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Mathematics Discourse in Instruction During Functions Lessons in Grade 10 Rural Classrooms

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Abstract: Despite the global significance of functions in mathematics education, there has been limited research on how South African rural teachers address specific mathematical concepts related to functions. This qualitative multiple case study, conducted within a post-structural paradigm, delved into the discourses of Grade 10 teachers during functions lessons in rural classrooms. The study focused on the teaching of mathematical concepts related to functions and involved five mathematics teachers from rural school sites in Acornhoek, Mpumalanga Province, South Africa. These teachers were selected based on their experience in teaching mathematics. The research employed semi-structured interviews, unstructured classroom observations, and Video-Stimulated Recall Interviews with all five teachers. By narrowing the focus to one teacher's lesson, this paper analyses the presentation of functions concepts using the Mathematical Discourse in Instruction (MDI) framework. The study revealed that clear teaching materials and learner-engaging instructional methods enable meaningful engagement and concept internalisation. Through the visible use of the components of the MDI framework, the teacher successfully engaged learners, clarified misconceptions, and encouraged learners' participation, ultimately enhancing the coherence, and understanding of mathematical concepts related to functions in the Grade 10 classroom.

Keywords: Functions lessons, mathematics discourse, mathematics instruction, rural teaching.

1. Introduction

Functions are fundamental mathematical concepts that describe the dependency of one variable on another. They are taught in schools to help learners understand the relationships between mathematical quantities. Functions are used to model real-world situations and help us understand the relationships between different variables. Therefore, they are considered one of the most important concepts in school mathematics. Denbel (2015, p. 77) stated that "by the end of high school, learners are expected to have a general understanding of the concept of functions and be familiar with specific types of functions, such as linear, quadratic, general polynomial, reciprocal, power, step, exponential, logarithmic, trigonometric, and piece-wise functions, in different representations."

Denbel's statement applies to the South African school mathematics curriculum. It is worth noting that several studies have highlighted the difficulties learners face in learning and understanding functions (Malahlela, 2017; Moeti, 2015; Mugwagwa, 2017; Ogbonnaya & Mushipe, 2020). Despite the significance of functions in mathematics education globally, limited research has been conducted in South African rural contexts to explore how teachers approach specific mathematical concepts related to functions. Learners often struggle with functional thinking, especially when it comes to interpreting linear functions, as pointed out by Ogbonnaya and Mushipe (2020) and Sehole et al. (2023). Our concern in this study is that there is a lack of research within the South African context on how teachers present function concepts and skills to learners, particularly in rural schools, due to

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the lack of Mathematics Education research in those contexts (Venkat et al., 2009). To address this research gap, we examined the discourses of Grade 10 rural mathematics teachers during functions lessons, aiming to understand how their pedagogical actions facilitate or hinder learners' access to function concepts. The study focused on the discourses of Grade 10 teachers during functions lessons in rural classrooms, emphasising the importance of functions in the South African curriculum.

The following research questions underpinned the current study:

- How do teachers introduce the object of learning during functions lessons within rural classrooms?
- What are Grade 10 rural teachers' explanatory talks during functions lessons?
- How do teachers facilitate learner participation during functions lessons?

The research questions were identified based on our understanding that learners acquire functional concepts and skills through the teacher's discourse during mathematics teaching in the classroom.

1.1 Understanding the teaching of functions in South Africa

A function, as defined by Ubah and Bansilal (2018), is an expression that describes the relationship between two or more variables, where an independent (input) variable has exactly one dependent (output) variable. The Curriculum and Assessment Policy Statement (CAPS) has shifted its focus from teaching learners how to sketch graphs of functions to analysing, interpreting, and performing calculations based on the information provided on function graphs (Mushipe, 2016). The Department of Basic Education (DBE, 2011) prescribes that teachers teach the concept of a function, where a certain quantity (output value) uniquely depends on another quantity (input value). They should also work with relationships between variables using tables, graphs, words, and formulas and be able to convert flexibly between these representations (p. 24). This means that learners are no longer expected to simply draw graphs but also to understand the underlying concepts of functions and use different modes of representation to make sense of relationships between given quantities. The curriculum specification above also highlights that functions have a significant feature of integrating various representations such as verbal, numerical, graphical, and symbolic descriptions. This allows the function to be interpreted, communicated, and discussed critically and effectively (Mellor et al., 2018). Therefore, it is crucial for teachers to prioritise teaching functions for conceptual understanding to help learners develop conceptual tools and links for the topic, which will be helpful in future learning (Mellor et al., 2018).

Ubah and Bansilal (2018) argue that teaching functions require teachers to provide learners with epistemological access to the concepts involved as well as the associated skills so that learners can construct meanings for themselves. Learners are expected to comprehend the effects of several parameters on the graphs of different families of functions, which puts further demands on teachers (Ubah & Bansilal, 2018). Thus, it is essential for teachers to shift learners' focus from seeing "x" and "y" as knowns and unknowns to conceive them as variables and constants for a meaningful understanding of functions (Mbhiza, 2021, p. 24). As indicated above, it is important for learners to understand that different representations of the same function, such as tabular, algebraic, and graphical, are equivalent forms of representing given quantities, and they should be flexibly translated between them to make sense of given relationships. Our concern is how the dearth of mathematics education research within rural and farm schools has not offered insights into how teachers in those contexts make mathematics knowledge available for learners to learn. Thus, we argue for the need to expand the research locale for mathematics education to include researching with rural mathematics teachers to determine whether those teachers have the necessary tools for effective mathematics teaching, particularly for functions which could significantly impact the quality of education provided to learners in these areas. In the following section, we discuss the theoretical framing that we espoused for the study, Adler and Ronda's Mathematical Discourse in Instruction (MDI), and its operationalisation in the current paper.

1.2 **Mathematics discourse in instruction**

The MDI framework is grounded in Vygotsky's (1978) sociocultural theory, specifically emphasising the principles of mediated learning and scientific knowledge. This framework, as described by Adler and Ronda (2015), consists of four essential components. The first component relates to the object of learning, which focuses on the educational goal and specifies what learners are expected to understand by the end of a particular instructional session. Adler and Ronda (2015) explain that the object of learning may include a procedural skill, conceptual understanding, or mathematical practice that educators aim for learners to comprehend and internalise. The second component, exemplification, involves the intentional selection and arrangement of illustrative examples, tasks, and representations used by teachers to highlight the object of learning for learners (Adler & Ronda, 2015; Adler & Ronda, 2017). This study gives attention to how educators utilise variations and invariances in their choice and sequence of examples to facilitate learners' comprehension and structuring of Functions concepts, and proficiencies at the Grade 10 level. To operationalise the third component, explanatory talk, Adler and Ronda (2015) draw upon Sfard's (2008) commognitive framework, which emphasises the importance of language use in mathematics instruction. This component includes teachers' verbal and written explanations during lessons, as well as the reasoning behind mathematical assertions (i.e., the construction of mathematical knowledge). The final component, learner participation, highlights the significance of providing opportunities for learners to actively engage in the co-construction of mathematical knowledge. This component focuses on learners' verbal and practical contributions regarding mathematical concepts and skills throughout instructional sessions, specifically during functions lessons for our study. The interplay among these four components and their interconnectedness is visually depicted in Figure 1.

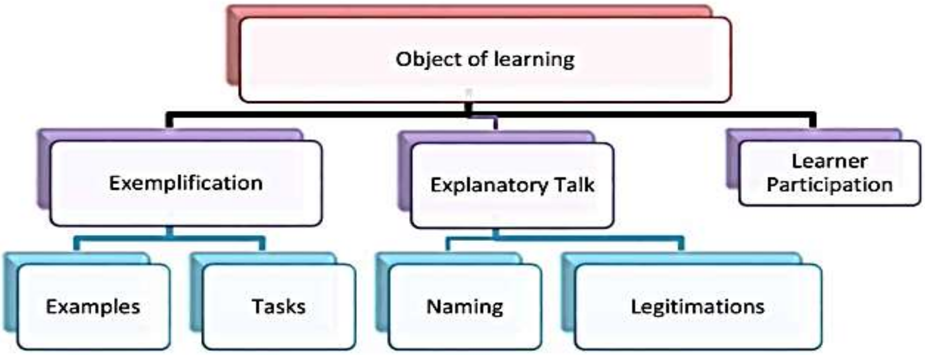


Figure 1. *MDI's components and how they interact (Adapted from Adler & Ronda, 2015, p.239).*

Due to the inherent static characteristics of visual representation, it is inadequate in depicting the temporal progression and sequential nature of teaching and learning processes. As a result, our analysis focuses on the longitudinal evolution of each observed instructional session and the specific aspects that can be illuminated through the application of the MDI framework.

2. Research Methodology

This qualitative, post-structural multiple case study investigated the discourses of Grade 10 teachers during functions lessons in rural classrooms. The study involved five mathematics teachers from distinct rural school sites in Acornhoek, Mpumalanga Province, South Africa. The research design, guided by Creswell (2013), aimed to explore the discursive practices of rural Grade 10 teachers within a specific context. The teachers were purposefully selected based on their experience in teaching mathematics and their current involvement in Grade 10 mathematics instruction. The study utilised semi-structured interviews, unstructured classroom observations, and Video-Stimulated Recall Interviews with all five teachers. For this paper, the focus is narrowed down to one teacher's lesson in order to critically analyse how mathematical concepts related to functions were presented to learners using the MDI framework. Before the study could commence, we applied for ethical clearance from the University of the Witwatersrand and the Mpumalanga Department of Education. The participants were assured anonymity and confidentiality, as well as the right to withdraw from the study for any reason.

2.1 Data analysis

After transcribing the observed lessons word for word, we employed a technique known as horizontalisation, which is defined by Creswell (2013) as understanding the general ideas, noting key points, and considering the overall organisation of the data. Our process involved a thorough review of video-recorded lessons from each teacher, including multiple viewings, detailed transcriptions, and summarising the content. By carefully examining the transcripts, we were able to identify discourses and strategies for teaching algebraic functions, which enhanced our understanding of the teaching process and communication effectiveness. In order to categorise and compare the data from the observed lessons and participants, we structured the classroom observations into episodes based on changes in topics or key learning concepts. This segmentation facilitated initial coding and led to a typological analysis for the development of distinct categories. Typologies, derived from theories, common knowledge, or research objectives, help to classify and clarify the data (Hatch, 2002). During the analysis phase, the teachers' teaching objectives and explanatory approaches for functions were extracted and visually represented in poster charts. Axial coding was used to establish codes and categories that were rooted in the teachers' instructional practices (Creswell, 2013). To assess the teachers' communication with learners, we applied the Mathematical Discourse in Instruction framework to identify the core components of the teachers' discourses and to evaluate teaching effectiveness across the lessons.

3. Findings and Discussion

The teacher started the lesson by writing "The Parabola" on the board (image 1) and introducing it as the topic for the day. He clarified the objective by stating, "Today, our focus will be on the Parabola," to the class.

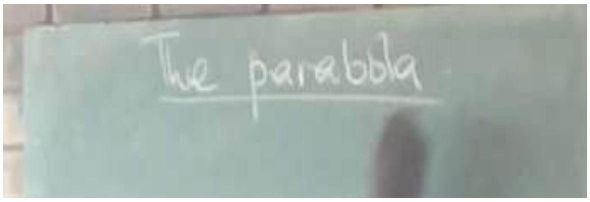


Image 1: *The object of learning*

The teacher introduced Grade 10 learners to parabolas in the form $y = ax^2$ and $y = ax^2 + q$. The emphasis was on investigating the parameters *a* and *q*. Clear learning objectives were set to guide the learners' focus and understanding (Adler & Ronda, 2015). By prompting learners to recall methods for sketching straight lines, the teacher aimed to connect the current and prior lessons, fostering continuity and coherence in mathematical learning.

1. *T: Now, today I want us to look at the Parabola, the parabola, now for you grade 10's we will be basically dealing will the parabola of the form $y = ax^2$ and the parabola of the form $y = ax^2 + q$. Basically, these are the types of parabola that we will be looking at, and dealing with the parabola basically we will be dealing with the investigation here, of these two parameters a and q. As to whether what effect does it have on the graph. Now, as we were sketching those straight lines, you still remember the methods that we applied in order for us to sketch those straight lines….*
2. *T: Which method did we use?*

3. *L1: (inaudible)*
4. *T: y is equal to x (revoicing what a learner said), I want the method, that is the graph, y is equal to x. Which method did we use in order to sketch the graph defined by that equation?*
5. *Ls: the table of points (inaudible)*
6. *T: the table of points, right*
7. *Ls: Yes*

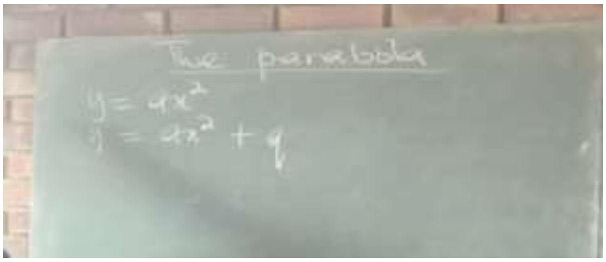


Image 2: *The general equation for parabolic functions*

In lines 2-7, the teacher asked the learners about the method for sketching linear functions. Despite one student's response being inaudible ("y is equal to x"), the teacher repeated it for the class. Instead of dismissing the answer, the teacher restated the question for clarity and explained that "y is equal to x" is an equation, not a technique for sketching. This rephrasing and clarification showed the teacher's engagement with student participation, which is crucial for understanding mathematical concepts (Dlamini & Essien, 2023). This interaction led to more student responses and involvement, and all students correctly identified the table of points. However, the teacher missed connecting this method to sketching parabola functions later and chose to discuss the impact of parameter "a" directly, which could potentially confuse the learners about the purpose of the table of points discussion.

In the excerpt below and image 3, the teacher informs the learners that they will explore the effects of parameter 'a' in quadratic equations. The teacher highlights the significance of parameter 'a' and not 'q', which helps the learners focus on the former. The teacher then asks the learners, "*If we say 'a' is greater than zero, what does that mean?*" and rephrases the response one of the learners gave. The teacher confirms the learner's response and then encourages more engagement. Another attempt to answer was made by the learners, but it was incorrect. The teacher asked the same question four times, "*What does it mean*?" In lines 8, 10, and 14, he continuously called on his learners to be co-constructors of mathematical knowledge relating to parabolas using prompting questions 'what does that mean?' and 'What does it mean?', calling on his learners to construct meanings and generalise the effect of parameter *a*. This is important to allow learners to make meanings and internalise the mathematical concepts (Adler & Ronda, 2015; Sfard, 2008). Then tries to relate it to the effects of the gradient 'm' in linear functions to help the learners link it to the coefficient 'a' in quadratic equations. However, the correct answer was not given by the learners, and the teacher then explained, "This simply means that 'a' is positive, right, because all the numbers greater than zero are positive, right? Now, if we say 'a' is less than zero, that means 'a' is negative." The learners responded in unison, indicating that they were paying attention and engaged in the lesson. Moreover, their high level of participation in this lesson shows that they were eager to learn and actively absorbing the material presented to them.

8. *T: Right, now what are we going to do today, I would like us to look at the effect of that parameter a, because we are going to deal with a situation here, where in it is possible that a will be greater than zero, or a is less than zero, If we say a is greater than zero, what does that mean? (silence), What does it mean?*
9. *Ls: speaking all at the same time but inaudible*
10. *T: Yah, if you say a is bigger to zero what does it mean?*
11. *Ls: giggling*
12. *T: It means value ya (of) a ke (is) zero (revoicing)*
13. *Ls: how? No*
14. *What does it mean? I explained that when we were dealing with the straight line there, wherein we are dealing with that parameter of m (writing on the board $m =$) which was the gradient. There is something which I said about the gradient, that if the gradient is greater zero and if the gradient if less than zero, and I explained this to you. If you say a is greater than zero, what does it mean?*
15. *Ls: mute*
16. *This simply means that a is positive, right, because all the numbers greater than zero are positive, right. Now, if we say a is less than zero, that means a*
17. *Ls: is negative (in chorus)*
18. *T: So, we are going to look at those two dynamics, what happens to the parabola if the value a is positive and what happens to the parabola if a is negative*
19. *Just like a straight line, we started with a simple equation there for straight line which was the equation y is equal to x ($y = x$), this is the parabola we started with, right.*
20. *Ls: Yes*
21. *And we ended up adding on those parameters so that we get to see how they affected the graph.*

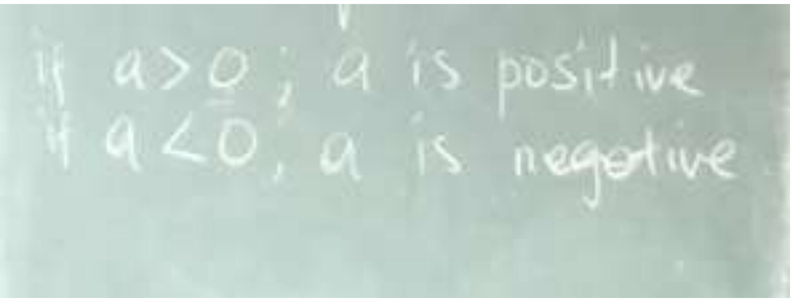


Image 3: *Teacher explaining the value(s) of 'a'*

In lines 22-25 of the text, the focus is still on the parameter 'a', which is defined as the coefficient of x squared in the graph of y equals x squared. The teacher introduces a mathematical term to refer to this parameter, as studies have shown that using the correct mathematical language helps learners understand mathematical concepts better (Adler & Venkat, 2014; Sfard, 2008). To test the learners' understanding of what the coefficient is, the teacher asks, "If I write something like $y = 2x^2$ (image 4), what is the coefficient of x squared?" The learners respond that the coefficient is two, demonstrating the importance of constantly assessing whether learners understand the concepts taught in mathematics. Also, the learners were given an equation $y = x^2$ and were asked to find the value of 'a'. They correctly answered that a = 1. However, when the teacher questioned them about how they arrived at the answer, they were unable to explain their reasoning. The teacher used a practical example involving chalks to help the learners understand the concept (in lines 31-40). It is important for teachers to ask "why" questions to encourage learners to speak mathematically and to display their mathematical reasoning skills (Adler & Ronda, 2015).

22. *T: Now, for us today I would like us to look at the graph of y is equal to x squared ($y = x^2$). Now, lets come back to this, lets look at that variable a, a is the co-efficient of x squared, right, so if I write something like y is equal to two x squared ($y = 2x^2$). What is the coefficient of x squared there?*
23. *Ls: is two (in chorus)*
24. *T: That means these two represent what? Our a, right, so if you have the equation y is equal to x squared ($y = x^2$), what is the value of a then?*
25. *Ls: One (in chorus)*
26. *T: Why do you say one?*
27. *Ls: because one it is not written in exponents (one of the learners responded)*
28. *T: It is only in exponents, Is it only in exponents?*
29. *S: mute*
30. *I normally do this, neh, what am I holding in my hand?*
31. *Ls: A chalk*
32. *I want you to listen to yourself as you answer me. What am I holding in my hand?*
33. *Ls: A chalk*
34. *T: You said it is a chalk neh, that means?*
35. *Ls: One (in chorus)*
36. *T: One, but now if I go on to put this one on, what am I having here?*
37. *Ls: Two chalks (in chorus)*
38. *T: Two chalks right, now if you say a chalk, that means it is one chalk right, so we are having this thing here, the co-efficient of x squared here will be………*
39. *Ls: One (in chorus)*
40. *T: One, right, and this one (1) which happens to be $1 > 0$, which means the value of a there is positive, greater than zero.*

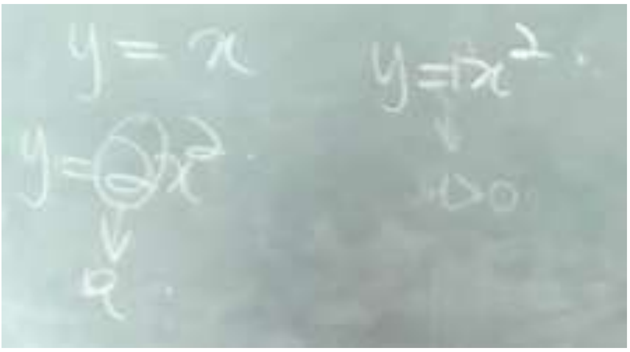


***Image 4**: identifying the value of 'a' in the given equations*

After giving learners a short activity to identify the value of 'a', the teacher stated that, "so what we are going to do now, just like a straight line y is equal to x we are going to design a table of points, right, and then out of the table of points then would have all those coordinates there and then we just plot on the Cartesian plan so that we get to see the shape of the graph, okay." The teacher is explaining how to plot a linear function by designing a table of points and plotting the coordinates on a Cartesian plane. He recaps the process for the learners, as this will be the same process to sketch a parabolic function of the form $y = x^2$ (Image 5). This statement explicitly highlights the purpose of the upcoming activity.

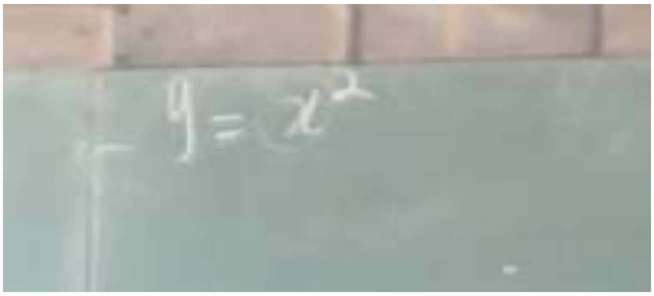


Image 5: *The equation of the parabola to be sketched*

After writing the equation on the board, he then asks his learners "*what do we call that variable x?*" then draws a circle around x, as indicated in image 6. There was no response from learners, and he then requested them to draw from their prior knowledge in straight lines, and asked again what is that x? Still no answer; after asking for the third time he then said, "*Is the independent variable or the impute value*." Thereafter asked, "*what about the y values? Learners mumbles something inaudible, and the teachers said "y* is the dependent variable or the output value." It could be possible that he was revoicing what his learners said and added by explaining further "*because the value y will depend upon the values of x*", as we observed that it is one of his good teaching practices to revoice and comment on learners' responses.

The teacher then asked about the y values and the learners mumbled something inaudible. As a part of his teaching practice, the teacher rephrased what the learners said and added further explanation. He explained that y is the dependent variable or the output value because its value depends upon the values of x. Therefore, the value of y would change based on the value of x. The teacher emphasised the connection between the x and y variables rather than simply labelling them as input-output or independent and dependent. This point was also made by Moalosi (2014), who argued that teaching and learning functions should focus on the relationship between variables instead of just determining the dependent values. Overall, the teacher's approach showed his commitment to engaging his learners in a meaningful way and ensuring that they understood the concepts being taught. Consider the excerpts below.

41. *T: So, what we are going to do now just like a straight line y is equal to x we are going to design a table of points, right, and then out of the table of points then would have all those coordinates there and then we just plot on the Cartesian plan so that we get to see the shape of the graph, okay*
42. *What do we call that variable x? (circling the x on the equation $y = x^2$)*
43. *Ls: mute*
44. *T: From your knowledge in straight lines*
45. *Ls: mute*
46. *T: What is that x?*
47. *Ls: mute*
48. *T: Ndlovu tsowa (wake up), what do we call that variable x?*
49. *Ls: mute*
50. *T: Is the independent variable or the input value*
51. *What about the y?*
52. *Ls: inaudible*
53. *T: y is the dependent variable or the output value, because the value y will depend upon the values of x, so what we re going to do now, we are going to design the very same table of points.*

The teacher started by sketching a table of points and allowed the learners to select their own data points. As the learners called out the points, the teacher noted them in the table, as seen in image 6. This method made the learners feel responsible for their own learning and motivated them to actively assist their classmates in learning too (Ayish & Deveci, 2019). As a result, this helped maintain student engagement and interest in the lesson.

54. *T: Let's select those input values*
55. *Ls: called out the values*
56. *T: Now, those are our input values, right, now from here what we are going to do now, we need to find out that for each value of x that we have here what will be the corresponding y values and how are going to find the corresponding y values here?*
57. *Ls: mute*

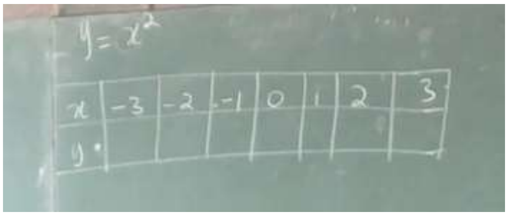


Image 6: *Table of points with input values*

In the dialogue below, the teacher gave the learners an opportunity to determine the output values of a linear function. However, upon noticing that the learners were unable to remember the process, the teacher reminded them of how they had calculated the output values in the previous lesson. The teacher explained that the process involved taking each value of x and substituting it in the equation, which was $y = x^2$ in this case. The teacher then demonstrated how to calculate the output values in line 62. He substituted the first input value, which was negative three in image 7, into the equation and asked what negative three squared was. The learners responded by saying negative nine in line 65 instead of positive nine. It is noteworthy that the teacher may have deliberately asked the learners this question, knowing that this is a common error that learners often make. It is crucial for teachers to understand the misconceptions that learners hold and be able to use relevant teaching strategies to eradicate them (Sarwadi & Shahrill, 2014). This approach is critical in ensuring that learners acquire a deep understanding of the concepts taught, and it helps in building their confidence in tackling more complex problems.

58. *Do you still remember how did we do that with a straight line?*
59. *Ls: Yes (in chorus)*
60. *T: How did we do that?*
61. *Ls: mute*
62. *T: We took each value of x and substituted on the equation, what is our core equation here? y is equal to x squared (underlines the equation $y = x^2$) that is our equation, then we are going to start, the first value of x is?*
63. *Ls: negative 3 (in chorus)*
64. *T: Negative 3, we are going to check here if x is equal to negative 3 then y will be equal to, then what is going to happen here you take that equation and substitute that x by negative 3, then we going to have $y = (-3)$ but the x you see is raised to the exponent of two, therefore we are going to raise this negative three the exponent of two $y = (-3)^2$ and what is negative three squared?*
65. *Ls: negative nine*

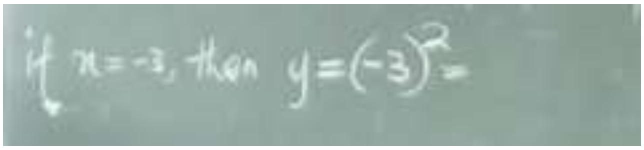


Image 7: *Teacher explaining how to find output values*

After learners provided an incorrect answer of negative 9 to a question. In response, the teacher giggled (line 66) and had a facial expression that suggested they anticipated the mistake, which could be interpreted as a friendly way to invite learners to rethink their responses and continue with active participation. The learner's answer was $(-3)^2$ =9, which is a common error among learners. The teacher promptly reminded the class that they had previously learned about exponents during the first term. Exponents play a vital role in mathematics, and they are introduced in earlier grades. However, the complexity of exponents increases as learners progress to higher grades. In Grade 10, learners study exponents in greater detail and receive more attention because they often make common errors when working with them (Pournara et al., 2016). These errors can sometimes lead to confusion and incorrect answers. Therefore, giving them extra attention and guidance is essential to ensure a strong foundation in this topic. By providing additional support and practice, learners can develop a better understanding and master the concepts of exponents. However, the teacher immediately addressed these misconceptions by making learners to identify the base and the exponent and then showed them that $(-3)^2 = -3 \times -3$ image 8. The teacher's prompt action in correcting the mistake will help the learners better understand exponents and avoid making similar mistakes in the future. Moreover, addressing learners' misconceptions as soon as they arise is crucial because they can persist for a long time if left uncorrected.

66. *T: Negative nine (giggling), no is not negative nine, look if we have a negative three all squared ($(-3)^2$), we have done exponents neh, first term, we have done exponents, so from the knowledge that we gained when we were dealing with exponents if you have a negative three, what is this negative three here?*
67. *S: mute*
68. *Is a base akere (right),*
69. *The two is the?*
70. *S: is the exponent*
71. *What does this exponent tell us about the base?*
72. *S: inaudible*
73. *Thank you very much for saying that, so what does negative three all squared mean? It means negative three multiplied by negative three (-3×-3), and you know that when you multiply negative by negative.*

Image 8: *Teacher explaining why $(-3)^2$ equals to positive nine.*

74. *Ls: Positive*
75. *T: You get a positive, then you take three multiplied by three*
76. *Ls: Nine*

77. *T: You get a nine, so which means is x is negative three then y will be equals to nine $y = (-3)^2 = 9$. Then we write that nine there, that the corresponding value of this negative three will be a nine, right.*

The teacher exemplified a learner-centred approach by actively involving the learners in the process of problem-solving. More specifically, in line 73 of the lesson, the teacher prompted the learners to engage in critical thinking by asking them to recall the rule that governs the multiplication of two negative numbers. The learners responded by correctly stating that the product of two negative numbers is positive, thus demonstrating their comprehension of the underlying concept. Subsequently, the teacher encouraged the learners to apply this rule to the given problem, which entailed multiplying negative three by negative three. The learners successfully solved the problem, arriving at the correct answer of nine. To document this solution, the teacher recorded the accurate answer in the table of points, which was further depicted in image 9.

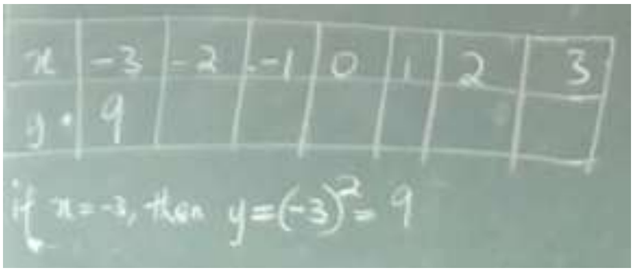


Image 9: *The first output value was recorded*

Following the explanation given by the teacher in image 8 above, the learners successfully determined the remaining output values. The learners called out the computed values one by one, and the teacher jotted them down in the table. This entire process was documented in Image 10, which visually represents the problem-solving process that occurred during the lesson. By documenting the solution, the teacher reinforced the learners' understanding of the concept.

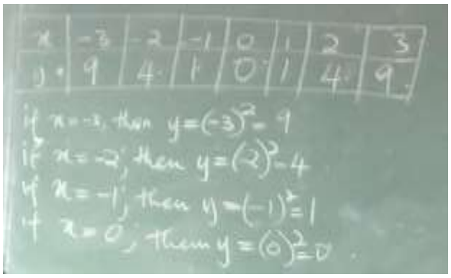


Image 10: *Table of values for* $y = x^2$

After completing the table of points, the teacher proceeded to sketch the Cartesian plane in image 11. The purpose of this was to visually demonstrate to the learners how to represent the input and output values on the plane. By doing so, the teacher provided a clear context for the learners and actively engaged them in identifying and plotting points on the plane, based on the completed table. The learners actively participated in the process by calling out input values and helping to locate the corresponding output values on the Cartesian plane in image 12. Line 85 of the text specifically highlights how the learners assisted in locating the negative x and positive y values by describing their position as halfway between the origin and two. The learners' involvement in translating the point values to the Cartesian plane was significant. The teacher's statement about representing the points effectively emphasised the relationship between the two entities, even though the teacher did not elaborate further on this relationship.

78. *T: Now we have completed our table, so what is going to happen next, now is that we have to represent on the table, on the Cartesian plane right (sketched the Cartesian plane). That's our Cartesian plane, now what we going to do now is to represent the input and the output values on the Cartesian plan, Ok fine, from the table what is the first point there? Three for x and*
79. *Ls: nine for y*
80. *T: Is it three?*
81. *Ls: No*
82. *T: Negative three right, here is out negative three for x (marking it on the cartesian plan) and where is our nine?*
83. *Ls: halfway between eight and ten*
84. *T: Between eight and ten, which means our nine will be somewhere there (marking it on the cartesian plane) so which means we need to plot that point, that will be the point (as indicated on the cartesian plane). And the second point, negative two for x and four for y, here is negative two for x and 4 for y (marking it on the cartesian plane). The next point, negative one for x and one for y. Where is the one? I don't see it there (calling a student by a name)*
85. *Ls: halfway between origin and two (other learners responded on behalf of the identified learners)*
86. *T: halfway between origin and the two, is there (marking it on the Cartesian), and the next point, zero for x and zero for y*

Image 11: *The Cartesian plane drawn*

Image 12: *Points plotted*

Lines 87- 98, after the process of plotting points on the Cartesian plane, to prompt the learners, the teacher then asked the learners what they did after plotting the points but was met with silence in the classroom. To give the learners a hint, the teacher made a leading statement by saying, "*After plotting the points, we just said, okay, that is fine, that is the graph 'line 89*'". This helped the learners remember that they needed to take the next step. One of the learners responded by suggesting that they join the line, but the teacher rephrased that statement, saying, "We do not join the line; we join the points." This clarification was important because it helped the learners understand the difference between the line and the separate points that form it. Then, the teacher joined the plotted points, forming a parabola on Cartesian plane image 13. After creating the graph, the teacher believed they were done, but a student disagreed, stating that something was missing. This prompted the teacher to ask the learners to identify what was missing, and a student correctly responded that the equation of the graph was missing. The teacher then wrote the equation of the graph $y = x^2$ next to the completed parabola image 14. This part of the lesson emphasised the link between the visual representation of a parabola on the Cartesian plane and its corresponding equation. It highlighted the process of joining plotted points to create the graph and emphasised the importance of recognising the graph and understanding its equation. The teacher's emphasis on clarifying the terminology and correcting the misconception of joining the line instead of the points helped the learners better understand the concepts involved.

87. *T: So, after plotting the points, just like with the straight line what do we do next?*
88. *Ls: mute*
89. *T: With a straight line we plotted points neh, after plotting the points we just said okay that is fine that is the graph,*
90. *Ls: No*
91. *T: What did we do?*
92. *Ls (a learner): we join the line*
93. *T: We join the points, eiiii we do not join the line, we join the points, right, thank you. (teacher joined the points to create a parabola)*
94. *T: After sketching a graph like that then we are done*
95. *Ls: No*
96. *T: What is missing?*
97. *Ls: The equation*
98. *T: The equation of the graph which is $y = x^2$ (labelling the graph).*

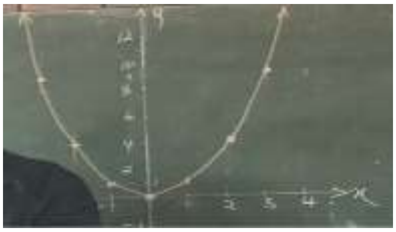


Image 13: *Parabola drawn.*

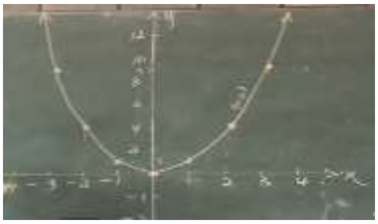


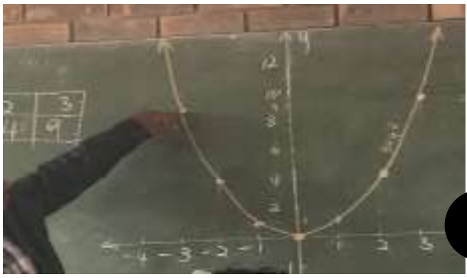
Image 14: *Parabola labelled $y = x^2$*

The exchange below describes a scenario where a teacher is using a visual mediator, a sketched parabola, to explain the effects of the parameter '*a*' in a graph to learners. The teacher first draws the learner's attention to the value of '*a*', which is the coefficient of x squared in line 99. The teacher then asks the learners to examine the graph and determine whether it is facing upwards or downwards. The purpose of this exercise is to explain how the positive value of '*a*' affects the graph of a parabola. The learners correctly responded that the graph was facing upwards. However, the teacher does not explicitly mention how the positive value of '*a*' affects the graph of a parabola. Instead, he discusses the relationship between points on the graph and their reflections. The teacher uses phrases like 'this one' and 'that one' (lines 101-107 and images 15 and 16) when referring to mathematical objects, which may not be clear to the learners and could limit their understanding of the correct mathematical language for talking about points on the graph.

To help the learners grasp the concept of reflection in line 105, the teacher draws an analogy between the graph and a mirror, using a ruler to represent a mirror image 17. The points and their reflections are compared to how one sees themselves in a mirror. This analogy is a useful tool to help learners understand the mathematical principles involved. The teacher's use of real-life examples to explain complex concepts is also worth noting. By using a relatable example, the teacher makes the content more accessible and easier for the learners to understand (Scott et al., 2011). Moreover, visual aids, such as the sketched parabola, can also aid comprehension. However, it is important to note that the teacher's use of ambiguous pronouns, such as 'this one' and 'that one', could hinder the learners' understanding of mathematical language. To ensure that the learners clearly understand the correct mathematical terminology, it is essential to use precise and consistent language when referring to mathematical objects.

99. *T: Now, there is something that I would like us to look at, at this graph the value of a here (pointing at the equation $y = x^2$) is positive neh, let's look at the graph is it facing upwards or downwards?*

100.*Ls: Upwards*
101.*T: It is facing upwards akere (right), there is another thing that I would like you to see, you see this point (pointing at the graph) and that one, do you see them?*
102.*Ls: Yes*
103.*T: Oaky, do you see this one and that one, this one and that one (point at a different point on the graph) (Continued pointing at different points along the graph)*
104.*Ls: Yes*
105.*T: Now, let me ask you this, when you stand in front of the mirror what do you see? When you stand in front of the mirror what do you see?*
106.*Ls: Myself, an image (in chorus)*
107.*T: So, you have a point here (pointing at the point) and this one becomes the image or the reflection or vice versa right*




Images 15 & 16: *Teacher indicating a point and its reflection, "this one" and "that one"*

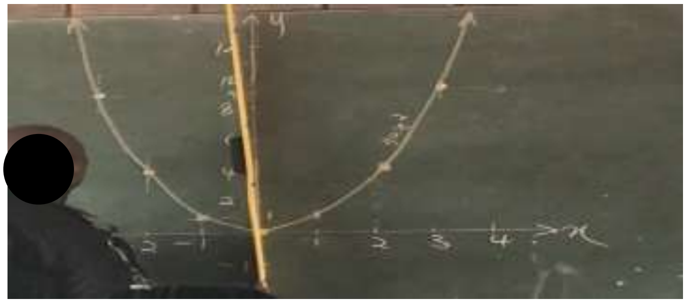


Image 17: *Using a ruler as a mirror to demonstrate a reflection line.*

108.*We are saying this one, because the value of* a *or the co-efficient of* x *squared is negative this graph is facing.*
109.*S: downwards*
110.*Downwards neh, that's the first thing about the graph right.*

In this excerpt, the teacher is discussing the negative coefficient of the squared variable in a parabolic equation. The teacher points to image 18, which depicts a downward-facing parabola, and says, "when we are saying this one, because the value of 'a' or the coefficient of the squared variable is negative, this graph is facing." The learners are expected to complete the teacher's sentence with a one-word answer (Adler & Ronda, 2015), which is "downwards." However, this approach has some limitations. The teacher's method does not allow the learners to actively engage with the content and construct knowledge effectively. The learners are not given the opportunity to compare the given parabola graph with other graphs to understand why it is facing downwards. This approach limits the learners' participation and engagement, which could result in a lack of understanding of the underlying concepts. Instead, the teacher could encourage the learners to explore and compare different parabolic equations with positive and negative '*a*' coefficient to develop a deeper understanding of the concept. By using an interactive and engaging approach, the learners will be better equipped to construct knowledge and make connections between mathematical concepts.

Image 18: *A decreasing parabola when the value of 'a' is negative*

The teacher proceeded to the second concept that he wanted the learners to focus on, which was the reflection points, in line 111. He pointed to the different points on the graph and said, "*This one is a reflection of that one, and this one is a reflection of that one, and then this one is a reflection of that one.*" The teacher used different tones while pointing to the graph to draw the learners' attention. He then asked the learners, "*So, what can you say about that*?" After making those statements, the teacher paused for a moment and asked the learners to comment on them, but they were silent. The teacher's question was ambiguous, especially after making such utterances. The learners seemed to be unsure of what to say or how to respond to the teacher's question. This led to the teacher to answer himself "that means the graph is symmetrical about the $y - axis$" in image 19 and extended his answer by referring to the mirror analogy, this time using learners' home language. The teacher continued the lesson by stating that, "*hotshwana leha ore $y - axis$ ya rona ke enye? Ke sepili, right (It is like saying that our $y - axis$ is a mirror, right), hewena o ema kamo, e eba image yao* (When you stand on this side, this one becomes your image/reflection)." This is important as Setati et al. (2002, p. 134) highlight that several studies have shown that "the use of the learners' first language in teaching and learning mathematics provides the support needed while the learners continue to develop proficiency in the language of learning and teaching." This enabled the learners to understand the concept of reflection. For instance, in line 114, the learners correctly completed the teacher's sentence with "the image". This demonstrates their understanding of the concept and proves this teaching method effectively enhances the learners' understanding of the topic.

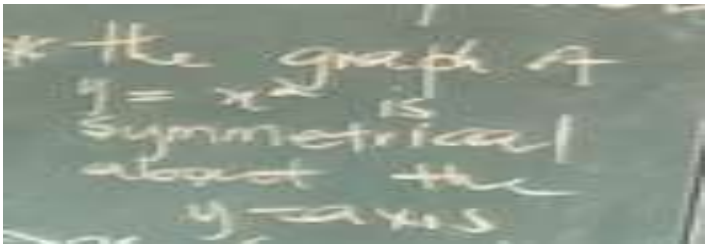


Image 19: *Teacher concluding that $y = x^2$ ia symmetrical about the $y - axis$*

111. *The second thing look at this points neh (pointing at the graph). This one is a reflection of that one right, this one a reflection of that one, and then this one a reflection of that one. So, what can you say about that?*
112. *S: mute*
113. *That means the graph is symmetrical about the $y - axis$, hotshwana leha ore $y - axis$ ya rona ke enye? Ke sepili, right (It is like saying that our $y - axis$ is a mirror, right),*

hewena o ema kamo e eba image yao (When you stand on this side, this one becomes your image/reflection). Hoema kamo (when you stand this side) this one becomes.

114.*S: The image*

Since the teacher used the analogy of a mirror to help the learners understand this symmetry, the teacher used the analogy of the y-axis being like a mirror. The teacher explained that just like how a reflection of a point on one side is seen when standing on the other side of a mirror, the graph has this same symmetry about the y-axis. This analogy helped the learners to visualise the symmetry of the graph. In the following excerpts. The teacher emphasised that, just like in the previous example where the graph was facing upwards, the two graphs are symmetrical about the y-axis but differ in the value of 'a.' In this case, the positive value of '*a*' results in an upward-facing or cup-shaped graph image 20. The teacher uses a creative mnemonic device to help the learners remember the shape of an increasing parabola (Adler & Ronda, 2015). Specifically, the teacher uses a cup as a cue to illustrate the shape of the parabola. The learners can more easily understand and remember the concept by comparing the shape of an increasing parabola to that of a cup. This is a great example of how a creative approach can help learners better grasp complex concepts. This was a crucial point in the lesson, as it helped the learners to understand the significance of the value of 'a' in graphing functions. Finally, the teacher checked if the learners knew what a cup was, and the learners responded affirmatively and with laughter. This teaching approach involves using real-world analogies and interactive questioning to help learners grasp mathematical concepts more intuitively. This approach can help make mathematics more engaging and enjoyable for learners of all levels.

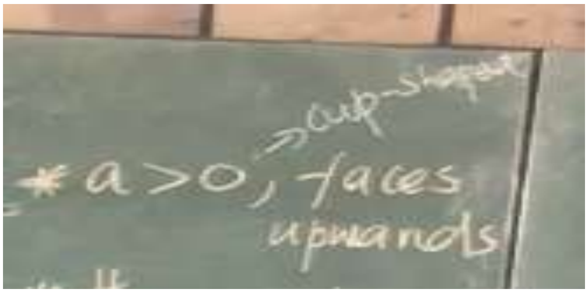


Image 20: *Increasing parabola compared to a cup*

115.*The image right, the same thing this one and that one. Just like the first one, the two graphs neh, are symmetrical about the $y - axis$ but they differ in terms of the value of a. With this one (pointing at the graph) the value of a was greater than zero which means it was positive akere (right) then the graph was facing upwards. If you don't want to say the faces upwards, you would like to say the graph is cup-shape. Do you know a cup*

116.*S: Yes*

117.*What is a cup?*

118.*S: laughing*

The teacher summarised the lesson and then invited learners to ask questions "if there is someone with something to ask, feel free to do so" line 119. At the end of a lesson, it is important to provide learners with a recap of the main concepts covered in the session. This will help them to consolidate and confirm their understanding of the material. By summarising the key points and ideas, learners can review the material concisely and organise, facilitating retention and recall. Additionally, a recap can help learners identify any areas where they may need additional clarification or support, allowing for a more effective and efficient learning experience. Then, he explains the next task. The teacher changed the value of '*a*' in the first graph they drew, which is a parabola facing upwards. This generated the second equation, which is y equals 2x squared. Then, he told the learners that their task now is to use this equation to either design or generate a table of points—then highlighted that this will help them to see what the graph of this equation looks like—alternatively, he suggested that they may use the same table of points that they have been using so far. However, this time, they will use the equation y equals 2x squared, where 'a' equals two. The teacher is using the same equation that was previously worked on but with a different coefficient. By doing so, the teacher is introducing the learners to a similar activity with varying aspects. This approach creates opportunities for the learners to identify the similarities and differences between the two equations. The learners can then make generalisations based on their observations. Furthermore, he mentioned that once they have generated and completed the table of points, they should plot it on the same cartesian plane where they sketched the first graph. Then, he emphasised that their task was to sketch the y equals 2x squared graph so they could compare the two graphs "you are going to sketch this graph (pointing to the equation $y = 2x^2$) so that we get to compare the two, if we increase the value of a what happens to the shape of the graph or what happens to the graph in general". By carefully analysing and contrasting the two graphs, learners can observe the differences in their shapes and overall appearance, as well as their similarities and commonalities. This, in turn, will help them draw more nuanced and insightful conclusions about the impact of the 'x' coefficient on the quadratic equation, as well as the broader implications and effects of 'a'. By paying close attention to the details, patterns, and trends present in the graphs, learners can gain a deeper understanding of the underlying relationships involved.

119. *T: So this one faces downward; that is the effect of that parameter a. Unless there is someone with something to ask, feel free to do so (paused to give learners a chance). Let me change for the first graph you have drawn this one neh (pointing at parabola facing up), let me make the value of a there to be two (pointing at $y = x^2$), just to generate the second equation akere (right), which means we will be having the equation $y = 2x^2$. Now, what I want you to do is to is to use this equation neh, you generate or you design table of points or maybe you maybe the very same table that we have neh, but this time the equation that we going to use is $y = 2x^2$, which means a here is two akere (right) and then you complete this table and thereafter on the same graph, on the same cartesian plane where in you sketched this one (pointing at the graph) you are going to sketch this graph (pointing the equation $y = 2x^2$) so that we get to compare the two, if we increase the value of a what happens to the shape of the graph or what happens to the graph in general, can you do that?*
120. *S: Yes*

After they confirmed that they could do it, the teacher provided them with some additional instructions. The teacher suggested that they could either use the same input values they had generated before or change them, depending on what they thought would be helpful for the task. After that, the teacher erased the output values on image 21 and asked the learners to find the output values or the values of the dependent variable, which is represented by the letter 'y', using the equation $y = x^2$.

Image 21: *Teacher erasing the output values*

By giving this instruction, the teacher was encouraging the learners to explore the graph of $y = x^2$ using different values and not restricting them to use only the values they had already generated. This approach promotes independent thinking and encourages the learners to apply their knowledge to new situations. Then the teacher noticed that even though he had a given a task some learners were folding hands not engaging with the given activity "*ketlore amen bana baka (I will say 'Amen', my kids) because some of you are folding your arms as if you are praying, Amen, let's work it out*". The teacher is calling out on the learners to bring their attention back to the task; Then re-explained the task to the learners. They must complete a table with points, plot the points on a Cartesian plane, and draw a graph. Then, they will compare their graph with these graphs to see what happens if they change the value of 'a' from one to two and observe how the graph changes. Will it become wider or narrower? Will it move up or down or shift to the left or right? Then, he emphasised that these are the things they will see as they sketch the graph. The bell rang, indicating the start of lunch break, but the teacher forbade the learners from leaving until they had finished their task "No, you not going for a break until you are done with this", causing some of them to sigh in response. This might explain why the learners were not fully engaged with the task, as they knew that lunchtime was approaching, which could distract their focus.

121. *T: let's do it; let's see, you will have the table; let's use the very same table if you like, you can reduce it using negative two, negative one, zero, one, and two if you like right. Those are your input values (erasing the output values). What I would like you to do is to find the output values here or find the values of the dependent variable which is y, using the equation $y = 2x^2$. Ketore amen bana baka (I will say 'Amen', my kids) because some of you are folding your arms as if you are praying, Amen, let's work it out. Complete the table after completing the table we take points from the table plot them on the cartesian plane, this one akere (right) (pointing the graph) and then we draw our graph and retlotla re e compare leyo (we going to compare it with this one), hore hare tlosa (that when we change) the value of a from one re (then) increase to two what happens to the shape of the graph? Is it going to be wide, closing, is it going to go up or down, shifting to the left or shifting to the left, those are the things that you will see as you sketch the graph (the bell rang, signalling lunch time). No, you not going for break until you are done with this.*
122. *Ls: aaaaaah (in chorus)*

The teacher insisted on seeing the learners' work before they could break for lunch. He emphasised the importance of accurate calculations to avoid messy graphs. He then walked around the classroom to check the learners' work. During his inspection, he noticed one learner's output value was twenty-four and exclaimed loudly, "Twenty-four, twenty-four, where did you get that from?" The teacher repeated the value twice and was astonished by the learner's answer of twenty-four. He asked the learner to review their calculations. Afterwards, he looked at another learner's work but noticed the learner was hiding it. The teacher asked, "What are you hiding?" The habit of shouting out learners' answers may have made other learners afraid to show their work, fearing embarrassment if their incorrect answers were announced.

4. Conclusion

The teacher effectively conducted the Grade 10 functions lesson by incorporating all components of the Mathematical Discourse in Instruction (MDI) framework. He began by clearly focusing on investigating the effects of parameters a and q in parabolas, aligning with the MDI component of specifying the learning objective. The teacher engaged learners by tapping into their prior knowledge and fostering connections between current and past lessons, as outlined in the MDI framework for learner participation. Despite a minor confusion regarding the method of sketching parabola functions, the teacher effectively revoiced learner responses, clarified misconceptions, and encouraged learner engagement, thereby demonstrating the MDI components of explanatory talk and learner participation. To further exemplify the concepts, the teacher introduced different values of parameter a to guide learners towards a general understanding of the effect of a being negative and the impact on the parabola when a is positive. By effectively utilising all components of the MDI framework, the teacher facilitated a meaningful and coherent learning experience for learners, enhancing their understanding of mathematical concepts related to functions.

5. Declarations

Author Contributions: Conceptualization (H.W.M. & M.S.); Literature review (H.W.M. & M.S.); methodology (H.W.M. & M.S.); software (N/A); validation (H.W.M.); formal analysis (H.W.M. & M.S.); investigation (H.W.M. & M.S.); data curation (M.S.); drafting and preparation (H.W.M. & M.S.); review and editing (H.W.M.); supervision (H.W.M.); project administration (H.W.M.); funding acquisition (H.W.M). All authors have read and approved the published version of the article.

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Conflict of interest: The authors declare no conflict of interest.

Data availability: Due to ethical standards and the conditions outlined in the consent agreement with participants, the data must remain confidential. However, interested persons may contact the corresponding author for more information.

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