

Rural Teachers' Teaching of Algebraic Functions through a Commognitive Lens

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Abstract: Rural contexts and their schools have continuously been overlooked by researchers of mathematics education in South Africa. This is despite the assumption that the educational landscape may vary markedly in rural areas compared to urban and township areas which have been solely researched in the post-apartheid dispensation. To address the dearth of mathematics education research located within South Africa's rural contexts, the study explored five Grade 10 rural mathematics teachers' discourses and approaches of teaching algebraic functions with five teachers from five different school sites. This qualitative multiple case study, using Sfard's commognitive theory, draws attention to rural mathematics teachers' classroom practices and views about the teaching of algebraic functions which is unexamined in the South African context. Three data generation tools were used to gain insight into teachers' discourses and approaches while teaching the topic. These are individual semi-structured interviews, classroom observations and Video-Stimulated Recall Interviews (VSRI). Research findings focus primarily on the data

generated through classroom observations. To analyse the data, I use Sfard's commognitive theory to give meaning to teachers' classroom practices. Focusing on the distinction between two tenets of commognitive theory, ritual and explorative routines, the findings demonstrate that four participating teachers acted in an extremely ritualised way. The other teacher was more explorative in her classroom observable actions. The findings illuminate that teachers need to move more towards the participationist approach during teaching to enable them to think, observe, and communicate about mathematical objects that commognitively link more with explorative routines.

Keywords: Algebraic function, Rural, Commognitive framework; Discourse, Teaching, Rural teachers

1. Introduction

This paper addresses the teaching of algebraic functions within rural classrooms, which has somewhat been underexplored within the South African mathematics education research. The concept of algebraic functions has received attention within the field of mathematics education (Kazima, 2008; Trigueros & Martinez-Planell, 2010; Moloj, 2013). Felix Klein in 1908 viewed functions as 'the soul of mathematics', and this view has since been discussed by various researchers (Afonso & Auliffe, 2019) to show the importance of effectively teaching the topic and enabling learners' epistemological access (Lotz-Sisitka, 2009). In relation to this, Sierpinska (1992, p. 32) stated that "functional thinking should pervade all mathematics and, at school, students should be brought up to functional thinking". This demonstrates that the concept of function is a 'bedrock' of school mathematics as it contains foundational content knowledge and skills to other content areas in Mathematics. This discussion resonates with Eisenberg's (1992, p. 153) iteration that developing learners' sense of functions "should be one of the school's main goals and collegiate curriculum" because developing learners' conceptual understanding of functions helps learners to work with other mathematics topics with ease. In view of the above discussion, functions can be considered a meta-discourse of algebra (Caspi & Sfard, 2012; Mudaly & Mpofo, 2019). My concern is that while such iterations are made about the importance of functions in the school curriculum, sparse research has been conducted within the South African context focusing on the teaching of the topic to explore the nature of teachers' classroom practices in order to understand how such practices enable and or constrain learners effective learning.

In addition to the above discussion, the paucity of research focusing on the teaching of mathematics within rural schools and classrooms has not offered accounts of teachers' teaching practices while teaching functions within rural classrooms, despite the

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abovementioned importance of the topic in school mathematics. It can be said that researching the teaching of functions with rural mathematics teachers is a matter of social justice, especially in the context of South Africa, where rural schools are continually represented as underperforming in mathematics (Nkambule & Mukeredzi, 2017). It is a significant concern that 26 years after South African democracy, the scope of mathematics education research locale has not shifted to include rural and farm schools, especially considering Mbata's (2014) statement that 62% of South African schools are located in rural areas. Accordingly, expanding the scope of mathematics education research to include researching with rural teachers could help us understand different ways of being, ways of thinking, ways of knowing, and ways of teaching mathematics, which could be different from the dominantly researched urban and township teaching and learning. As an attempt to diversify the research locale for mathematics education, the current study intended to answer the following research questions:

- a) What are rural mathematics teachers' discourses while teaching algebraic functions in Grade 10?
- b) What is the nature of Grade 10 rural teachers' mathematical discourse relating to the different representations of functions?

Suppose the need for social transformation in rural schools is to be seriously considered. In that case, we need to expand and diversify the scope of mathematics education research to include researching with rural teachers, to understand the nature of their classroom practices and how such practices facilitate and or constrain learners' understanding of the subject. In this study, I consider teacher thinking and communication as important components in facilitating learners' mathematical discourse and in turn conceptual development in mathematics, especially in enabling learners' learning outcomes. Teacher thinking and communication about mathematical objects influence learner learning outcomes (Sfard, 2008; Shabuddin, 2016). If teachers' communication skills are low and learner involvement during learning and teaching is limited, it can result in failure of learner learning outcomes. Thus, the teacher must be able to communicate mathematical ideas effectively and have adequate knowledge while teaching in the classroom.

2. Review of previous research

2.1 Sparse mathematics education research in rural schools

The teaching of mathematics in rural South African schools and classrooms is often cited as sub-standard, especially compared to their urban and township counterparts (Spaull, 2013). However, the sparsity of mathematics education research located within rural schools has not been able to offer insights into the nature of teaching and learning of mathematics in those contexts as well as teachers' and learners' lived experiences. There is an anthology (Berger, 2013; Brodie, 2014; Gcasamba, 2014; Mudaly & Mpofu, 2019), focusing on many aspects of mathematics teaching at different levels of schooling generally. However, there is sparse research that has explored the actual practice of mathematics teaching within rural and farm areas (Nkambule, 2017; Mbhiza, 2019). Equally, as far as it can be determined, research on the teaching of algebraic functions at South African secondary schools is rare, particularly within rural schools setting. This is concerning, especially considering Venkat, Adler, Rollnick, Setati and Vhurumuku (2009, p. 11) acknowledgement that the dearth of mathematics research "done in rural schools is problematic given that the majority of South African learners are educated in these contexts, as urban contexts continue to be explicitly and solely focused upon...". Despite this reflection by the pioneers of mathematics education research within the South African context, the research locale bias is still eminent a decade later, even from the work of Venkat et al. (2009). Thus, this research addresses the research locale gap by expanding the scope of mathematics teaching research to include researching with rural teachers.

2.2 Research on algebraic functions

Algebraic functions is one of the important concept in the South African Curriculum, Assessment Policy Statement (CAPS) as signified by the prioritisation given to it in terms

of the teaching time allocated across the Further Education and Training (FET) phase (Department of Education (DoE), 2011). While this is the case, there is sparse research that explored the teaching and learning of the topic within the South African context, especially focusing at Grade 10 level. Of concern for the current study is that previous studies were conducted at Grades 11 and 12 (Gcasamba, 2014; Moeti, 2015; Mugwagwa, 2017; Malahlela, 2017) as well as tertiary levels (Vinner & Dreyfus, 1989; Viirman, 2014), particularly within urban and township schools, overlooking Grade 10 as the 'foundational grade' for the topic and perpetuating the marginalisation of rural education research respectively. That is, rural areas have not been focused upon in general, especially within the South African context, to understand teachers and learners' discourses when engaging with the topic. It is also notable that previous studies on algebraic functions has predominately focused on learners' difficulties related to learning the topic (Malahlela, 2017; Mpofo & Pournara, 2018), overlooking researching with teachers who are the primary source of mathematical knowledge.

Within the concept of the algebraic function in school mathematics, both the importance and problems relating to its teaching and learning have been researched and documented in mathematics education research (Moloi, 2013; Moalosi, 2014). Swarthout, Jones, Klespis and Cory (2009) posits that functions is a very important topic in the mathematics curriculum, claiming that it is necessary for all learners to understand the topic in order to use the knowledge of functions in further learning other topics in school mathematics. In view of this, Moalosi's (2014) study findings on mathematics teacher knowledge demonstrate that learners experience difficulties in learning and understanding the topic, and teachers' classroom practices while teaching the topic could be a factor at play in reinforcing the associated learners' encountered difficulties. I believe it is essential to focus on the teachers' classroom practices during the teaching of algebraic functions as a starting point to understand how such practices enable and/or constrains learners' effective learning of the topic. This position is informed by Sfard and Caspi's (2012) argument that learners' development of mathematical discourse is inextricably dependent on the mathematical discourse that the interlocutor brings to the fore during teaching and learning.

3. Commognition as theoretical framework

Teachers' presentation of mathematical ideas and concepts in the classroom involves communication that is distinctive for mathematics teachers. For the study of rural teachers' teaching of algebraic functions, I have chosen to use Sfard's commognitive theoretical framework since it provides conceptual tools for exploring and capturing mathematical discourse and social participation patterns during teaching (Sfard, 2008; Sfard, 2015). The commognitive theoretical framework focuses on the "inseparability of thought and its expression, either verbal or not" (Sfard, 2015, p. 132). In this paper, the teaching of mathematics includes engaging in a well-defined type of communication, an expression of mathematics at an intrapersonal (cognition) and interpersonal (communication) level as teachers teach algebraic functions. Thus, cognition and communication are regarded as different expressions of the same phenomenon (Sfard, 2008). The unit analysis for commognitive theory is the discursive activity, which refers to the "patterned, collective doings" within an activity (Sfard, 2006, p. 157). In the current paper, I focus on algebraic functions' discourse, as it is manifested in the rural Grade 10 teachers' communicative practices.

The commognitive theory presents four distinguishing characteristics of mathematical discourses: words and their uses; endorsed narratives; visual mediators, and routines (Sfard, 2008). Word use entails the words that are specific to the discourse or common words teachers and learners use in discourse specific ways during teaching and learning in the classroom. Visual mediators refer to the visual objects with mathematics that are operated upon as a part of the discursive practice. Examples from mathematical function discourse could be table of values and special symbols. Narratives are about the patterns of utterances referring to mathematical objects, relations between processes that are

performed upon objects, subject to either rejection or endorsement within the mathematics community. The examples includes mathematical definitions, theorems and equations. Lastly, routines refer to repetitive actions of the discourse. Within the mathematical context, routines are for instance the performance of mathematical calculations and drawing of graphs.

A more comprehensive understanding of Sfard's commognitive theory of mathematics teaching and learning is beyond the scope of the current paper, but few concepts about the notion of mathematical routines and mathematical rules are needed in analysing teachers' classroom practices during algebraic functions lessons. Sfard (2008) distinguishes between ritual and exploration routines. The former refers to "a routine whose goal is social approval that create and sustain a bond with other people", which necessitate the imitation of rule applications as the interlocutor (Berger, 2013, p. 3). On the other hand, exploration routines are those whose "goal is to produce new narratives" (Sfard, 2015, p. 131) and are linked to the routines that focus on the analysis of repetitive patterns of mathematical discourse that underlie participatory regularities. My focus in the current paper is on the nature of narratives and routines characterising rural teachers' discourses while teaching algebraic functions at Grade 10 level. I will therefore omit the analysis of visual mediators and the words that teachers used. Observations regarding visual mediators and word use will be referred to whenever I want to make commognitive elaborations and where they are relevant to the analysis of the teachers' teaching.

4. Research methodology

The empirical data in the current paper consists mainly of videotaped lessons presented by five mathematics teachers at five different school sites in rural Mpumalanga Province of South Africa representing multiple cases. As reported in this paper, a qualitative research approach was espoused (Creswell, 2013). The qualitative approach entails "a systematic subjective approach used to describe life experiences and situations to give them meaning" (Burns & Grove, 2003, p. 19) and is appropriate for this study. This approach allowed me to gain insight into rural teachers' teaching practices in their uniqueness, the nature and influence of rurality in their teaching as well as what it means for them to live and teach mathematics within a rural context and schools. To understand the teachers' lived experiences, I immersed myself into the lives of the teachers to explore and understand the teaching of algebraic functions as experienced by teachers.

In addition, the current study used a multiple case study design. This design enabled me to understand the nature of mathematics teaching, specifically the teaching of algebraic functions within a bounded context and bounded activity (Creswell, 2013). For the current study, the bounded context is rural schools in Acornhoek and mathematics classrooms in the schools, and the bounded activity is the teaching of algebraic functions at Grade 10 level. The study was conducted with five (5) Grade 10 mathematics teachers at five (5) secondary schools in rural Acornhoek, Mpumalanga Province of South Africa, forming multiple cases. The schools and participating teachers were selected purposively, based on their participation in the Wits Rural Teaching Experience (WRTE) project. Also, teachers needed to possess experience and knowledge of teaching Grade 10 mathematics within rural classrooms—tables 1 below presents participating teachers' biographical information. To conceal and protect teachers' true identities, I use pseudonyms, as shown in Table 1 below. The use of pseudonyms helps in ensuring anonymity in the study.

Table 1: Teachers' biographical information

Pseudonym	Gender	Mathematics Education qualifications	Number of years teaching	Institution trained at to become a teacher
Zelda	Female	Bachelor of Education	5 years	North-West University, South Africa
Mafada	Male	Honours in Mathematics Education	20 years	Giyani College of Education, South Africa
Tinyiko	Female	Bachelor of Education	5 years	University of Venda, South Africa
Mutsakisi	Female	Bachelor of Education	30 years	University of Zimbabwe
Jaden	Male	Bachelor of Education	17 years	College of Education in India

The empirical data in the current study was generated by means of semi-structured interviews, unstructured non-participatory classroom videotaped observations and Video-Stimulated Recall Interviews (VSRI). In this paper, I mainly focus on the teachers' classroom practices as videotaped during observations to discern their discourses during the lessons. Johnson, Johnson and Cristensen (2012, p. 206) defines an observation technique to refer to "the watching of behavioural patterns of people in certain situations to obtain information about the phenomenon of interest". In the context of the current study, the definition suggests that classroom observations can be used to explore and generate in-depth understanding of the nature of teachers' classroom practices related to the teaching of algebraic functions (Guthrie, 2011). The nature of my participation in the observations was non-participatory, I adopted a "passive, non-intrusive role" during teaching in all the classrooms that I observed (Cohen, Manion & Morrison, 2011, p. 459). One way of ensuring the trustworthiness of data was through peer scrutiny of the research processes. During the course of the study, I welcomed scrutiny of the project by peers, colleagues and academics at conferences, which allowed me to address biases and assumptions relating to my interpretations of teachers' classroom practices during the lessons.

4.1 Data analysis

According to Nieuwenhuis (2007, pp. 99-100), "... qualitative data analysis tends to be an ongoing and iterative process, implying that data collection, processing, analysis and reporting are intertwined, and not necessarily a successive process". In the current study, the analysis of observed lessons commenced during data collection and units of analysis were created through ascribing codes to the teachers' observed practices during teaching (Muir & Beswick, 2007). After transcription, the recorded lessons were analysed with the purpose of segmenting and distinguishing the discursive activities characterising the teachers' respective discourses of algebraic functions. I first analysed each lesson for individual teachers separately, paying attention to repetitive patterns and characteristics of the use of different modalities of mathematical representations and narratives. I then compared the different lessons, searching for similarities and differences and using the identified nuances to inform and re-shape my separate lessons analyses. In the current study, I have intentionally adopted an outsider position to view the discourses unfolding from the different teachers' teaching in as unbiased a way as possible. Equally, it is important to acknowledge that I am aware of and also making use of my own mathematical knowledge, which indirectly makes me an insider to the discourse.

In addition to the above discussion, analysing teachers' classroom discourses while teaching the topic under study required an approach that allowed me to look within and across the different teachers' lessons to create a picture of each teacher's teaching quality. Thus, I overlaid the commognitive theory's tenets for both structure and generality about the teachers' discourses during the observed lessons. I initially chunked lessons into episodes on the basis of what activities set and their related examples for each lesson. Within the episodes, I then noted the nature of teachers' mathematical discourse as framed by the four components of commognitive mathematical discourse. The following section details the ethical considerations I made before, during and after the data generation processes for the study.

4.2 Ethical Consideration

Attention on society's ethical conduct has increased and broadened in response to expectations for greater research participants' protection (Zegwaard, Campbell, & Pretti, 2017). Accordingly, before the study could commence, ethical clearance was granted by the University of the Witwatersrand and access to the schools was permitted by the Mpumalanga Department of Education. All teachers were informed of the purpose, confidentiality, and voluntary nature of participation in the study before any data generation processes commenced, and all participating teachers signed informed consents. I also adhered to the importance of ensuring that the identity of participants is protected, both in terms of keeping the information they provided confidential and using pseudonyms to conceal their true identities as well as those of their respective schools. The assurances for confidentiality and anonymity in this study extended beyond protecting the teachers' names and those of their schools to also include the avoidance of using self-identifying statements and information. In the following section, I organise the analysis of teachers' teaching practices in terms of the characteristics of mathematical discourse (Sfard, 2008).

5. Findings and discussion

According to Renkl (2017), teachers should choose appropriate examples to facilitate the learning and understanding of the concepts. Thus, the findings presented in this paper focuses specifically on how teachers worked with the examples during the lessons to help learners understand the critical features for different families of functions. The discussion is in line with the South African CAPS curriculum recommendations regarding the teaching of algebraic functions at Grade 10 level, that teachers teach learners about the critical features for the different families of algebraic functions, such as the effect of different parameters, domain, range, intercepts, Turning points (DoE, 2011). Specifically, the discussion addresses the curriculum standard that stipulates that teachers should provide learners with opportunities to make conjectures and prove them and formulate generalisations, especially with different parameters for different functions. In this paper, I focus on two discourses of algebraic functions that emerged from the analysis of the five teachers' observed lessons: *'Generalising from worked examples'*, and, *'the participationist approach to generalisation'*. The former addresses the lack of learning opportunities for learners to observe the behaviour of mathematical objects to make their own meanings of the topic and generalisations about the effect of different parameters and resonate more with ritualistic discourse. The latter highlights how one of the participants moved more towards the participationist approach to help learners to observe the effect of different parameters for the different families of functions, which *commognitively* links more with explorative routines.

5.1 Generalising from worked examples

While it is expected for the teachers to dominate at the beginning of the lesson when they introduce a new topic and explain concepts, learners should be given a chance to learn for meaning-making than teachers learning for them. The findings indicate that Mafada, Tinyiko, Mutsakisi and Jaden provided limited opportunities for learners to use their cognitive skills to engage with the maths content and concepts critically. As will be seen

in the next section, of the five participating teachers, only Zelda guided learners to construct and verbalise their observation about different parameters' effect to create narratives. Mafada Tinyiko, Mutsakisi and Jaden on the contrary, used the 'transmission style' to narrate the procedures that learners ought to memorise and the generalisations about the effect of different parameters for different families of functions covered during the lessons. Their teaching approaches could be linked with the Freirean (1970) banking model of education, which reinforces a lack of knowledge ownership and critical thinking in learners.

The following excerpts are examples of the four teachers' generalisation statements relating to the effect of different parameters of algebraic functions, and the numbering will be used in the subsequent discussion of the narratives:

1 "now we have seen that when the sign is positive, where is our graph facing? Our graph is facing up".

2 "... if a is positive the graph faces up, so the graph smile, when a is positive, our graph is a smile".

3 "because you were given the function as ax^2 , the coefficient of your x squared is positive, it simply tells you that your graph will go up".

4 "... it is because of that number that when it is increasing, the graphs are coming closer to the x -axis".

Of importance to note is that the above statements were uttered without giving learners opportunities to observe and explore the behaviours of the functions when values of the parameters are changed to construct their own meanings about the concepts. Thus, the statements represent teachers' dominance and non-dialogic/authoritative approach during the lessons, which links closely with ritualisation (Caspi & Sfard, 2012). The narratives "our graph is facing up", "... now we have seen that when the sign is positive ... our graph is facing up" (Mafada), "if a is positive the graph faces up ..." (Mutsakisi), "... it simply tells you that your graph will go up" (Tinyiko) and "... the graphs are coming closer to the x -axis" (Jaden) illustrate that teachers were *telling* learners what the effect of the different parameters are rather than *teaching* them (Hawes, 2004). Telling entails the teachers' simply feeding learners with the information relating to the effect of parameters on the different families of functions without necessarily understanding what the teacher is doing and why (Hawes, 2004; Sullivan, 2011). From the focus on "what" facts learners need to focus on without the "how" they are important and "why" is important to know them, the teachers did not help learners understand the inherent principles of what they are to do, why they should use a specific procedure and how they should make meaning of the topic irrespective of the questions. Schoenfeld (2012, p. 592) states that it is important for teachers to enable learners' "predilection to explore, to model, to look for structure, to make connections, to abstract, to generalise, to prove" to ensure that learners co-construct mathematical meanings for themselves. Thus, of concern is there were no "opportunities for learners to explore, reflect upon and share their developing understanding" about the effect of the parameters (Killen, 2015, p. 47). One could argue that the teachers' teaching of algebraic functions therefore, does not allow learners to engage in meaningful mathematical learning that supports progression towards formal knowledge set out in CAPS.

Furthermore, Mafada, Tinyiko, Mutsakisi and Jaden generalised the effect of the parameters from new examples without allowing learners opportunities to experiment with different parameters and formulate interpretations and generalisations for themselves. Suppose the purpose is to ensure learners' conceptual understanding. In that case, it is crucial that teachers create opportunities for learners to observe the effect of parameters and construct conjectures and prove them for themselves as this is foundational in facilitating internalisation of contents (Vygotsky, 1978; DoE, 2011). Hawes (2004, p. 47) posits that "rather than telling students what we know, we need to ask questions that lead students along the path of learning". The extremely ritualised ways teachers used during the observed lessons is problematic, especially the verbalisation of

the generalisations without allowing learners to make sense of the changes in the parameters and their effect thereof. As also prescribed by CAPS (DoE, 2011), teachers should allow learners to engage in mathematical reasoning, observe the effect of parameters, formulate conjectures, and prove them to guide learners towards generality about the effect of different parameters for different families of functions. Accordingly, instead of dominating the teaching and learning processes, teachers need to create effective learning opportunities to enable learners' epistemological access to mathematical concepts and meanings. In the following section, I present Zelda's teaching approach, which links more with the participationist way of working with learners.

5.2 The participationist approach to generalisation

In contrast to the four teachers in the above sub-theme, Zelda used a more participationist discourse and approaches during the lessons, which is seen as "patterned collective doings", whereby learners are active participants in the mathematisation processes (Sfard, 2013, p. 6). According to Caspi and Sfard (2012), Zelda's teaching resonates well with the explorative routines, to allow learners to construct their own meanings relating to the concept of functions. Considering that learners are formally learning the concept of algebraic functions for the first time in Grade 10, they ought to experience instances where teachers allow them to make their own observations, conjectures, prove them and construct generalisations about functions. In the study reported in this paper, Zelda's interactions with the learners helped learners to exhibit functional thinking and verbalised narratives about the behaviour of given functions. Consider the following extract, for example:

Let us go back to the three graphs, what can you say about the effect of a? What can you see from the graphs? now, let us talk, we talk about the white and the blue, in Grade 9, I remember teaching you about the Cartesian plane, I said (drawing a sketch of a Cartesian plane), this is quadrant one, this is quadrant two, third quadrant and fourth quadrant. Let us check the white arcs, the white arcs are in which quadrant? First and third quadrant, okay let us talk now, going back to our a, I want to talk about when a is greater than zero and when a is less than zero (she writes a greater zero and a less than zero). What is happening when a is greater than zero? Greater than zero is one up.

This statement illustrates the learning opportunities Zelda created for learners to make conjectures and generalisations about the effect of parameter a on the functions. According to Molefe and Brodie (2010, p. 9), "creating room for learners to explain themselves is a practice that goes with reform approaches". Of importance to emphasise is that the above was not an isolated event, but across the different lessons I observed Zelda teach, the participationist approach for generalisation was evident. There were many opportunities that the teacher created to allow learners to observe and interpret the relationship between quantities using different forms of representation, the graphical representation in the above statement. The following classroom exchange between learners and the teacher further illuminates the dialogic-non-authoritative approach Zelda used during teaching:

5 Zelda: *Before I go to the Cartesian plane I forgot something, let's look at the table, as the x values are increasing, what is happening, let's start with this one (pointing at the y values for $y = \frac{1}{x}$)*

what is happening here? let us check the table, as these x values increase, what is happening with these y values?

6 Learners: *(chorusing) increasing!*

7 Zelda: *it is also increasing neh! And then, this one? (pointing at y values for $y = -\frac{1}{x}$).*

8 Learners: *(chorusing) decreasing neh!*

9 Zelda: *let us put this in our heads, with the first one, if x is increasing, even the y is going to increase. This one if x is increasing, this one is decreasing (pointing at y values for $y = -\frac{1}{x}$,*

we will come back and speak about this, remember yesterday I gave you a general form where I talked about y equals to a over x or y equals to negative a over x (writing the equations on the board), we will see the graph and talk akere (isn't).

Zelda's classroom observable actions demonstrated that she views mathematical meaning as "a product of social processes, in particular as a product of social interactions" (Voigt, 1994, p. 276). The repetition of the question, "*what is happening?*" in line 5 and the choice of words "*let us put this in our heads*" in line 9 could be interpreted as inviting learners to be active co-constructors of mathematical meanings, thereby eluding to the explorative way of teaching and learning. To this effect, the 'participationist pedagogy' in Zelda's classroom during the teaching of the function concept presents learners with learning affordances to make sense of reasoning and internalise mathematical meanings. I position this teacher to be explorative in her approach because learners were given opportunities to observe the changes brought by the variation in parameters for themselves and were encouraged to verbalise their observational statements and move towards generality about such parameters. As suggested by Accordingly, the instances of generalisation in Zelda's lessons highlights that there is presence of mathematically productive interactions in her teaching, as well as presence of mathematical activities, which includes analysing and justifying the nature of the parameters, of which from the commognitive perspective links more with explorative routines (Caspi & Sfard, 2012).

In view of the above discussion, what sets Zelda's teaching practices from the other teachers' is the thoughtful participation and the fact that her learners responded positively to prompts about the effect of parameters and the teacher made good strides in helping learners to make sense of their routines and guiding them to use endorsed narratives. The nuances that emerged between Zelda and the other teachers' teaching practices, are a possible lever to shifting learners towards explorative discourse and enable learners to make their own meanings. It follows from the Vygotskian constructivist position and the commognitive theoretical view (Caspi & Sfard, 2012), that the teachers' role is not to deposit mathematical contents into the minds of the learners. Instead, suppose the urgency to enable learners' problem-solving skills is seriously considered. In that case, teachers should create teaching and learning situations in which learners construct interpretations about the effect of parameters for themselves (Omodan, 2019). To be clear, I am not positing that the learners are expected to discover all, or even most, of the mathematical knowledge for themselves during learning. Teachers should structure classroom activities that facilitate learners' active participation during learning.

6. Conclusion and recommendation

In this paper, I have used commognition (Sfard, 2008) as theoretical framing to examine five teachers' teaching practices during algebraic functions lessons. The tenets of commognitive theory adopted in this article and the characterisation of mathematical discourse, especially routines, helped understand the nature of teachers' thinking and communication about algebraic functions concepts. In particular, I have used the analytic constructs of ritual and explorative routines to discuss how teachers taught the concept of functions. Suffice it to say that in teaching algebraic functions, it is essential for teachers to allow learners to actively observe the effect of parameters on the behaviour of functions, interpret what they observe, make conjectures and prove them as a way of helping them towards generality about the functions. The findings presented in this paper demonstrate that drill and practice is the most common teaching approach adopted by the participants. From the classroom observations and as anticipated, traditional forms of mathematics teaching predominated teachers' teaching, especially for Mafada, Tinyiko, Mutsakisi and Jaden. The observations were not surprising since traditional forms of teaching are common in South Africa and worldwide (Nachlieli & Tabach, 2019). I argue that if the goal to enable learners' mathematical thinking and, in turn, improve learners' performance in the subject is seriously considered, teachers need to create more explorative classroom situations to ensure epistemological access.

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