e-ISSN: 2789-5661
2024: Vol 4(s1), pp 1-13. https://doi.org/10.38140/ijss-2024.vol4.s1.02
Interdisciplinary Journal of Sociality Studies

Pedagogical responsiveness: Focus on the Ukuqonda Institute's mode of engagement in mathematics

Satsope Maoto[1*]
Manare Setati[2]
Gabriel Mphuthi[3]

AFFILIATIONS

[1&2]Department of Mathematics, Science and Technology Education, University of Limpopo, Polokwane, South Africa.
[3]Department of Mathematics Education, University of South Africa, Pretoria, South Africa.

CORRESPONDENCE

Email: tmambiravana@yahoo.com*

EDITORIAL INFORMATION
Received: 19 April 2024
Revised: 02 October 2024
Accepted: 08 October 2024
Published: 03 November 2024

DOI: 10.38140/ijss-2024.vol4.s1.02

Abstract: This paper was prompted by mathematics teachers' challenges in implementing the proposed framework that centres on teaching for understanding, as opposed to teaching aimed at producing marks as evidence of achievement. Teaching for understanding in mathematics, among other factors, requires the creation of engaging and inclusive learning environments underpinned by teachers' pedagogical responsiveness to the diverse needs of learners. We investigated pedagogical responsiveness, focusing on the Ukuqonda Institute's mode of engagement in mathematics. We specifically pursued two research questions: 1) What are the key elements of the Ukuqonda Institute's mode of engagement in mathematics? and 2) What characterised the pedagogical responsiveness of mathematics educators at the Ukuqonda Institute? We adopted collaborative autoethnography and used two data sets generated from audio recordings captured by the authors. We employed narrative analysis and organised the findings using the characteristics of pedagogical responsiveness as heuristic devices. The findings show that the participants' pedagogical responsiveness was stimulated by the iterative process of team planning, deliberate implementation, and reflection. The prevalent characteristics were learner interactions, learner focus, inclusivity, dialogue and relationality, knowledge work, social justice, and equity. We recommend that similar studies be conducted targeting pedagogical responsiveness with a focus on other mathematical themes. Furthermore, this study suggests that, to remain pedagogically responsive, there should be constructive alignment among the types of tasks, questions, classroom interactions, and targeted content that underpin mathematics teaching and learning across different educational levels.

Keywords: Collaborative autoethnography, equivalence, mode of engagement, narrative analysis, pedagogical responsiveness.

1. Introduction

Teaching mathematics constitutes a complex engagement, fundamentally centred on learner engagement, which is itself a multidimensional construct predictive of learning performance (Maamin et al., 2022). Learner engagement in their own learning of mathematics and learner attitudes towards mathematics represent critical dimensions of the learning process (Irvine, 2020). Consequently, there remains an ongoing necessity for the development of pedagogical tools that empower mathematics educators to cultivate dynamic pedagogical content knowledge and practices tailored to meet the educational needs of diverse learners. Educators who exhibit pedagogical responsiveness possess the agility to engage with learners, comprehend the subjects being taught, and promote learner-centred learning (Jayabalan, 2023). They assist learners in applying knowledge to real-life contexts, building upon prior learning, developing skills, shaping attitudes, and fostering independent learning. By adopting a learner-centred approach, these educators effectively enhance learner engagement and comprehension. Their teaching practices play a significant role in improving student learning outcomes by nurturing independent learners and critical thinkers—elements that are essential for responsive and responsible learning (ibid.).

How to cite this article:
Maoto, S., Setati, M., & Mphuthi, G. (2024). Pedagogical responsiveness: Focus on the Ukuqonda institute's mode of engagement in mathematics. *Interdisciplinary Journal of Sociality Studies, 4*(s1), 1–13. https://doi.org/10.38140/ijss-2024.vol4.s1.02

Alongside other national priorities, South African mathematics teachers are mandated to implement a framework for the teaching and learning of mathematics (Department of Basic Education (DBE), 2018; Gaillard, 2019). This framework, proposed by the DBE (2018), centres on the principle of teaching mathematics for understanding. As explicitly articulated in the current South African mathematics curriculum, learners are required to engage with mathematics as a human activity and to develop mental processes that enhance logical and critical thinking, accuracy, and problem-solving skills, all of which contribute to informed decision-making (DBE, 2011). Consequently, mathematics teachers across all educational levels are expected to respond pedagogically to these expectations. Furthermore, teachers' robust content knowledge remains a critical factor influencing the success of mathematics learning. In response to the DBE's challenge of facilitating learner engagement in mathematics as a human activity, the Ukuqonda Institute has assisted the education sector in interpreting the implied pedagogical demands by modelling such practices within classroom settings. However, there exists limited scientific published research that documents the work of the Ukuqonda Institute in addressing the challenge posed by the DBE. Achieving improved educational outcomes is essential (Gaillard, 2019) and represents a natural component of any concentrated academic endeavour. Nevertheless, it is imperative that we ensure our learners are provided with opportunities to comprehend what they learn, thus acquiring knowledge with understanding throughout their educational journey (Doyle, 2023). It is the existing gap between the teaching of mathematics for understanding and the actual classroom practices of teachers that has led us to pursue the two formulated research questions. Thus, this paper investigates pedagogical responsiveness, specifically focusing on the Ukuqonda Institute's mode of engagement in mathematics. We particularly pursue two research questions:

- What are the key elements of the Ukuqonda Institute's mode of engagement in mathematics?
- What characterises the pedagogical responsiveness of mathematics educators at the Ukuqonda Institute?

1.2 The Ukuqonda Institute's mode of engagement in mathematics

Ukuqonda is a Zulu or Xhosa word meaning 'to understand'. The Ukuqonda Institute was founded by a group of dedicated mathematics educators who created a community focused on in-service and pre-service practices to engage in mathematics education reform and related issues. The overarching perspective informing the vision of the Ukuqonda Institute is a shared approach to teaching and learning that enables and promotes sense-making, conceptual understanding, and the development of valuable know-how and strategic skills (see Figure 1).

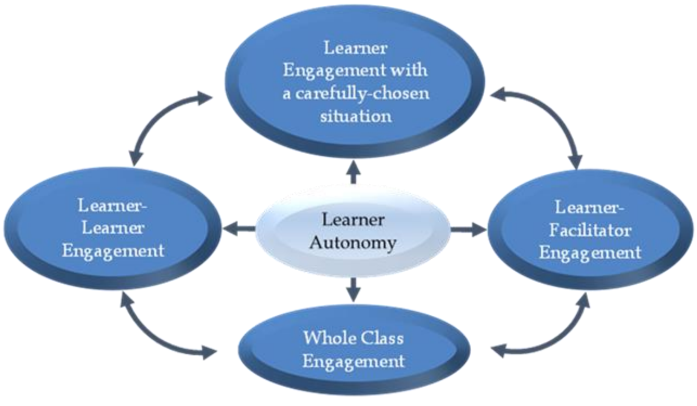

Figure 1: *The Ukuqonda Institute's mode of engagement in mathematics*

The four stages of the strategy depicted in Figure 1 represent a non-linear, holistic heuristic that embodies the conceptual vision rather than mechanical technicalities. This approach to learner engagement creates opportunities for a dynamic mode of mathematical learning to emerge in context-specific situations, facilitating variable outcomes within the DBE's vision already described. In other words, this mode of mathematical learning embraces spontaneity and creativity during learner-learner engagement while also providing spaces for further refinement of thought catalysed by learner-facilitator engagement. Additionally, learner-facilitator engagement offers opportunities to engage individual learners by setting up situations in which they can participate productively, followed by reflection, knowledge construction, and the development of capacities to think and act mathematically (Durksen et al., 2017). It provides a platform for learner-learner or whole-class engagement. Whole-class mathematics engagement serves as a forum to clarify and consolidate thoughts by nurturing and refining learners' confidence, intuitions, clarity of thought, and technical skills (Dunning, 2023).

This novel change in the dynamics of a mathematics classroom appears to significantly encourage Ukuqonda's mathematics staff to initiate and promote intellectual autonomy in their learners (McConney & Perry, 2011; Waghid et al., 2022). This approach to learner engagement in mathematics is possible only when facilitators become pedagogically responsive, using a variety of instructional strategies and learning pathways that accommodate the diverse cultures and learning styles of all learners (Walton & Osman, 2022).

2. Pedagogical Responsiveness

In this paper, we investigate pedagogical responsiveness, focusing on the Ukuqonda Institute's approach to mathematics engagement. As interpreted by Walton and Osman (2022, p.7), pedagogical responsiveness suggests "a disposition or orientation to pedagogy that is sensitive, open and empathetic, not only to individual students or groups of students but to wider factors in that community and context." Walton et al. (2019) view pedagogical responsiveness as more than mere awareness of students' needs, the demands of the content, or the influence of the environment; it also requires professional judgment that leads to action. Therefore, pedagogical responsiveness is characterised by inclusivity and a focus on students, knowledge work, dialogue and relationality, community orientation, social justice, and equity (Walton & Osman, 2022). As such, pedagogical responsiveness represents a collaborative form of relational agency, epistemic engagement, contextual sensitivity, technology, institutional capacity, and Ubuntu.

Krull (2022) reported on the pedagogical responsiveness of a traditional residential university in South Africa as it faced both the challenges of the COVID-19 pandemic and the local circumstances of student inequities. In this context, as in all other universities in South Africa, academics were challenged to rethink their teaching, learning, and assessment approaches for a remote environment. Nkambule and Mbhiza (2022) interrogated the nature of knowledge and debates in their third-year pedagogy course. They argued for the importance of exposing pre-service teachers to pedagogical knowledge and contexts that deviate from both their professional and personal comfort zones. They subsequently reported that their students' culture of disengagement with readings for lectures and tutorials changed as students were motivated by the additional materials introduced into the course; so, too, did their participation in lectures, tutorials, discussions, and critical dialogues.

The impact of humanising pedagogy (HP) on mathematics education in Technical and Vocational Education and Training (TVET) colleges in South Africa was explored by Vimbelo and Bayaga (2024). Their focus was on teaching strategies, student engagement, and real-life examples. Prior to the implementation of HP, traditional teaching methods were prevalent, resulting in limited student engagement. However, the adoption of HP led to a shift towards student-centred teaching, which significantly enhanced both student engagement and the relevance of mathematics to students' lives. The study underscored the practical importance of providing professional development for lecturers and adapting the curriculum to support the implementation of HP in TVET colleges. The aim was to create more engaging and inclusive learning environments for students, demonstrating the real-world impact of their research.

The study by Dunning (2023) focused on selecting student strategies for whole-class discussions, with teachers working towards a vision of instruction that is responsive to students' mathematical thinking. After exploring the teaching and learning of fractions, his findings led to the creation of a two-level framework that not only provides guidance for the purposeful selection of strategies but is also accessible and useful to teachers at any phase of their development in responding to students' mathematical thinking. According to Waghid et al. (2022), educators in African higher education play a crucial role in promoting autonomy, deliberation, and diffractive actions among teachers and students. These pedagogically responsive educators aim to cultivate democratic citizenship education by encouraging human relations to address societal challenges. By integrating the concept of bare life into educational interactions, educators can navigate complex situations that lead to genuine human co-existence and acknowledgement on the African continent. Ultimately, these educators establish an environment that fosters the development of students' critical thinking skills, meaningful dialogue, and positive societal contributions, thereby improving student learning outcomes.

3. Research Methodology

We adopted collaborative autoethnography, a qualitative research design, to investigate pedagogical responsiveness, focusing on the Ukuqonda Institute's mode of engagement in mathematics (Adams & Herrmann, 2020; Adams et al., 2021; Mbhiza et al., 2022). Mbhiza et al. (2022, p. 165) argue that the "collaborative autoethnographic approach views the research processes as socially just, political, and socially conscious acts of creating knowledge through group interaction." Adams et al. (2021, p. 2) state that "autoethnographers mindfully mine the past, attend to the present, and chart paths toward more humane and just futures." They further explain autoethnography from the perspectives of the "micro" (personal), "meso" (relational), and "macro" (structural). They argue that all lived experiences are situated within relationships and systems; therefore, there is no truly "lone" autoethnographer. It is against this background that we deemed collaborative autoethnography appropriate for this research.

3.1 Brief background of the participants

In this paper, we report on six participants (PH, GM, MaV, MS, NM, and EM) who all taught mathematics at the Ukuqonda Institute and collaborated for a five-day teaching session. PH was a Professor of Mathematics Education at a university for more than seventeen years. He stepped out of academic life in 1996 to devote himself full-time to providing second-chance opportunities for matriculants who were disadvantaged by poor schooling. GM worked as the principal of a university preparedness programme and as a mathematics educator at the Ukuqonda Institute. Before joining, he was a secondary school mathematics teacher for three years. He holds a Bachelor of Science Honours in Computational and Applied Mathematics and a Secondary Teachers Diploma. MaV worked as a lecturer in Mathematics, Chemistry, and Physical Science for the university preparedness programme and as a teacher trainer. MS was a manager of special projects, coordinating one of the school's projects and serving as a mathematics teacher trainer at the institute. Prior to this, he worked as both a primary and secondary school mathematics teacher for seven years, followed by eleven years as a mathematics teacher trainer at two other not-for-profit organisations. He holds a Bachelor of Education Honours in Mathematics Education. Before joining the Institute, NM was a mathematics teacher at a secondary school and holds a Bachelor of Education Honours in Mathematics Education. EM worked as a mathematics and technology teacher trainer and was a secondary school mathematics teacher for twelve years. He also holds a Bachelor of Education Honours in Mathematics Education.

3.2 Data sets

The first data set emanated from transcriptions of one of the Ukuqonda Institute's staff planning meetings that targeted teaching algebra, embracing a concept of equivalence, in a Grade 10 class. More often than not, when vacation classes were to take place, the Ukuqonda Institute team that was going to teach would meet to develop the learning trajectory of the content to be taught. This included discussions on possible conceptions, alternative conceptions, and misconceptions that learners might bring to class, as well as past experiences from the teaching content in other Ukuqonda Institute teaching sessions. This also included reviewing a learning trajectory that was previously applied. The team leader (PH) would present the intended trajectory and provide inputs based on past experience/s and research literature. Other team members were expected to give inputs to strengthen the intended learning trajectory. Audio and/or video recordings were used to capture the proceedings of the planning meetings and were subsequently used as sources for reflections and further planning. During the teaching sessions, at the end of each teaching day, the team assembled to share challenges or successes that they had experienced, and then the trajectory may be reshaped based on critical reflections on the feedback.

The planning meeting discussed here was based on an introductory task on a module "Equivalent Expressions" for a Grade 10 class. The purpose of the tasks was to prepare the learners' mindset for the nature of mathematical engagement that is necessary to make sense of algebraic transformation, which includes making connections between mathematical ideas, making sense of mathematical representations, and mathematical modelling (Ardiansari et al., 2020; Palatnik & Koichu, 2017).

Task 1: **David's Spaza Shop**
Menu for party items.

Pizza	**R18**
Cooldrink	**R4**
Fruit juice	**R6**
Packet of chips	**R7**
Ice-cream	**R12**

25 people are coming to a party.
Each person must get one of each item.
How much will it cost in total?

Figure 2: *David's spaza shop activity*

This activity presented an arithmetic problem, yet the focus task was an algebraic problem.

The second data set originated from a focus group's reflection meeting at the end of a five-day teaching session on the same concept, targeted at the same grade, as discussed during a planning meeting. The focus group's reflection meeting was led by the same team leader (PH) from the planning meeting, assisted by one of the facilitators (GM). The purpose of this focus group's reflection meeting was to determine the point at which learners became aware of the equivalence of expressions and to establish whether learners recognised the value of using a simplified equivalent expression to evaluate the value of expressions. An audio recording was made of the learners' interview session for analysis at a later stage.

3.3 Data analysis

The audio recordings of the data sets were listened to repeatedly and ultimately transcribed verbatim. The transcriptions of both the planning and focus group reflection meetings were read multiple times, during which the data were first coded and grouped, guided by the emerging characteristics of pedagogical responsiveness as identified by Walton and Osman (2022). We employed Polkinghorne's (1995) conception of narrative analysis to give meaning to our interpretations. According to Polkinghorne, narrative analysis is the procedure through which the researcher organises the data elements into a coherent developmental account. This process involves synthesising the data rather than separating it into its constituent parts. To construct our narratives, we found the characteristics of pedagogical responsiveness to be a useful heuristic device for understanding participants' pedagogical responsiveness (Bleakley, 2005; Soliva, 2007). Verbatim quotes from critical incidents were used to verify the emergent data set throughout (Tripp, 2012).

3.4 Ethical clearance

As stated by Adams and Herrmann (2020, p. 3), autoethnographers take ethics seriously: "they must worry about how they implicate and represent themselves, others, and the happenings of a group." Permission and consent were granted by the Ukuqonda Institute board of directors to use its documents and data sets. We agreed on the anonymity of the participants from the outset with the board; thus, pseudonyms were used, and no individual identities were divulged.

3.5 Quality criteria

Prolonged engagement, ongoing probing, peer debriefing, and member checks provided sufficient opportunities to accurately interpret the data sets, contributing to the credibility of this study (Bitsch, 2005; Guba & Lincoln, 1989). Recursive discussions with four of the six participants, along with sufficient descriptive data, enhanced the confirmability and transferability of this study (Guba & Lincoln, 1989). To further satisfy member checking, the analysis was shared with two participants who conducted the interviews to verify the authenticity of the findings. This allowed for additional inputs from these participants to be incorporated into the final analysis.

4. Findings and Discussions

Pedagogical responsiveness was investigated with a focus on our interpretation of what constituted the Ukuqonda Institute's mode of engagement in mathematics. We analysed the critical incidents from the planning and focus group reflection meetings to examine the key elements of the participants' classroom engagement and to infer the characteristics of their pedagogical responsiveness (Walton & Osman, 2022). The characteristics of pedagogical responsiveness were found to be a useful heuristic device for understanding participants' approaches (Bleakley, 2005; Soliva, 2007). Additionally, they assisted us in constructing our narratives. We first zoom in on the participants' planning meeting to trace alignment with the implementation discussed in the focus group's reflection meeting before reflecting further on the analysis.

4.1 Dialogue and relationality, inclusivity and student-focused

The planning meeting began with PH opening a dialogue by reminding others of their previous reflections on teaching a similar mathematical idea. MS pointed out after their first attempt that, "*instead of quickly learning cross multiplication to add fractions, learners should replace the fractions with their equivalents*". PH agreed with MS that this shift added value and was thus to be integrated into the curriculum from the outset. PH intended to caution other facilitators that, in their next lessons, instead of teaching the algorithm for adding fractions, it would be important to engage learners in understanding that when adding fractions with denominators that are multiples of each other, one must replace a fraction that is difficult to work with its equivalent—while maintaining the same fraction name—and then add them.

For example, the expression $\frac{1}{2}+\frac{1}{4}$ *is difficult to calculate and must therefore be replaced with the expression* $\frac{2}{4}+\frac{1}{4}$ *because this latter expression is easier and/or user-friendly.*

A further example was given that they claimed learners had a misconception, also in terms of elementary information, like how multiplication is done.

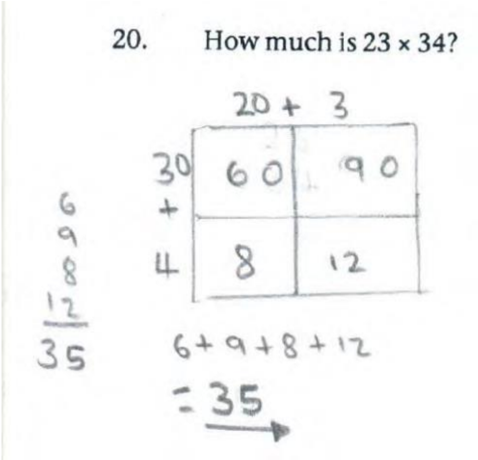

Figure 3: *Learner's work as captured in the Ukuqonda Institute's baseline study report (2019)*

PH emphasised that the aim here should be to focus on remedial education because something did not go as planned; thus, educators should try to rescue the learners to correct the misconception. For him, it was not just the notion of equivalence that could have been developed. It was also the fact that learners should make sense of algebraic manipulation not as producing equivalent formulas but as calculating with letters to produce a solution.

Similarly, the expression 23×34 *can be replaced with* $(20 + 3) \times (30 + 4)$ *which may also be replaced by* $20 \times 30 + 20 \times 4 + 3 \times 30 + 3 \times 4$.

The idea of replacing a troublesome expression with a user-friendly expression was therefore at the core of David's Spaza Shop task, and teachers were advised to introduce and reinforce the concept during classroom discussions. Equivalence, as a mathematical concept discussed during the planning meeting on fractions, was referenced so that learners could notice that mathematical ideas can be useful in arithmetic expressions and also in algebraic expressions.

Emphasis was also placed on an awareness of the equivalence concept, which should not just be taught for its own sake but rather used in basic algebraic expression manipulation, allowing learners to autonomously choose to use equivalent simplified "user-friendly" expressions to evaluate long or complicated algebraic expressions. It was also noted that in an attempt to develop learners' understanding of equivalence, there may be other things that learners grapple with – such as the appropriate meaning of symbols used – and it is thus important to be aware that such concepts may inhibit the smooth development of the intended idea to be learned. Mathematical ideas that are underdeveloped may, therefore, sub-optimally influence the learning of a new mathematical construct.

PH also made connections between mathematical representation, "*… these are two ways, different ways in which you can calculate the cost of the party… This is the way in which we want to get learners to have a richer representation of expressions*". This was with reference to $25 \times (18 + 4 + 6 + 7 + 12)$, that is, the product of the sum and $25 \times 18 + 25 \times 4 + 25 \times 6 + 25 \times 7 + 25 \times 12$, that is, the sum of the products. The two calculation methods represent two different ways of thinking about the same situation. PH, in this case, encourages teachers to prompt learners to analyse the two expressions to find relationships between them. At a later stage, towards the end of the meeting, PH talked about "…given an algebraic model and then to decide that the model is inconvenient to analyse, *so we are going to replace it with an alternative model*". This is related to thinking about an alternative model.

Three critical areas relating to the participants' pedagogical responsiveness can be deduced from their initial planning dialogue. Firstly, they succeeded in creating a platform to empower each other pedagogically, enabling the engagement of learners in a learning situation that was deliberately designed to provide them with opportunities to reconstruct specific aspects of their knowledge. Secondly, they established an engaging and inclusive environment for sense-making with respect to algebraic expressions (Vimbelo & Bayaga, 2024). Thirdly, by remaining learner-focused, the participants were sufficiently equipped to provide learners with opportunities to explore different methods of calculating the same outcome in a real-world context.

The choice of activity was inclusive, as its design allowed each learner entry, enabling them to conceive equivalence in terms of different sets of computations producing the same quantity. It became evident that the participants' rationale behind the activity was to help learners perceive a set of computations as an 'object' in the sense of a computational plan. Additionally, it aimed to convey that different computational plans can serve the same purpose or achieve the same result. From the participants' discussions, it also emerged that they provided learners with opportunities to reflect on the merits of various computational plans that accomplish the same task. Furthermore, they offered learners the chance to inscribe these different computational plans as expressions, initially using conventional notation and later as algebraic expressions. Their hypothesis was that, in this way, learners would have the freedom to make sense of algebraic expressions as inscriptions of computational plans in ways that resonated with them.

4.2 Knowledge work

The proceedings of both the planning and the focus group's reflection meetings uncovered the participants' knowledge of the concept of equivalence, moving from a real-life context into and operating within the world of mathematics. This movement appeared to have been enabled by the participants' critical and participatory dialogue (Nkabule & Mbhiza, 2022). Mathematical representations are not only tools that are used to express one's ideas but can also help to develop an understanding of mathematical ideas. Appropriate use of mathematical representations, such as the use of the equal sign, has a significant effect on learners' understanding of mathematics (Ardiansari et al., 2020). There was a critical consciousness among the participants that during their teaching sessions with either learners or teachers, they should draw attention to the most common use of the equal sign as *"… to say I have calculated something and this is my answer, that's how an equal sign is introduced."* This meaning of the equal sign, which is prevalent among many learners, as opposed to the meaning of the equal sign as equivalent (what is on the left-hand side of the equal sign has the same value as what is on the right-hand side), is in most instances a hindrance to arithmetic and algebraic manipulation. Suggestions were made that maybe the use of a different symbol to denote the answer to a particular calculation could be used to preserve the real meaning of an equal sign. They thought that may be useful.

> ***PH:*** *…So at the beginning they are taught that that's its meaning and that … remains with them, ok!*
>
> *And I think to solve this issue would be, not to use an equal sign there but to use an arrow or something else and to reserve an equal sign for equivalence*
>
> ***NM:*** *But wouldn't it help Prof. if .. if, if, if … this other aspect of the equal sign is introduced earlier?* $23 + 7 = 4 + \cdots$
>
> ***PH:*** *Absolutely, and I think an equal sign should be reserved for that. It should not be in primary mathematics for any other purpose than to express equivalence or the other way.*

As the participants further deliberated, it became evident that their handling of emerging mathematics concepts was critically scrutinised in preparation for their future teaching. In addition to focusing on the learners, they emphasised mastery of content. They were aware that in order to respond pedagogically, they needed to strive for conceptual understanding. Consequently, knowledge work emerged as integral to their pedagogical responsiveness.

4.3 Student-focused, social Justice and equity

There were several instances discussed during the planning meeting that focused on knowledge of conceptions and misconceptions that learners bring to class. One such incident discussed earlier in this paper was the misconception held by many learners of the meaning of an equal sign (Ardiansari et al., 2020). There was a reflection on other failed attempts made to address this misconception. Further attempts were made to assist learners in interpreting an equal sign differently from the way they normally do. The second misconception discussed was the interpretation of the acronym 'BODMAS' for the order of operations, in which B means "brackets out" rather than "do calculations of numbers that are in brackets first." A systematic way to develop awareness of the order of operations without using the acronym was suggested. The third, which is not a misconception, was that teachers were to be warned that it does not mean that once learners have noted the need to replace an expression with its equivalent, then they will consistently do it whenever a need arises. The habit of mind relating to the 'practice makes perfect' mentality was found not to be in place in learners' minds. Learners' view of mathematics was interpreted to be that if they were asked to evaluate an expression, they must just substitute the variable with the number and do calculations without finding an equivalent or a user-friendly expression before evaluating.

The team leader, PH, also brought forth knowledge or lack of knowledge that learners bring to class: *"what we found was the ability to interpret simple figures was non-existent; another thing that's non-existent is the idea of a formula… The whole idea of evaluating expressions doesn't exist in their minds… both the idea of evaluating and the idea of making a formula don't exist for them, it just doesn't exist."* After identifying these knowledge gaps, the dialogue took a direction towards volunteering suggestions about how to assist each learner to realise sense-making in algebra. Thus, there was emphasis on scaffolding learners through a sense-making process that would enable them to formulate and justify their claims; generalise conjectures, find the mechanisms behind the algebraic objects (that is, answer why-questions), and establish coherence among the explored objects (Palatnik & Koichu, 2017).

4.4 Focus on learners' interactions

In this section, we report on our interpretation of the participants' focus group reflection meeting that took place at the end of a five-day teaching session on the same concept in the same targeted grade, as discussed during a planning meeting which involved five learners (Nathan, Mpho, Godfrey, David, and Kabelo). The participants focused on how learners interacted with the facilitator (GM) about their choices for preferred expression between the two equivalent expressions and reasons for such choices. This focus was constructively aligned with the participants' planning meeting, which was on developing awareness of equivalence and choosing the simplified version when substituting. Interest in this case was on how GM responded pedagogically post the participants' planning meeting

The participants opened their dialogue by first reflecting on Nathan's response to the given question. Efficiency was not an issue for him. At a later stage, Nathan realised the tediousness in some expressions, particularly long expressions, as explained by Mpho when substituting the value of x without simplifying. He indicated that when substituting in such expressions, you may not get the *"same answer"*, meaning the wrong answer, because of mistakes. At this stage, Nathan said: "*So, because the expression is the same, then we solve it first and then replace with x?"*. With this, he suggested that it was better to simplify before substituting. For Nathan, an alternative model was only necessary when the one at hand was problematic.

Godfrey also demonstrated an awareness of the equivalence of expressions but chose to use the long expressions because *"some people who do not know why I did this 5x will ask questions"*. This related to the fact that 5x is an expression that resulted from the simplification of the original expression, and according to Godfrey, other people might reject the use of 5x as its origin is unknown to them.

Question 1 asked for simplification of expression $3x(4x + 5) - 2x(6x + 5)$

Questions 4 and 5 asked for the value of the expression when x is 18,5 and 66 respectively, without suggesting the use of the simplified expression.

Godfrey preferred to substitute in the original expression even if he was aware of the equivalence and efficiency of using $5x$. It also emerged that this may be due to his lack of confidence with algebraic manipulation, which was evident when he said: *"I didn't know how I can simplify because there were no brackets"*. This was in response to the question asked by the facilitator as to why he did not simplify the expression first. It could be interpreted as a lack of confidence that inhibited Godfrey from using equivalent expressions because he was, however, aware that they could be useful.

Mpho was one learner who, from the beginning, said, "… because this is a long-expression, *we have simplified ...*". For Mpho, the two questions, one asking for substituting in the simplified expression and another asking for the value of the long-expression given the value of x, were the same as the two expressions are equivalent. She did not change from substituting in the simplified expression because the simplified is an equivalent of the long expression. She said: *"I feel that this is the same question, I can just simplify it"*. This was in response to being pressed by the facilitator to explain if she can do it differently. For Mpho, to be asked if she could change from using the simplified expression $28x - 5$ and $12x(3x + 7) - 4x(9x + 14) - 5$ was asked to use one expression rather than the other and is asking the same question as these expressions are equivalent. Interestingly, Mpho did not see the need to first simplify $7x + 32 - 5x + 12 + 12x + 23 + 8x - 2x$, before finding the value of the expression for specific values of x. She said that it was a different question. Perhaps, it was because the expression was easy to evaluate for Mpho.

It emerged as Mpho continued to interact with the facilitator that she worked with both simplified and original expressions. She, without provocation, said *"when it looks like the expression can lead to silly mistakes then you must first simplify the expression before you substitute"*. She also highlighted the influence of social interaction. She indicated that during their discussions when dealing with corrections, others used shorter methods. The facilitator did not intervene and that prompted Mpho to resort to a shorter method. Mpho ultimately saw the value of a user-friendly yet equivalent expression and consistently used simplified expressions.

David used the original and longer expression $3x(6x + 10) - 9x(2x + 3)$ to check the correctness of the simplified expression. Initially, in question 1, he simplified the expression and got $3x$. Question 2 required them to respond to how much will the expression $3x(6x + 10) - 9x(2x + 3)$ be if x is equal to 3,2? David used the calculator to find the value of $3x$ when $x = 3.2$ and substituted 3.2 in the original expression to check if the solution will be the same. He was yet to develop confidence in using simplified version. He was, however, aware of the value of using the simplified version. For Kabelo, different expressions meant expressions are not the same (equivalent). He became aware of different but equivalent expressions.

Deduced from the focus group interactions, it was noted that the facilitator created a classroom learning environment that allowed for interactions among learners themselves and with him through speaking, listening, critically reflecting and talking back, thus furthering the conceptual understanding in multiple ways. In the simplest of terms: the facilitator provided an opportunity for learners to grapple with, critically reflect on and gradually learn complicated concepts in variable ways that made unique sense to each individual learner throughout the process.

5. Conclusion and recommendations

In this paper, we investigated pedagogical responsiveness, focusing on the Ukuqonda Institute's mode of engagement in mathematics. Two research questions were pursued: *"What are the key* *elements of the Ukuqonda Institute's mode of engagement in mathematics?"* and *"What characterised the pedagogical responsiveness of mathematics educators at the Ukuqonda Institute?"*

As illustrative cases, we analysed data focusing on one planning meeting and the focus group's reflection meeting after a teaching session by the Ukuqonda Institute's mathematics facilitators that embraced the concept of equivalence in a Grade 10 class. Both meetings emerged as strongholds that supported all participants in concentrating on learners' conceptual understanding rather than a narrow focus on preparing learners for tests and examinations. It became evident that this created a community of mathematics in-service and pre-service practitioners, which significantly influenced all participants to engage learners during their teaching sessions with meaningful tasks. We inferred that the key elements of the Ukuqonda Institute's mode of engagement were the continued encouragement of each other through team planning and reflection on/for action; a sense-making process connecting previous experiences and mathematical ideas; and the creation of an engaging and inclusive learning environment that allowed all learners to express and explain their mathematical thinking and reasoning.

During the focus group's reflection meeting, the Ukuqonda Institute team members reflected on and encouraged each other to consciously try to use equivalence—specifically, replacing an expression with its equivalent in the same way it was discussed in the previous session on fractions. This was observed during the planning meeting when PH, who led the discussions, began by reminding others of their previous reflections on teaching similar mathematical ideas. The participants' critical, concept-focused feedback dialogue during both the planning and focus group reflection meetings enriched each other's responsive pedagogy in various ways. This continued practice offered the participants opportunities to develop responsibility for their own learning and self-reliance. They became empowered to develop practices that addressed self-regulation and self-efficacy in dialogues with their learners. As expected, they were challenged to transfer their learning dialogue into their teaching sessions.

The findings show that the participants' pedagogical responsiveness was aroused by their iterative process of team planning, deliberate implementation, and reflection. The prevalent characteristics were learners' interactions, learner focus, inclusivity, dialogue and relationality, knowledge work, social justice, and equity. Participants' productive interactions during both the planning and focus group reflection meetings contributed significantly to being prepared to afford learners opportunities to independently evaluate whether the simplified or original version of the expression would be fit for purpose. Informed by these findings, we recommend that similar studies be conducted that target pedagogical responsiveness focusing on other mathematical concepts. Furthermore, this study recommends that to remain pedagogically responsive, there should be constructive alignment among the types of tasks, questions, classroom interactions, and targeted content that ground mathematics teaching and learning within and across different educational levels. Implementation, accompanied by ownership for all involved in the reform, is an essential value.

5. Declaration

Authors contributions: Conceptualisation (S.M., M.S., & G.M.); Literature review (S.M., M.S., & G.M.); methodology (S.M., M.S., & G.M.); software (N/A); validation (S.M., M.S., & G.M.); formal analysis (S.M., M.S., & G.M.); investigation (S.M., M.S., & G.M.); data curation (S.M.) drafting and preparation (S.M., M.S., & G.M.); review and editing (M.S.); supervision (N/A); project administration (S.M.); funding acquisition (N/A). All authors have read and approved the published version of the article.

Funding: This research did not receive any external funding. The Article Processing Charge (APC) was covered by the University of Limpopo.

Acknowledgements: We would like to acknowledge the Board of Directors of the Ukuqonda Institute for allowing us to source data generated by its staff members for publications.

Conflicts of Interest: The authors declare no conflict of interest.

Data availability: In accordance with ethical standards and the stipulations set forth in the consent agreement with participants, the data must be maintained as confidential. Nevertheless, individuals seeking further information may contact the corresponding author.

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