ism' that functionalizes entities (the opposite of hypostatization or reification – treating a function as if it is an entity, such as when biologists speak of the origin of 'life' instead of the origin of *living things*).

In line with this new approach Stafleu discerns in my own approach to the foundations of mathematics something essentialist. In fact I was surprised to read: "Strauss appears to start from a moderate intuitionist and maybe essentialist worldview that I do not share" (Stafleu, 2006:163, 170-172). Until 1972 I did not have a proper understanding of what the so-called actual infinite entails. In 1973 I developed a deepened understanding of this kind of infinity and at that stage of my development I designated it as *completed infinity* (see Strauss, 1973:184 ff.). Since 1981 I introduced and consistently employed the idea of the *at once infinite* as the best designation of the meaning of what traditionally was known as the actual infinite.

3.3 The philosophical background of Stafleu's (anti-essentialist) emphasis on relations

Stafleu's new emphasis on relations – also reflected in the title of his 2002 work – substantiated by 'degrading' a mode of speech in which it will be possible to mention the *nature* of things, may actually continue a long standing functionalistic approach particularly prominent in the entire intellectual development of the natural sciences since the Renaissance. He mentions, as an example of a 'mistaken' understanding of 'zin' (meaning), Dooyeweerd's designation of the meaning-kernel of the numerical aspect as discrete quantity. For Stafleu the only acceptable connotation of meaning ('zin') is given in a reference to the origin. This referentiality of meaning – i.e. that meaning is relational and ultimately refers to the origin of creation – brings to expression his preference for *relations* and *relational concepts*. Dooyeweerd has an integral understanding of 'zin' (*meaning*). Creation in all its dimensions and entities *is meaning*, i.e. whatever God has created exists from, through and to God. For this reason Dooyeweerd's philosophy is permeated by the qualification 'zin' – just think of phrases such as '*zin*-structuur', *zin*-zijde', *zin*-samenhang', *zin*-verband', *zin*-systase', *zin*-synthesis', *zin*-ontsluiting' and not the least: *zin-kern* (meaning-nucleus). The very meaning of 'zin' precludes an essentialist interpretation of it.

For example, when it is asserted that the meaning-nucleus of the arithmetical aspect is *discrete quantity*, the way in which Stafleu understands *zin* disqualifies this assertion as essentialistic, because according to him it does not contain a reference to the origin. In fact Stafleu does not hesitate to call upon the development of modern natural science in its reaction to the essentialistic philosophy of Plato and Aristotle. He says that the question regarding the essence disappeared from modern natural science

and that therefore it also should not find shelter in a "relational philosophy". This is nothing but a deviation (or even: derailment) of the original idea of Dooyeweerd. Since the early twenties of the 20th century the latter took a principled stance in opposition to both the substantialistic ('essentialistic') orientation of Greek-Medieval philosophy and the functionalistic ('relationalistic') orientation of modern natural science. An integral cosmonic idea, i.e. an encompassing idea of creation in its unity (coherence/ relatedness) and diversity (uniqueness/ irreducibility) has to affirm both sides of the coin – uniqueness and coherence. Meaning comes to expression in the coherence ('relation') between distinct (unique) aspects of reality and likewise different entities and processes are intertwined in multiple ways.

Reference (relatedness/ relation) depends on uniqueness and uniqueness depends on coherence. In the sense of concept-transcending knowledge the ideas of uniqueness and (inter-modal) coherence explore modal numerical and spatial terms stretched beyond the boundaries of these aspects. It is therefore not a sign of 'essentialism' when the uniqueness of aspects and entities is acknowledged. However, not being willing to speak of the 'nature' of things does not avoid references to 'de-natured' things, explaining why Stafleu nonetheless still has to speak of the relations of (or: between) THINGS!

The (early 20th century) Neo-Kantian philosopher, Rickert, continues the mentioned functionalistic tradition with his view that the natural sciences have to proceed in a generalizing fashion, in opposition to the individualizing mode of thought predominant in the (historical) humanities (Rickert, 1913:68-69, 173). Rickert initially develops this perspective by binding the natural sciences to the ideal of transforming all thing concepts into concepts of function (explicitly designated by him as *concepts of relations*). This Neo-Kantian view of the natural sciences remains completely faithful to the aim of the classical science ideal, namely to reduce all of reality to some or other modal aspect, function or relation. According to Rickert the (functionalistic) logical ideal of the natural sciences finds its limit in the uniqueness (individuality) of experiential reality itself.[11]

Rickert holds:

> Whatever the role the category of a thing may fulfill in a theory of the thing world, envisaged as closed, at bottom there is no doubt that the natural sciences have to strive to resolve the rigid and fixed things

11 That which poses an inaccessible limit to natural scientific concept formation is nothing else but unique empirical reality itself, as we intuitively experience it in the immediacy of its individuality (Rickert, 1913:197).

> increasingly, … and this means nothing else but transforming as far as possible all thing concepts into relation concepts. … Our theory is valid for the logical ideal of natural scientific concepts, because this ideal solely concerns relation concepts (Rickert, 1913:68-70).[12]

Moreover, highlighting the functionalistic background of an emphasis on relations is further supported by the fact that Stafleu views laws as timeless. "Individual things and events are intrinsically temporal, ... The timeless character conditions the existence of individuals concerned in their temporal circumstances" (Stafleu, 2002:14).[13] (Rickert held the view that values have an ideal, timeless being.)

As soon as Stafleu has to articulate more precisely what characters are all about, he takes recourse to the precision provided by modal terms, thus leaving behind suggestive metaphors (such as 'characters'). He then offers a description that looks like a quasi-compound basic concept: "A character determines an unlimited complete class of temporal subjects" (Stafleu, 2002:14). The term 'determines' is derived from the modal meaning of the physical aspect, the terms 'unlimited' and 'complete' from the spatial mode, and 'class' from a combination of the numerical and the spatial aspects. The use of a metaphor, such as figuratively designating a type law as a character, in the final analysis therefore requires modal terms in order to obtain a precise meaning.

Stafleu says that he defines a character as a cluster of immutable ('onveranderlijke') natural laws, instead of rather speaking of their *constancy*, because when anticipatory meaning moments are disclosed on the law side of an aspect, then the law side itself indeed *changes*. In the discussion between Van Peursen and Dooyeweerd, the latter responded to the accusation that his conception of law is static, and he did that by explaining that the meaning-dynamics of reality manifests itself also on its law side (see Dooyeweerd, 1960:109).

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12 "Welche Rolle also auch die Kategorie des Dinges in einer abgeschlossenen gedachten Theorie der Körperwelt noch spielen mag, so unterliegt es doch jedenfalls keinem Zweifel, dass die Naturwissenschaften danach strebt und streben muss, die starren und festen Dinge immer mehr aufzulösen, . und das heist nichts anderes, als die Dingbegriffe so weit wie möglich in Relationsbegriffe umzuwandeln. ... Für das logische Ideal der naturwissenschaftlichen Begriffe in einer 'letzten' Naturwissenschaft ist unsere Theorie gültig, denn es handelt sich bei diesem Ideal nur noch um Relationsbegriffe."

13 "A class is just as timeless as the natural laws determining the class" (Stafleu, 2002:14).

3.4 The mathematical context of infinity

Stafleu gives an explanation of the correlation between law and subject that is also mathematically acceptable – at least as long as we restrict ourselves to the finite case. But when he categorically states: "I do not consider Cantor's transfinites to be numbers, for these do not confirm to arithmetical laws for addition *etc.* (Stafleu, 2006:168), it is clear that he rejects the foundation of Cantor's transfinite number theory and arithmetic. Interestingly, a contemporary of Cantor, Gutberlet, although accepting *actual infinity*, also rejected the notion of infinite *numbers* (see Cantor, 1962:394 and the discussion of Meschkowski, 1967:65 ff.).

In his *Gesammelte Abhandlungen* Cantor argues against such an understanding of his theory. Cantor points out that the material of traditional arithmetic was obtained by abstracting the finite concept of number from finite sets. However, merely acknowledging the actual infinite in the form of actually infinite *sets* stops short of Cantor's achievement, because he considers the demand to acquire a *number concept* of the actually infinite (through similar abstractions pertaining to the nature of the issue) as inescapable (Cantor, 1962:411).[14] Cantor distinguishes between a *set* and its *power* or *cardinality* (*cardinal number*). The latter is obtained when a double abstraction is performed, first of all from the character (*Beschaffenheit*) of the elements and secondly from any order in which the elements are given (Cantor, 1962:282; cf. 329, 388, 411).[15] It is therefore not correct to question Cantor's transfinite arithmetic by measuring it against the laws for finite arithmetic (Stafleu, 2006:168). In other words, Cantor accounted for the difference between finite and infinite sets, but also for the difference between finite and transfinite *numbers*.

Perhaps the most important implication of Cantor's employment of the actual infinite for modern mathematics is its significance for the theory of real numbers. Cantor specifically mentions the "theory of irrational numbers" and adds that without some or other form in which the actual infinite is employed, an adequate foundation for the irrational numbers cannot be carried through.[16]

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14 "Hat aber hiermit das Aktual-Unendliche in Form actual-unendlicher *Mengen* sein Bürgerecht in der Mathematik geltend gemacht, so ist die Forderung eine unabweisliche geworden, auch den aktual-unendlichen *Zahlbegriff* durch geeignete *natürgemäße Abstraktionen* auszubilden, ähnlich wie die endliche Zahlbegriffe, das Material der bisherigen Arithmetik, durch Abstraktion aus endlichen Mengen gewonnen worden sind. Dieser Gedankengang hat mich auf die *transfinite Zahlenlehre* geführt, ..."

15 " '*Mächtigkeit*' *oder* '*Kardinalzahl*' *von M nennen wir den Allgemeinbegriff, welcher mit Hilfe unseres aktiven Denkvermögens dadurch aus der Menge M hervorgeht, daß von der Beschaffenheit ihrer verschiedenen Elemente m und von der Ordnung ihres Gegebenseins abstrahiert wird.*"

Moreover, in a specific sense Cantor actually sees in *transfinite numbers* themselves "new irrationals," providing the best method to define the *finite* irrational numbers. Introducing irrational numbers, according to Cantor, in principle follows the same method as introducing transfinite numbers. Without any qualification one can say that the transfinite numbers *stand or fall* (*stehen oder fallen*) with the finite irrational numbers, for they are similar according to their innermost essence: "both are determinate and delimited manifestations or modifications of the actual infinite" (Cantor, 1962:395-396).[17]

It is therefore mathematically strange to read that the cardinality of a set is not a number (see Stafleu, 2006:169) – as if Cantor's transfinite arithmetic does not exist. Has Stafleu decided to leave the "paradise" created for us by Cantor (just compare Hilbert's positive assessment of Cantor's transfinite arithmetic, Hilbert, 1925:170).[18]

3.5 Intuitionism and constructivism

Stafleu believes that I am a "moderate intuitionist" – and he adds the remark: "Like all mathematical intuitionists, Strauss appears to be a constructivist" (Stafleu, 2006:169). In order to support this interpretation of my view Stafleu refers to an article that appeared in 1996 in Dutch (see Strauss, 1996:158-159). However, on these pages exactly the opposite is found – of course for the simple reason that my views are not intuitionist and constructivist, let alone essentialist. In the light of Cantor's proof of the non-denumerability of the real numbers I here remark that constructionist interpretations are inadequate to reach a non-denumerable conclusion, because there is no constructive transition from the uncompleted infinite to completed infinitude.[19] This entire article could be seen as a plea for the acceptance of the at once infinite (my designation of the actual infinite) on the basis of an account of the anticipatory coherence between the aspects of number and space (see Strauss, 1996:164 ff. & 168

16 "Hier ist in erster Linie die Theorie der irrationalen Zahlgrößen anzuführen, deren Begründung nicht durchführbar ist, ohne daß das A.-U. in irgendeiner Form herangezogen wird" (Cantor, 1962:410).

17 "Die transfiniten Zahlen sind in gewissem Sinne selbst *neue Irrationalitäten* und in der Tat ist die in meinen Augen beste Methode, die *endlichen* Irrationalzahlen zu definieren, ganz ähnlich, ja ich möchte sogar sagen im Prinzip dieselbe wie meine oben beschriebene Methode der Einführung transfiniter Zahlen. Man kann unbedingt sagen : die transfiniten Zahlen *stehen oder fallen* mit den endlichen Irrationalzahlen; sie gleichen einander ihrem [396] innersten Wesen nach; denn jene wie diese sind bestimmt abgegrenzte Gestaltungen oder Modifikationen (*ajfwrismevvna*) des aktualen Unendlichen."

18 "Aus dem Paradies, das Cantor uns geschaffen [hat], soll uns niemand vertreiben können."

ff.). I am afraid that my friend, Stafleu, did not read this article properly – there is no intuitionist or constructivist that will be satisfied with my positive account of the actual infinite.

At this point we have to return to the functionalist slant present in Stafleu's accusation that my understanding of the core meaning of number (and space) is essentialist. It coheres with his views on discreteness, sets and continuity.

3.6 Discreteness and continuity

In 2006 Stafleu continues his original stance, found in our 1971 correspondence and in his 1972 article. He writes:

> On the one side the projection of spatial relations on quantitative ones (requiring the continuous set of real numbers) is *retrocipatory.* On the other hand, the definition of a continuous set by the projection of real numbers on a spatial line is *anticipatory,* requiring the continuity of spatial figures. In Strauss's words, the former requires the *concept* of a measure, the latter the *idea* of a continuous set (Stafleu, 2006:166).

The term 'continuous' is introduced as a property of *sets*. Stafleu uncritically follows the mathematical practice by primarily distinguishing between *discrete* (denumerable), *dense* and *continuous* sets. Whenever the elements of a set could be mapped one-to-one on the set of natural numbers, it is said to be *denumerable*. Stafleu mentions the set of integers and the set of rational numbers as examples of *denumerable* sets (Stafleu, 2006:164-165). A significant element of ambiguity accompanies this practice, because the feature of *denumerability* on the one hand embraces both the natural numbers and the rational numbers, while on the other hand the natural numbers are said to constitute a *discrete set* and the rational numbers a *dense set*. What is meant by mathematicians when they call the rational numbers *dense* is not that these numbers are *less distinct* – for that would have cancelled their status as numbers (note the plural – multiplicity). The only adequate reason for calling the set of rational numbers *dense* is because this set 'imitates' a property of spatial extension, namely the infinite divisibility of any spatial interval: the numerical difference between any two rational numbers can also be 'bisected' by further rational numbers *ad infinitum*. Since the infinite divisibility of any extended spatial subject in itself refers back (retrocipates) to the order of

19 My words are: "Hoewel het intuïtionisme dit bewijs als geldig accepteert, doet het dat in constructivistische zin. Echter, alle constructivistische interpretaties zijn inadequaat om een niet-aftelbare conclusie te verkrijgen, eenvoudigweg omdat er geen constructieve overgang mogelijk is van het onvoltooid-oneindige naar het voltooid-oneindige" (Strauss, 1996:156).

succession (on the law-side) of the numerical aspect, and since the numerical difference between two rational numbers analogically reflects (as an anticipation from number to space) this infinite divisibility of spatial continuity, one may observe in the rational numbers an anticipation to a retrocipation. This configuration makes possible what is known as the *denseness* of the set of rational numbers. Yet there can be no doubt that – given the (denumerable) *distinctness* of every rational number – the rational numbers are still discrete in the sense that each one is distinct from every other one (every rational number is unique).[20]

Stafleu proceeds by explaining a "continuous set" as follows: "It turns out that the only possible way of defining a continuous set (including that of the real numbers) is by reference in one way or another to the spatial relation frame,[21] for instance applying the axiom that the set of real numbers can be projected on a spatial line" (Stafleu, 2006:165-166). I have (partially) to agree and disagree with him in this context. I am also convinced that one cannot (properly)[22] account for the real numbers without employing the inter-modal connection between number and space. Yet I believe that one should invoke the inter-modal connection between number and space in a different way, namely by explicitly pointing out that the *at once infinite* is needed – keeping in mind that I have extensively argued that the at once infinite presupposes the irreducibility of the spatial aspect (with its irreducible time-order of *simultaneity*, of *at once*).

Stafleu and I agree that the *whole-parts relation* is something original within the aspect of space and on this basis my claim is that set theory is a *spatially deepened numerical theory*.[23] But I cannot see why this anticipatory theory should entail the loss of the discreteness of the meaning-nucleus of the arithmetical aspect. Recently I discovered the important work of Laugwitz (1986) and I am pleased to note that he simply cuts through the entire arithmeticistic legacy that reduces the

20 Laugwitz mentions in connection with the view of Euclid that a point in the "geometrical continuum" does not display a characteristic property – it is not possible to distinguish one point (line or surface) from another one: "aber eines ohne charakteristische Eigenschaften; es ist nicht möglich, einen Punkt vom andern zu unterscheiden, keine Gerade oder Ebene ist vor einer anderen Gerade oder Ebene ausgezeichnet" (Laugwitz, 1986:9).

21 "Relation frame" represents the way in which Stafleu refers to modal aspects.

22 I consider the intuitionistic theory of the real numbers – in terms of rational numbers and infinitely proceeding sequences – as representing a semi-disclosed approach. Only when the actual infinite (the at once infinite) is employed do we encounter a fully disclosed approach.

23 Hao Wang reports that Kurt Gödel speaks of sets as being "quasi-spatial" and then adds the remark that he is not sure if Gödel would have said the "same thing of numbers" (Wang, 1988:202).

original spatial meaning of continuity to the opened up meaning of the numerical aspect as it is prominent in the ideal of a *continuous* set of *numbers*. In spite of Cantor's claim that he developed a purely arithmetical concept of a continuum of points (Cantor, 1962:192), Laugwitz makes an appeal to Cantor's original definition of a set which contains the key phrase that a set is constituted by "wohlunterschiedener Dinge" (properly distinct entities) and then emphatically states: "das Diskrete herrscht" ("the discrete reigns") (Laugwitz, 1986:10).

The entire sphere of number (the numerical or quantitative aspect) is qualified by its meaning-nucleus, *discrete quantity*. For that reason every number – be it natural numbers, integers, rationals, real numbers, imaginary numbers – will exhibit its *distinctness* as some or other *number*. Moreover, also each *real number* remains characteristically *distinct*, i.e. qualified by the meaning-nucleus of *discrete quantity* – for without being characterized by this core meaning the real numbers will loose the numerical property of being a *multiplicity* (albeit non-denumerable). Already in 1918 Hermann Weyl has remarked that one should not forget that in the 'continuum' of real numbers the individual elements are *just as* isolated to each other as for example the integers.[24] The mathematician Laugwitz, who developed a non-standard analysis in 1958 (before Robinson published his article on non-standard analysis in 1961 – see also Robinson, 1966), makes a similar remark in respect of non-standard analysis. He commences with a reference to what K. Mainzer writes in an authoratative *Lexikon der Philosophie*: "The continuum is an ordered Archimedean field which is complete in the sense of Cauchy's convergence criterium, the interval principle of Weierstrass or Dedekind's infimum theorem." The non-Archimedean field *IR of non-standard analysis is contained in IR as a sub-field and within it there are within the infinite proximity of every real number other elements (there are infinitely small numbers). But also here every real number remains distinct: "Here the individual real number lies isolated from all the others" (Laugwitz, 1997:266).[25] The term 'isolated' reflects the *disctinctness* of every individual real number, i.e. it reflects the qualifying role of the core meaning of number.

The basic quantitative question, disclosing the modal *functional* character of this aspect, is: how many? Whatever the answer may be, it will always

24 "Man vergesse nicht, dass im 'Kontinuum' der reellen Zahlen die einzelnen Elemente *genau* so isoliert gegeneinander stehen wie etwa die ganzen Zahlen" (Weyl, 1918:69, footnote).

25 "In diesem hat man u.a. in unendlicher Nähe zu jeder reellen Zahl noch weitere Elemente, und es gibt darin unendlich kleine Zahlen. Hier liegt die einzelne reelle Zahl isoliert von allen anderen."

reveal an awareness of a *multiplicity*. None of the different types of numbers, such as the natural numbers, integers, fractions or real numbers – can escape from this (universal, modal) numerical property. Modern mathematics had a solid awareness of two related states of affairs: (1) On the one hand it never questioned the numerical quality of any type of number (reflecting an implicit insight into the core meaning of number: discrete quantity). (2) On the other it employed designations that reveal an implicit awareness of the fact that the introduction of new types of numbers *imitates*, *analogically reflects* (in an anticipatory sense) structural features of the spatial aspect that is distinct from the quantitative aspect. But the lack of a theory of modal aspects and their inter-modal connections prevented it from advancing a lucid account of what is at stake here.

In the Appendix I have captured the relevant structural features of the spatial aspect and their correlation with different types of numbers.[26] Since I have explained the two criteria for continuity in detail in Strauss (2006: 26-32), showing that Aristotle and the Cantor-Dedekind accounts employ the same criteria in spite of their disagreement about the actual infinite, I shall leave this issue out of discussion here.

4. Concluding remark

The fact that the contributions to the *Festschrift* made possible a constructive and on-going discussion ought to be appreciated as a good sign of the vitality of the reformational philosophical tradition, also underscored by the fact that more issues will be discussed in subsequent articles.

Bibliography

BOTHA, M.E. 2006. Metaphor, Embodiment and Fiduciary Beliefs in Science. In: *Time and context relevant philosophy*. Festschrift dedicated to D.F.M. Strauss. (Special edition of the Journal for Christian scholarship), pp.17-36.

CAMERON, A. 2006. Integrity as a Jural Concept. In: *Time and context relevant philosophy*. Festschrift dedicated to D.F.M. Strauss. (Special edition of the Journal for Christian scholarship), pp.37-48.

CANTOR, G. 1962. *Gesammelte Abhandlungen Mathematischen und Philosophischen Inhalts*. Hildesheim: Oldenburg Verlag (1932).

DOOYEWEERD, H. 1960. Van Peursen's critische vragen bij A New Critique of Theoretical Thought. *Philosophia reformata*, 25(1&2):97-150.

DOOYEWEERD, H. 1997. *A new critique of theoretical thought*. Collected Works of Herman Dooyeweerd, A-Series Vols. I-IV, Gen. Ed. Strauss, D.F.M. Lewiston: Edwin Mellen.

26 The circularity entailed in Grünbaum's attempt to develop a consistent conception of the extended linear continuum as being as an aggregate of unextended elements is discussed in Strauss, 2002 and Strauss, 2006.

GRÜNBAUM, A. 1952: A consistent conception of the extended linear continuum as an aggregate of unextended elements. *Philosophy of science*, (19)2, April. pp.288-306.

HASSE, H. & SCHOLTZ, H. 1928. Die Grundlagenkrisis der griechischen Mathematik. *Kant-studien*, 33:4-35.

JANDL, M. 2006. Das Loib-Seele-Problem im Spannungsveld von modaler Ontologie und inferentialistischer Sprachfilosophie. In: *Time and context relevant philosophy*. Festschrift dedicated to D.F.M. Strauss (Special edition of the Journal for Christian scholarship), pp.75-94.

LAUGWITZ, D.D. 1986. *Zahlen und Kontinuum*. *Eine Einführung in die Infinitesimal-mathematik*. Mannheim: B.I.-Wissenschaftsverlag.

LAUGWITZ, D.D. 1997. Mathematische Modelle zum Kontinuum und zur Kontinuität. *Philosophia naturalis*, 34:265-313.

LORENZEN, P. 1968. Das Aktual-Unendliche in der Mathematik. In: *Methodisches denken*, Frankfurt am Main: Suhrkamp Taschenbuch Wissenschaft (73), pp.94-119 (also published in Meschkowski, 1972:157-165).

LORENZEN, P. 1972. Das Aktual-Unendliche in der Mathematik, In: Meschkowski, 1972:157-165.

MAIER, A. (1964): Diskussion über das Aktuell Unendlichen in der ersten Hälfte des 14.Jahrhunderts. *Ausgehendes Mittelalter*, Vol.I. Rome.

MESCHKOWSKI, H. 1967: *Problemen des Unendlichen*. Braunschweig: Friedrich Vieweg & Sohn.

MESCHKOWSKI, H. (Editor) 1972. *Grundlagen der modernen Mathematik*. Darmstadt: Wissenschaftliche Buchgesellschaft.

RICE, M. 2006. The Logic of Materialism: A Test Case for the Principium Exclusae Antinomiae. *Time and context relevant philosophy*. Festschrift dedicated to D.F.M. Strauss (Special edition of the Journal for Christian scholarship), pp.123-138.

ROBINSON, A. 1961. Non-standard analysis. *Indagationes Mathematica*, 23:432-440.

ROBINSON, A. 1966. *Non-standard analysis*. Amsterdam: North Holland.

STAFLEU, M.D. 1972. Metric and Measurement. *Philosophia reformata*. 42-57.

STAFLEU, M.D. 1980. *Time and again*. *A systematic analysis of the foundations of physics*, Toronto: Wedge.

STAFLEU, M.D. 2002. *Een wereld vol relaties*. Amsterdam: Buijten & Schipperheijn.

STAFLEU, M.D. 2006. Infinity and Continuity. *Time and context relevant philosophy*. Festschrift dedicated to D.F.M Strauss (Special edition of the Journal for Christian scholarship), pp.163-174.

STRAUSS, D.F.M. 1973. *Begrip en idee*. Kampen: Kok.

STRAUSS, D.F.M. 1996. Filosofie van de Wiskunde, In: Van Woudenberg, (Red.),. *Kennis en werkelijkheid*. 143-176.

STRAUSS, D.F.M. 2002. Philosophical reflections on continuity. *Acta academica*, 34(3):1-32.

STRAUSS, D.F.M. 2006. Infinity and continuity. Presented on Thursday, October 12, at the Vrije Universiteit of Brussels (VUB) – an invitation from the Faculties of Philosophy and Liberal Arts as well as the Faculty of Natural Science. [Available on the WEB at: http://www.freewebs.com/dfmstrauss/articles.htm.]

VAN WOUDENBERG, R. (Ed.) 1996. *Kennis en werkelijkheid*. *Tweede inleiding tot een Christelijke filosofie*. Kampen: Kok.

VERELST, K. 2006. *De ontologie van den paradox*. Dissertation submitted for obtaining the Degree of Doctor in Philosophy, Brussels: Free University.

VISAGIE, J. 2006. Key Theory and Philosophy of Mind. In: *Time and context relevant philosophy*. Festschrift dedicated to D.F.M. Strauss (Special edition of the Journal for Christian scholarship), pp.201-217.

WANG, H. 1988: *Reflections on Gödel*. Cambridge, Massachusetts: MIT Press.

WEIDEMAN, A. 2006. A Systematically Significant Episode in Applied Linguistics. In: *Time and context relevant philosophy*. Festschrift dedicated to D.F.M. Strauss (Special edition of the Journal for Christian scholarship), pp.231-244.

WESSELS, D.C.J. 2006. Die Uniekheid van Getal en Ruimte as Grondslag vir Besinning oor die Elementêre Grondbegippe in die Wiskunde. *Time and context relevant philosophy*. Festschrift dedicated to D.F.M. Strauss (Special edition of the Journal for Christian scholarship), pp.245-267.

WEYL, H. 1918. *Das Kontinuum*. Berlin. (Later editions: Weyl, H. 1960. Das Kontinuum und andere Monographien.) (Das Kontinuum, von H. Weyl. Mathematische Analyse des Raumproblems, von H. Weyl. Darstellung und Begruendung einiger neuerer Ergebnisse der Funktionentheorie, von E. Landau. Ueber die Hypothesen, welche der Geometrie zugrunde liegen, von B. Riemann.) Reprint in one volume of several books on mathematics; Weyl, H. 1987. The continuum: a critical examination of the foundation of analysis / Hermann Weyl ; translated by Stephen Pollard & Thomas Bole. Kirksville, MO: Thomas Jefferson University Press.