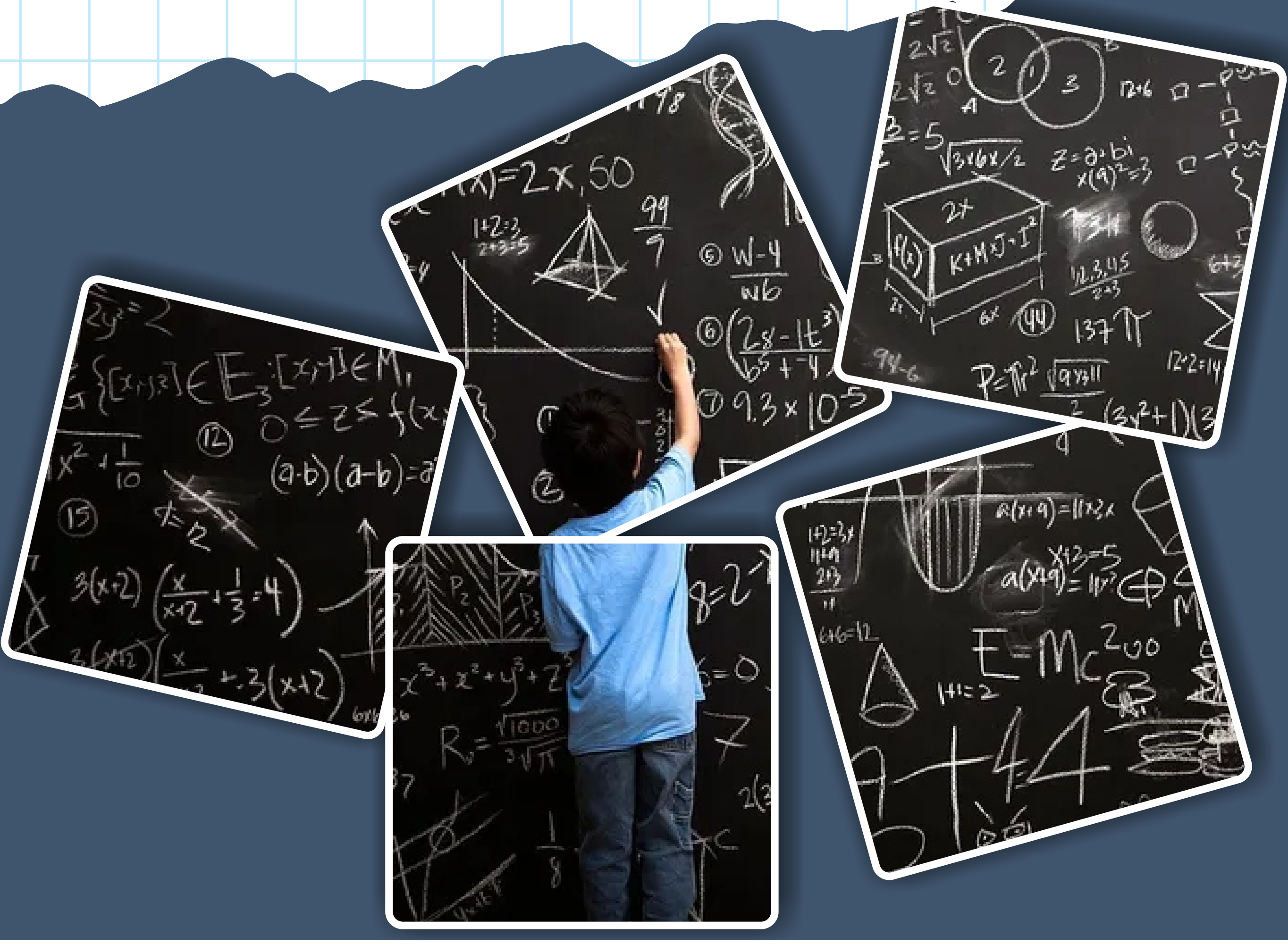
BUILDING THE FOUNDATIONS

Effective Approaches in

Mathematics

TEACHER PREPARATION



ERRCDF
Rethinking Education 4 Inclusive Development

B.E. Olawale

BUILDING THE FOUNDATIONS

Effective Approaches in Mathematics Teacher Preparation

Editor
B. E. Olawale

ISBN: 978-1-0370-5389-4(e-book)
Chapters' DOI: https://doi.org/10.38140/obp3-2025c
Main DOI: https://doi.org/10.38140/obp3-2025
Publication date: March 2025

Copyright and Licensing

Disclaimer

All chapters in this book have undergone a Turnitin review process to ensure quality and prevent possible plagiarism. Every effort has been made to contact all copyright holders to secure copyright permissions where necessary. The views, perspectives, information, and data contained within the publications are exclusively those of the respective author(s) and do not represent or reflect the positions of ERRCD Forum and/or its editor. ERRCD Forum and its editor expressly disclaim responsibility for any damages to persons or property arising from ideas, methods, instructions, or products referenced in the content.

Peer Review Confirmation

The publisher, ERRCD Forum, and the editor affirm that every chapter included in this book underwent a rigorous double-blind peer-review process. This process ensured that each submission was independently reviewed by two experts in the field, with the identities of both the authors and reviewers kept confidential to maintain impartiality. Prior to the peer review, the editor assessed each chapter's quality and relevance to the book's scope. A total of 24 chapters were submitted for consideration in the book, of which 10 were ultimately accepted for publication.

PUBLISHER'S INFORMATION

ERRCD Forum
Harrismith, South Africa

This edited book is published via Open Books and Proceedings, an extension of the ERRCD Forum, committed to advancing knowledge through the publication of high-quality academic books, edited volumes, conference proceedings, and other scholarly monographs. Founded by a collective of experienced scholars, our mission is to ensure that cutting-edge research and evidence-based literature are accessible to all. ERRCD Forum is a registered private and independent publishing house that specialises in open-access publications. Our dedication to open access means that all our published works are freely available to readers, students, and researchers worldwide. We believe in the power of knowledge sharing and are passionate about making scholarly work accessible to a broader audience.

OBP is hosted, archived and maintained by the University of the Free State Library and Information Services and published by the ERRCD Forum. ERRCD Forum is based in South Africa and publishes open-access academic and peer-reviewed books and journals in education, rural and community studies, management and social sciences, among others.

EDITOR'S PROFILE

Babawande Emmanuel Olawale
Faculty of Education
Walter Sisulu University
bolawale@wsu.ac.za
ORCID: https://orcid.org/0000-0001-5265-1583

Babawande Emmanuel Olawale is an esteemed researcher and educator specialising in mathematics education at the Faculty of Education, Walter Sisulu University, South Africa. He obtained a PhD in Education with a focus on mathematics education from the University of Fort Hare. His research delves into the intricacies of teaching and learning mathematics, with a particular emphasis on the training of mathematics teachers and the integration of ethnomathematical perspectives. In the domain of mathematics teacher preparation, his work examines the enhancement of teachers' mathematical knowledge, pedagogical beliefs, and classroom practices. His research contributes to the development of teacher education programmes that cultivate reflective and effective mathematics educators. Over the past five years, he has published over 50 journal articles, conference proceedings, and book chapters with reputable publishing houses. He serves as an editor for the Journal of Matematika dan Pembelajaran at Institut Agama Islam Negeri, Indonesia. Additionally, he is an active reviewer for reputable journals such as the South African Journal of Education (SAJE), Perspectives in Education (PiE), and Cogent Education (Taylor & Francis). He is a member of esteemed scientific associations, including the South African Education Research Association (SAERA), the Southern African Association for Research in Mathematics, Science and Technology Education (SAARMSTE), and the Mathematics Association of Nigeria (MAN). His dedication to excellence in teaching and research has garnered him commendations and recognition within the field.

AUTHORS' PROFILES

Joshua Abah Abah
Department of Mathematics, Science and Technology Education
Faculty of Education
University of Zululand, South Africa
ORCID: https://orcid.org/0000-0001-5485-9396

Dr ABAH is an astute academic with an interest in Mathematics Education, Technology of Mathematics Education, Ethnomathematics, Instructional Design, and Statistical Computing. He holds a First-Class Honours Bachelor of Science (Education) in Statistics and Computer Science, a Master of Education in Mathematics Education, and a PhD in Mathematics Education, all from the prestigious Federal University of Agriculture, Makurdi, Nigeria. He is presently a Postdoctoral Research Fellow at the University of Zululand, South Africa, where he researches culture-based instructional design.

Zaheera Jina Asvat
Wits School of Education
University of the Witwatersrand, South Africa
ORCID: https://orcid.org/0000-0002-1095-2449

Dr Zaheera Jina Asvat is an esteemed researcher and educator specialising in mathematics education at the Wits School of Education, University of the Witwatersrand, South Africa. She obtained a PhD in Education, focusing on mathematics education, from the University of the Witwatersrand. Her research explores teacher identity, agency, and belonging in mathematics education, particularly how these factors influence teaching practices and educational outcomes for pre-service teachers.

Frans N. Haimbodi
Faculty of Education & Human Sciences
University of Namibia, Namibia
ORCID: https://orcid.og/0000-0002-7596-6999

Dr Haimbodi is a Senior Lecturer in Mathematics at the School of Education, University of Namibia. He holds a PhD in Mathematics Education from the University of the Western Cape, South Africa. His research interests encompass mathematics education, science education, indigenous knowledge systems, and postgraduate research supervision. He has successfully supervised numerous postgraduate students and has contributed to academia through publications in refereed journals and conference proceedings, thereby advancing research in his field.

Hesekiel K. Iilonga
Ministry of Education, Arts & Culture, Namibia
ORCID: https://orcid.og/0009-0005-6528-0204

Dr Iilonga is the Head of the Department for Mathematics and Sciences in the Ministry of Education, Arts and Culture in Namibia. He holds a PhD in Mathematics Education from the University of Pretoria. His research interests focus on the teaching and learning of mathematics, as well as teacher education. He is passionate about improving mathematics instruction, developing curricula, and enhancing teacher training programmes to strengthen mathematics education in Namibia and beyond.

Winston Hendricks
Faculty of Education
University of Fort Hare, South Africa
ORCID: https://orcid.org/0000-0002-1308-9521

Prof Hendricks is an Associate Professor of Mathematics Education in the Faculty of Education at the University of Fort Hare, South Africa. He obtained his PhD in Education from Nelson Mandela Metropolitan University, focusing on how self-efficacy beliefs impact teachers and learners in relation to their mathematics curricula. His involvement in teacher training provides an excellent foundation for sharing his research on effective academic practices and for developing competent pre-service mathematics teachers.

Sakyiwaa Boateng
Faculty of Education
Walter Sisulu University, South Africa
ORCID ID: https://orcid.org/0000-0002-1308-9521

Sakyiwaa Boateng holds a PhD in Science Education from the University of the Witwatersrand, Johannesburg, South Africa. She is currently a senior lecturer at Walter Sisulu University in Mthatha. Sakyiwaa is an established researcher with a passion for science teacher professional development, particularly focusing on the affective aspects that teachers consider when evaluating the worth of teaching as a profession. She is also working to bridge the gap between science, culture, and meaning, while emphasising 21st-century teaching, science education, and teacher training. Her current research addresses mathematics and science anxiety, aiming to develop interventions that build resilience in mathematics and science among pre-service science teachers and high school science learners.

Abongile Ngwabe
Faculty of Education
Walter Sisulu University, South Africa
ORCID ID: https://orcid.org/0000-0002-8915-5521

Abongile Ngwabe is a passionate Mathematics Education Lecturer at Walter Sisulu University in Mthatha, South Africa. She holds a Master's in Education from Nelson Mandela University in Port Elizabeth. Her research focuses on innovative teaching methods, technology integration in education, and enhancing student learning experiences. Committed to shaping future educators, she explores dynamic approaches to mathematics instruction, including the use of digital tools and pedagogical strategies, to improve teaching and learning in diverse educational settings.

Buhle Stella Nhlumayo
College of Education
University of South Africa, South Africa
ORCID: https://orcid.org/0000-0002-0598-3988

Dr Buhle S. Nhlumayo is a Senior Lecturer in the Department of Educational Foundations at the University of South Africa (UNISA). She has served in the Department of Basic Education for nearly three decades as a Mathematics teacher and Head. Dr Nhlumayo obtained her PhD in Education Leadership, Management, and Policy from the University of KwaZulu-Natal (UKZN). Her research focuses on educational leadership, teacher professional development, comparative education, and ICT in education, all within the context of rural settings.

Kereng Gilbert Pule
College of Education
University of South Africa, South Africa
ORCID: https://orcid.org/0000-0002-6038-4398

Dr. Kereng Gilbert Pule is a Senior Lecturer in the Department of Mathematics Education at the University of South Africa (UNISA). He was a Mathematics teacher in various secondary schools across South Africa for almost three decades. His research focuses on effective mathematics teaching and learning, with particular attention to the transformative aspects of multilingualism and multiculturalism in education. Dr. Pule has contributed to the field through publications that explore effective mathematics teaching strategies.

Khalid Saddiq
Faculty of Education
University of Tasmania, Australia
ORCID: https://orcid.org/0000-0002-3918-8667

Khalid Saddiq is a PhD research student jointly affiliated with the Mathematics Education, Faculty of Education, University of Tasmania, and the Federal College of Education (Special), Oyo. His work centres on online mathematics teaching, e-learning, mathematics education, curriculum and pedagogy, teacher education, the creation of instructional materials, and the development of effective pedagogical practices for primary and secondary school mathematics, as well as tertiary education. Currently, Khalid is investigating Nigerian undergraduate mathematics lecturers' conceptions of online teaching, contributing valuable insights into the evolving landscape of digital instruction and mathematics pedagogy.

Felix Oromena Egara
Faculty of Education
University of the Free State South Africa
ORCID: https://orcid.org/0000-0002-6249-9615

Dr. Egara is a Postdoctoral Fellow at the University of the Free State, South Africa. His research focuses on mathematics teaching and learning, ethnomathematics, ICT in mathematics education, and teacher education. He also explores mathematics anxiety and psychotherapies, including Rational Emotive Behaviour Therapy (REBT), Mindfulness-Based Cognitive Therapy (MBCT), and Systematic Desensitisation (SD).

Moeketsi Mosia
Faculty of Education
University of the Free State, South Africa
ORCID: https://orcid.org/0000-0002-7189-0018

Prof. Mosia is an Associate Professor of Mathematics Education and the ETDP SETA Research Chair at the University of the Free State, where he also serves as Vice Dean (Teaching and Learning). His research focuses on applied mathematics, statistics, ICT in education, and innovative teaching methods, contributing to the advancement of mathematics education through research-driven methodologies.

Linda van Laren
School of Education
University of KwaZulu-Natal, South Africa
ORCID: https://orcid.org/0000-0002-1282-810X

Dr Linda van Laren is an honorary senior lecturer in mathematics teacher education at the Edgewood Campus of the University of KwaZulu-Natal in Durban, South Africa. She holds a PhD in mathematics education, and her research focuses on the integration of HIV and AIDS education across disciplines. Her published research includes a variety of arts-inspired self-study methods. As a member of many self-study transdisciplinary collaborative research teams, she explores social justice issues in the South African teacher education context.

RESEARCH JUSTIFICATION

Teacher preparation programmes play a crucial role in shaping the knowledge, skills, and attitudes of prospective mathematics teachers. However, there is considerable variation in the methods used to train these educators, raising concerns about the effectiveness and fairness of these programmes. While extensive research has been conducted on teacher preparation in general, the specific field of mathematics education remains relatively unexplored. Additionally, existing research offers some insights into the overall impact of teacher preparation programmes on student outcomes, but there is limited information on how different programme components—such as coursework, field experiences, forms of assessment, and mentorship—contribute to teachers' development and readiness to teach mathematics. In essence, this book provides a deeper understanding of how these elements shape the preparedness and effectiveness of future mathematics educators.

There is also a significant gap in the literature regarding the incorporation of cultural competence and diversity training in mathematics teacher preparation programmes. In an era when classrooms are becoming increasingly diverse, it is imperative to investigate how these programmes integrate culturally responsive teaching practices, address implicit biases, and promote equity and social justice in mathematics education. Many teacher preparation models fail to adequately equip pre-service teachers with the skills necessary to navigate diverse learning environments, which leads to persistent disparities in student achievement. This book seeks to bridge this divide by exploring the intersections of mathematics education, social justice, and culturally relevant pedagogy, thereby offering a framework for fostering more inclusive teaching practices that meet the needs of all learners.

To address these pressing concerns, this book presents a comprehensive overview of diverse approaches to mathematics teacher preparation. It offers a collection of empirical research, conceptual analyses, and theoretical contributions that examine innovative strategies for equipping pre-service and in-service mathematics teachers with the necessary knowledge and pedagogical competencies. This volume provides educators, researchers, and policymakers with evidence-based insights into effective mathematics teacher preparation. Furthermore, it highlights the role of technology, mentorship, and field-based learning experiences in shaping effective mathematics instruction, ensuring that teachers are better prepared to engage with contemporary educational challenges.

Ultimately, this book fills a critical gap in the discourse on mathematics teacher education by addressing the evolving needs of the profession and the students it serves. It offers a roadmap for rethinking traditional teacher preparation models and fostering a more equitable and effective landscape for mathematics education. As the global educational community grapples with issues of equity, inclusion, and technological transformation, this book serves as a timely contribution to the ongoing discourse on preparing mathematics teachers for the future. By fostering dialogue among scholars and practitioners, it aims to inspire innovative approaches to teacher preparation that will ultimately lead to improved learning experiences for students worldwide.

PREFACE

The field of mathematics teacher preparation is at a pivotal moment in its evolution. As educational landscapes continue to shift, there is an urgent need for research and pedagogical strategies that embrace inclusivity, social justice, and innovation in teacher training. This edited volume presents a comprehensive exploration of contemporary challenges and solutions in preparing mathematics educators for diverse and dynamic classrooms. Incorporating perspectives from scholars across various contexts, this book provides invaluable resources for educators, policymakers, and researchers committed to improving mathematics instruction at all levels.

This book comprises ten chapters, each addressing a critical aspect of mathematics teacher education, including equity and social justice, self-efficacy, culturally relevant pedagogy, technological integration, and sustainable educational practices. Through a blend of empirical research and theoretical discourse, the contributors highlight the evolving nature of teacher preparation and the necessity of equipping future educators with the skills and mindsets required to navigate complex learning environments. Therefore, foregrounding issues of diversity and inclusion, this book underscores the transformative potential of effective teacher preparation in fostering equitable educational outcomes.

Hence, the book serves as both a catalyst for meaningful change and a reference point for future research in mathematics teacher education. As readers engage with the insights presented in these chapters, they are invited to reflect on how these perspectives can inform their own practices and policies. The collective responsibility is to ensure that mathematics education remains a space where every learner can thrive, and it is inspired that this book contributes meaningfully to that mission.

TABLE OF CONTENTS

CHAPTER ONE

Equity and Social Justice in Mathematics Teacher Preparation: Diving into the Nitty-gritty

Babawande E. Olawale[1]

AFFILIATIONS

[1]Faculty of Education, Walter Sisulu University, Mthatha, South Africa.

CORRESPONDENCE

Email: bolawale@wsu.ac.za*

Reference Formats (APA 7th Edition)

Olawale, B. E. (2025). Equity and social justice in mathematics teacher preparation: Diving into the nitty-gritty. In B. E. Olawale (Ed.), *Building the Foundations: Effective Approaches in Mathematics Teacher Preparation* (pp. 1-21). ERRCD Forum. https://doi.org/10.38140/obp3-2025-01

Abstract: In contemporary education, the principles of equity and social justice have emerged as critical components that shape the pedagogical landscape, particularly in mathematics teacher preparation programmes. This chapter examines how mathematics teacher education programmes prepare pre-service mathematics teachers for equitable and socially just classrooms. It seeks to understand how these principles are integrated into teacher education programmes and their impact on the development of future educators to address diverse student needs in mathematics classrooms. This study employs a qualitative research design, utilising semi-structured interviews with mathematics teacher educators, pre-service mathematics teachers, and heads of departments. Data were collected from two traditional institutions to capture varied perspectives on the incorporation of equity and social justice within mathematics teacher preparation. Thematic analysis was employed to identify recurring themes and patterns in the data, enabling a comprehensive understanding of the participants' experiences and insights regarding the integration of these critical principles in their training. The findings indicate that integrating culturally relevant teaching, enhancing various field experiences, and developing community engagement can enable teacher preparation programmes to produce a new cohort of educators dedicated to social justice and equity in their classrooms. This comprehensive approach is essential for reforming educational processes and ensuring that all students have access to equitable learning opportunities.

Keywords: Equity, inclusive pedagogy, professional development, reform, social justice.

1. Introduction

The educational environment is widely regarded as a vital domain for addressing equity and social justice challenges, particularly in mathematics teaching and learning. As societies become increasingly diverse, the need for teachers who are knowledgeable in their fields and attuned to their students' varied cultural, social, and economic contexts is essential. Consequently, the requirement for mathematics teacher education programmes to effectively prepare pre-service teachers for equitable and socially just classrooms has emerged as a central theme in research and practice (Ladson-Billings, 1995; Gutstein & Peterson, 2005; Ladson-Billings, 2006; Gutiérrez, 2013; Ladson-Billings, 2021; Ladson-Billings, 2023). The foundation of equitable mathematics instruction is grounded in diverse theoretical frameworks that guide teacher education programmes. Critical pedagogy, as defined by Paulo Freire, emphasises education's role as a vehicle for social reform (Freire, 1970). This perspective is particularly relevant in mathematics education, as conventional approaches often perpetuate structural disparities

(Olawale, Mncube, & Harber, 2021; Olawale, 2022; Ladson-Billings, 2023). Thus, Gutiérrez (2013) builds upon Freire's concepts by advocating for a critical mathematics teaching approach that challenges the status quo and empowers students to use mathematics as a tool for social justice.

Furthermore, the paradigm of culturally responsive pedagogy, established by Ladson-Billings (1995), offers a framework for analysing teacher education. Culturally relevant pedagogy highlights the importance of integrating students' cultural backgrounds into the curriculum, fostering a sense of belonging, and promoting academic success. Studies show that pre-service teachers trained in culturally responsive methodologies are more adept at meeting the diverse needs of their learners (Villegas & Lucas, 2002; Premier & Miller, 2010; Skepple, 2014; Olawale, Mncube, & Harber, 2021; Davis, 2022). Therefore, equity and social justice in education involve recognising and addressing systemic disparities that affect student learning experiences and outcomes (Gutstein, 2007; Ladson-Billings, 2021). In the context of this study, the concept of equity in mathematics education refers to ensuring that all students have access to high-quality mathematical instruction, resources, and support, regardless of their background, identity, or circumstances. This notion encompasses equal access to advanced courses, the provision of appropriate resources, and the adoption of inclusive teaching approaches that address diverse learning needs.

Similarly, social justice is a comprehensive concept that includes equity and seeks to rectify the systemic barriers and injustices preventing many students from achieving mathematical excellence. Consequently, equity primarily focuses on establishing equitable access and opportunities in mathematics education (Gutiérrez, 2013; Civil, Hunter, and Crespo, 2019), while social justice encompasses a broader range of societal concerns within the realm of mathematics education (McGee & Hostetler, 2014; Bartell, 2018). In mathematics education, these ideas challenge conventional teaching methods that often sustain gaps in achievement and engagement among students from diverse backgrounds. Therefore, incorporating equity and social justice into mathematics teacher preparation is essential and aligns with comprehensive educational reforms aimed at promoting inclusive and equitable learning environments.

However, despite an expanding body of literature emphasising the significance of equity and social justice in mathematics education (Gutiérrez, 2011; National Council of Teachers of Mathematics, 2014; McGee & Hostetler, 2014; Leonard, 2018), notable research deficiencies persist regarding the pedagogical practices employed in the preparation of mathematics student teachers for equity and social justice classrooms within teacher training programmes. Moreover, the current literature often overlooks the perspectives of mathematics student teachers, who may provide essential insights into their readiness to foster equitable learning environments. Hence, the need for this chapter.

3.2 The concept of equity in the preparation of mathematics teachers

The training of mathematics teachers has received significant attention in educational research, particularly regarding the notion of equity. Equity in mathematics education refers to the fair allocation of resources, opportunities, and support, ensuring that all students can excel in mathematics, regardless of their backgrounds (National Council of Teachers of Mathematics, 2014). According to Gutiérrez (2011), equity in mathematics education is often contextualised within social justice, emphasising the need to confront institutional inequities that affect students' learning experiences (Gutiérrez, 2009). Gutiérrez (2013) asserts that equity is not merely the allocation of equal resources but also the recognition and appreciation of the diverse cultural and linguistic backgrounds of students. This perspective aligns with Ladson-Billings' (1998) advocacy for culturally relevant teaching, which empowers students by connecting mathematical concepts to their personal experiences. Thus, the notion of equity in mathematics education is grounded in multiple theoretical frameworks. A significant framework is the social justice approach, which posits that education must actively address systemic injustices (Gutiérrez, 2011). This perspective underscores the necessity for mathematics educators to recognise the socio-cultural contexts of their pupils and to adapt their instructional methods accordingly. Gutiérrez (2013) contends that equity in mathematics education transcends mere resource accessibility; it includes the quality of educational experiences provided to students.

Therefore, the culturally relevant pedagogy paradigm emphasises the significance of integrating students' cultural origins into the curriculum (Ladson-Billings, 1995). This approach promotes the establishment of inclusive educational environments that affirm students' identities and experiences. Studies demonstrate that culturally relevant pedagogical methods can markedly improve student engagement and success in mathematics (Tate, 1995; Bonner, 2009; Hodge & Lawson, 2018; Marshall, 2023). Thus, the training of mathematics educators is essential for promoting equitable educational settings within teacher education programmes. The National Council of Teachers of Mathematics (2014) underscores the necessity for teacher educators to equip pre-service teachers to identify and rectify disparities in mathematics education. This entails preparing future educators with the expertise and competencies to implement equitable teaching methodologies that accommodate diverse learners (Villegas & Lucas, 2002; Ladson-Billings, 2006; Gutiérrez, 2013). Consequently, Civil, Hunter, and Crespo (2019) highlight the importance of incorporating equity into the curricula for mathematics teacher preparation. The authors advocate for a holistic strategy that involves education on equity, practical experiences in diverse contexts, and opportunities for critical reflection. This approach enables pre-service teachers to develop a profound understanding of the barriers faced by disadvantaged children and to cultivate the skills necessary to address these challenges in their teaching (Civil, Hunter, & Crespo, 2019; Olawale, Mncube, & Harber, 2021; Olawale, 2024). Therefore, the training of pre-service mathematics teachers is essential for promoting equitable educational settings.

Numerous studies have investigated the integration of equity-focused frameworks within teacher education programmes (Achinstein & Athanases, 2005; Bancroft & Nyirenda, 2020; Grudnoff, Dixon, & Murray, 2021; Liao et al., 2022; Olawale, 2024). This entails preparing future educators with the expertise and competencies to apply equitable teaching methodologies that assist diverse learners (Civil, Hunter, & Crespo, 2019; Grudnoff, Dixon, & Murray, 2021). However, despite the increasing acknowledgment of the significance of equity in mathematics teacher education, numerous problems still exist. A major difficulty is the absence of agreement on the definition of equitable teaching practices. Thus, Leonard (2018) asserts that many teacher preparation programmes struggle to define and execute equity-focused methods effectively. This uncertainty may result in discrepancies in the implementation of equity among various programmes (Ladson-Billings, 1995; Ladson-Billings, 2006; Ladson-Billings, 2021). Similarly, the structural nature of educational disparities presents a challenge for teacher preparation programmes. Ladson-Billings (2014) asserts that mathematics teacher educators must navigate intricate socio-political environments that affect their capacity to implement equitable practices. This entails rectifying biases in the curriculum, tackling institutional obstacles, and promoting governmental reforms that foster equity in mathematical education. In addition, Ladson-Billings (2014) notes that professional development is essential for assisting in-service mathematics educators in their endeavours to foster equity within their classrooms. As such, Liao et al. (2022) highlight the significance of continuous professional development centred on equity-oriented approaches, stating that professional development should offer educators opportunities for collaborative learning, reflection on their instructional methods, and exploration of solutions to meet the varied needs of their students (Ladson-Billings, 2014; Ries et al., 2024).

1.2 The concept of social justice in the preparation of mathematics teachers

The notion of social justice in education, particularly in the training of mathematics teachers, has received significant attention in recent years. Social justice is a complex concept that encompasses multiple elements, including equity, access, participation, and rights (Bell, 2016). In the field of education, social justice highlights the necessity of addressing systematic disparities that affect underrepresented populations. Ladson-Billings (1995) asserts that culturally relevant pedagogy is crucial for advancing social justice in education, as it compels educators to connect content to students' cultural backgrounds and experiences. This approach is especially pertinent in mathematics education, where conventional teaching practices often overlook the diverse cultural backgrounds of students.

Furthermore, the concept of critical pedagogy, as defined by Freire (1970), provides a fundamental framework for understanding social justice in education. Freire advocates for a dialogical method of instruction, in which students are encouraged to interrogate and challenge prevailing norms. In mathematics education, this can take the form of activities that enable students to view mathematics as a means of social transformation, rather than merely a collection of procedures to memorise (Gutstein, 2006; Olawale, 2021).

Thus, incorporating social justice themes into mathematics teaching is essential for preparing future educators to address disparities in the classroom. Research indicates that mathematics is often perceived as a neutral and objective field; however, this perspective may obscure the social and political implications of mathematical knowledge (Skovsmose, 1994; Skovsmose, 2019). If this misconception is not addressed, the practice of mathematics instruction may perpetuate stereotypes and reinforce existing power dynamics, particularly for students from disadvantaged communities (Skovsmose, 2019; Skovsmose, 2020).

Studies such as those by Gutstein (2003), Spielman (2012), McGee & Hostetler (2014), and Bartell (2018) have highlighted the significance of integrating social justice into mathematics education. Gutstein (2007) advocates for the incorporation of real-world issues in mathematics education, allowing students to analyse topics such as poverty, race, and gender using mathematical frameworks. This method not only improves students' understanding of mathematics but also cultivates critical awareness, enabling them to identify and confront social injustices (Gutstein, 2007; Bell, 2016; Olawale, Mncube, & Harber, 2021).

To adequately equip mathematics teachers for socially equitable practice, teacher education programs must integrate pedagogical practices that foster equity and inclusivity. One approach employed is culturally responsive teaching, which recognises and appreciates students' cultural backgrounds while incorporating them into the educational process (Gay, 2015; Gay, 2018). This strategy enables mathematics educators to cultivate a more inclusive classroom atmosphere that acknowledges and leverages students' varied backgrounds. Furthermore, collaborative learning environments can significantly contribute to advancing social justice in mathematics education (Gay, 2018).

Similarly, Vygotsky's (1978) social constructivist theory underscores the significance of social interaction in the learning process, positing that students can collaboratively create knowledge (McLead, 2023). Thus, mathematics educators should promote collaborative problem-solving exercises to allow students to exchange viewpoints and learn from each other, thereby questioning prevailing narratives and cultivating a sense of community (Gay, 2018; Skovsmose, 2019).

Furthermore, the implementation of critical mathematics education (CME) has emerged as a viable method for incorporating social justice into mathematics instruction. CME urges students to rigorously analyse mathematical concepts and their practical applications, enabling them to utilise mathematics as a means for advocacy and social transformation (Skovsmose, 1994). Teacher preparation programmes that integrate CME concepts help prepare future educators with the competencies and knowledge essential for promoting a socially equitable mathematics education. Despite the increasing acknowledgment of the need for social justice in mathematics teacher training, numerous problems persist. A major obstacle is the dominant focus on standardised testing and accountability in education, which may restrict teachers' capacity to adopt socially equitable practices (Gonzalez, 2010). Furthermore, many teacher education programmes may be deficient in the resources and training required to proficiently incorporate social justice themes into their curricula. Additionally, the reluctance to change within educational institutions can obstruct the implementation of socially equitable practices in mathematics teaching. Educators may also experience a lack of preparation or support in tackling intricate social issues in their classrooms, resulting in hesitance to discuss social justice topics (Cochran-Smith, 2004; Olawale, Mncube, & Harber, 2021). To address these problems, it is imperative for teacher education programmes to offer a comprehensive teacher education programme for pre-service teachers and continuous professional development and support for educators aiming to promote socially equitable practices. Hence, this chapter seeks to examine how mathematics teacher education programmes prepare pre-service mathematics teachers for equitable and socially just classrooms.

2. Methodology

This study is grounded in an interpretivist paradigm, indicating that individuals develop an understanding of their environment through interactions with the external world (Ormston, Spencer, Barnard, & Snape, 2014). A qualitative methodology was employed to gain deeper insight into the preparation of pre-service mathematics teachers for equitable and socially just classrooms within mathematics teacher education programmes. A case study research design was utilised. As stated by Yin (2014), case studies are the most appropriate method when the contextual conditions are relevant to the phenomenon under investigation. The application of a multiple case study approach allowed the researcher to select research sites and participants from various backgrounds, reflecting a spectrum of experiences related to equity and social justice in classrooms. Consequently, two traditional South African universities offering theoretically oriented degree programmes were purposively chosen for this investigation. These universities were expected to provide unique and compelling data concerning equity and social justice issues in their pre-service teacher training programmes.

The study primarily focused on mathematics teacher educators, pre-service mathematics teachers, and department heads at the two selected universities. Each university comprised a sample of twelve pre-service mathematics student teachers, two mathematics teacher educators, and one department head. The sample, utilising a purposive sampling technique, consists of twenty-four pre-service mathematics teachers, four mathematics teacher educators, and two department heads, totalling thirty participants. The selection of the study participants was based on the fact that they possess in-depth knowledge and experience relating to equity and social justice within the mathematics teacher education programmes. Data for this study were gathered via semi-structured interviews lasting roughly 12 to 20 minutes per participant. One-on-one interviews were conducted with the study participants at each university; the conversations were framed by informal dialogue and audiotaped. Although the interviews were conducted in English, participants were allowed to use vernacular if they wished to express anything in their home language. The interviews were held in lecture halls, outside the building, and in any location where the interviewees felt at ease on their respective campuses, ensuring a tranquil environment free from disruptions. The nature and objective of the research were disclosed, and confidentiality and anonymity were guaranteed prior to and after the interviews. Verbal consent was obtained to record the interviews. The semi-structured interview approach provided participants with significant opportunities for self-expression; however, greater emphasis was placed on the predetermined questions to prevent meandering discourse (Datko, 2015; Olawale, 2021).

The data obtained from these participants were subjected to thematic analysis. The thematic data-analysis technique, as delineated by Braun and Clarke (2006), consists of six stages. During the initial step, the collected data were transcribed, systematically categorised, and thoroughly examined to discern relevant themes and categories. In the subsequent step, classifications, themes, and patterns were established. During the third phase, coding was conducted using names, and colours were allocated to participant responses for the systematic management of information. In the fourth step, emergent comprehension was assessed by employing colours to code content according to the subheadings, to evaluate the data's use. In the fifth stage, the content was thoroughly reviewed, quotations were incorporated as needed, and subheadings were substituted with comments highlighting contrasts and similarities. In the final phase, the researcher undertakes the writing of the report, striving to diminish the authorial voice. For data presentation, the two traditional universities that participated in this study were designated with fictitious names: University A and University B. The data obtained from the interviews are organised according to the interview questions and the raw responses provided by the participants associated with those universities. This was conducted to achieve a comprehensive understanding of the participants' responses. The two mathematics teacher educators from each of University A and B were designated as MTE1 and MTE2, while the mathematics student teachers were labelled as PST1 through ST12, and the Head of the Department as HOD, corresponding to their respective universities A or B.

To ensure data trustworthiness, emphasis was placed on the triangulation of the unit of analysis. Triangulation entails utilising diverse data sources, several locations, and various data-gathering methods while examining analogous subjects to enhance the trustworthiness of research outcomes (Cohen et al., 2018). Cohen et al. (2018) assert that researchers who depend solely on a singular method of data collection or unit of analysis exhibit bias in assessing the actuality or truthfulness of the phenomenon being studied. Thus, the utilisation of several units of analysis (pre-service mathematics teachers, mathematics teacher educators, and Heads of Department) enabled the researcher to triangulate data to derive valid results. As such, the research findings were analysed for similarities among the data-gathering sources to enhance the study's validity. Furthermore, the researcher involved participants in member-checking from the initial phases of data processing, including the establishment of codes and categories, the development of themes, and the analysis of outcomes. To do this, individuals were contacted via telephone and email, while meetings were organised with other participants according to their availability. The researcher solicited feedback from the participants regarding the raw data, codes, themes, and interpretations of the data. This strategy was considered beneficial for maintaining internal validity (Oats, 2014).

3. Presentation of Results

This section presents the findings from the study on how mathematics teacher education programmes prepare pre-service mathematics teachers for equitable and social justice classrooms. The results are organised into two major themes:

- Curriculum design and pedagogical approaches.
- Practical experiences and community engagement.

Each theme is examined in detail to elucidate how these programmes equip future educators with the necessary tools and perspectives for fostering equity and social justice in their teaching practices.

3.1 Curriculum design and pedagogical approaches

To gather stakeholders' perspectives on the preparation of pre-service mathematics teachers for equitable and social justice classrooms, participants were asked, "How do curriculum design and pedagogical approaches enhance the preparation of pre-service teachers for equity and social justice?" Research findings revealed that the integration of an equity and social justice framework and an emphasis on culturally relevant pedagogy are ways teacher education programs promote equity and social justice in classrooms.

3.1.1 Integration of equity and social justice frameworks

To create a fairer and more inclusive classroom, mathematics teacher preparation programs must incorporate a social justice and equity perspective. In today's increasingly varied world, pre-service teachers must possess a deep understanding of the social variables that impact student learning in addition to mathematical prowess and pedagogical knowledge. Improved instructional efficacy, more equitable access to mathematical knowledge, and better-prepared educators to fight for institutional reform are all outcomes of this integration. For instance, a participant stated:

> *Incorporating equity and social justice frameworks into our curriculum has significantly shaped my understanding of the diverse experiences and backgrounds of my future students. By analysing real-world issues related to race, gender, and socio-economic status through mathematical concepts, I feel better equipped to create lessons that resonate with my students' lives. This approach encourages me to use inclusive pedagogy, making sure every voice is heard and valued in my classroom. (MST 9 – University B)*

Similarly, another participant stated:

The focus on social justice in my mathematics training has encouraged me to think critically about the societal implications of mathematical practices. It has helped me realize that math is not just abstract numbers but can be used to tackle issues such as income inequality and access to education. This perspective prepares me to guide my students in using math as a tool for social change, promoting critical consciousness about their contexts. (MST 3 – University A)

Another participant also asserts that:

Equity and social justice frameworks have taught me the significance of building relationships and a sense of community within my classroom. Creating a space where all students feel safe to express their thoughts and contribute to discussions is vital. This preparation enables me to foster an environment where students can collaborate on math problems that address social justice issues, helping them to connect mathematical thinking with collective action. (MST 7 – University A)

The above research findings reflect a thoughtful and systematic integration of equity and social justice in mathematics education, highlighting the importance of relevance, inclusivity, community, critical thinking, and empowerment in preparing future teachers. Similarly, mathematics teacher educators also iterate that:

By integrating equity and social justice frameworks, I focus on contextual learning that relates mathematics to real-world issues affecting marginalised communities. This approach encourages future teachers to see mathematics not just as abstract concepts but as tools for social change. During our curriculum design, we incorporate projects that require students to analyse data on social issues, enabling them to recognise the role of math in understanding and addressing disparities. (MTE 2 – University A)

In addition, another mathematics teacher educator adds:

We encourage critical inquiry and self-reflection among our pre-service teachers regarding their biases and assumptions about mathematics and education. Integrating social justice frameworks prompts our students to question the status quo and consider how traditional curricula may perpetuate inequities. Assignments that require them to reflect on their mathematical identity and societal impact foster a mindset geared toward challenging inequitable practices in their future classrooms. (MTE 1 – University B)

Thus, these responses illustrate the diverse strategies that mathematics teacher educators might use to incorporate equity and social justice into their teaching, preparation, and curriculum design. Integrating these frameworks helps shape future teachers who are mindful of the impact of their teaching on their learners, particularly those from underrepresented groups. Similarly, incorporating equity and social justice frameworks into mathematics curriculum design and pedagogical approaches is essential for preparing students for an equitable world, as noted by a head of department:

………Our curricula incorporate critical mathematics education, which challenges students to explore the social implications of mathematical concepts. We engage students in discussions about data representation, statistical literacy, and how mathematics can be used to both oppress and empower communities. By analysing real-life scenarios such as income inequality through data analysis, students learn to critically assess and use mathematics as a tool for social justice, preparing them to advocate for equitable solutions. (HOD – University B)

The research findings reflect a commitment to equity and social justice in mathematics education, emphasizing the importance of relevance, collaboration, critical thinking, diverse assessment, and community connection in preparing students for a more equitable future. Additionally, the findings revealed that mathematics teacher education programs strive to integrate equity and social justice frameworks into their curricula. These efforts manifest through teachingthat addresses issues of race, class, and gender in mathematics education, providing pre-service teachers with theoretical foundations and practical strategies to recognize and challenge inequities in the classroom.

3.1.2 Emphasis on Culturally Relevant Pedagogy

Achieving educational equality and social justice requires mathematics teacher preparation programsto incorporate culturally relevant pedagogy. To foster inclusive learning environments that empower all learners, it is crucial for mathematics teacher educators to acknowledge and appreciate the cultural backgrounds of pre-service mathematics teachers. In addition to improving classroom instruction, a commitment to culturally relevant pedagogy can drive broader reforms in teacher education programs. Thus, a participant stated:

Culturally relevant pedagogy enables us to highlight diverse mathematicians and their contributions, showcasing role models who reflect the backgrounds of our students. This not only helps break down stereotypes around who can excel in mathematics but also fosters a sense of belonging. By seeing themselves represented in mathematical narratives, students gain the confidence to pursue their interests in math, ultimately supporting an equitable academic environment. (MTE 2 – University of B).

Similarly, a participant added:

By embracing culturally relevant pedagogy, we actively engage with our students' communities, collaborating with families and local organisations to inform our teaching practices. This relationship-building helps us understand the unique challenges and strengths our students face, equipping us to better support their learning needs. Such an approach not only enhances equity but also empowers students to see mathematics as a tool for social change in their communities. (MTE 1 – University A).

The above responses highlight how culturally relevant pedagogy plays a vital role in fostering an equitable and socially just mathematics education. Similarly, mathematics pre-service teachers added:

> *As a future mathematics teacher, I've come to appreciate how culturally relevant pedagogy emphasises the importance of understanding students' backgrounds. By integrating diverse cultural perspectives into math lessons, I feel better equipped to create an inclusive learning environment. This approach allows me to connect mathematical concepts to students' real-world experiences, making math more relatable and empowering for all learners, especially those from marginalised communities. (MST 2 – University B).*

Similarly, another participant added:

> *Having a strong emphasis on culturally relevant pedagogy in my training has highlighted the significance of student voice in the learning process. I've learned to value the mathematical knowledge that students bring from their own cultures. For instance, when teaching geometry, I can include traditional architecture from various cultures. This not only validates students' experiences but also fosters a deeper understanding of mathematical concepts, thereby preparing me to teach in a way that honours diversity. (MST 6 – University A)*

In addition, another participant stated that:

> *The training I've received on culturally relevant pedagogy has taught me the importance of building relationships with my students. Understanding their cultures, interests, and experiences allows me to tailor my teaching approach and create a culturally responsive classroom. By doing so, I can address the varying needs of my students, ensuring that everyone has a fair chance to succeed in mathematics. This preparation is vital for fostering an equitable classroom environment where all voices are heard. (MST 11 – University B)*

The research findings highlight how culturally relevant pedagogy prepares pre-service teachers to create equitable and socially just mathematics classrooms by emphasising inclusivity, critical reflection, student engagement, and community connection. Similarly, the findings reveal that culturally relevant pedagogy stresses the importance of incorporating students' cultural orientations in all aspects of learning. For instance, the head of the department added:

> *In our preparation programs, we emphasise culturally relevant pedagogy and encourage educators to draw connections between mathematical concepts and the cultural practices/experiences of their students. For instance, relating geometry to architecture in various cultural contexts or using statistical analysis to examine local issues allows students to see the relevance of math in their everyday lives. By preparing teachers to implement these connections, we foster a sense* of *belonging and relevance that can motivate all students, especially those from underrepresented backgrounds. (HOD – University A)*

Through the reflections above, it is clear that culturally relevant pedagogy plays a vital role in preparing pre-service mathematics teachers to create equitable and socially just classrooms, ultimately benefiting both teachers and their students. Research findings also revealed that mathematics teacher education programs employed teaching methods that connected mathematical concepts to students' cultural backgrounds and real-world experiences. Additionally, teacher educators utilised culturally relevant examples and problems that resonated with the diverse experiences of their students, making mathematics more accessible and engaging.

3.2 Practical experiences and community engagement

To gather stakeholders' perspectives on the preparation of mathematics teachers for equitable and socially just classrooms, participants were asked, "How do practical experiences and community engagement enhance the preparation of pre-service teachers for equity and social justice?" Research findings revealed that exposure to field experiences in diverse settings, as well as partnerships with community organisations, are ways in which teacher education programmes promote equity and social justice in classrooms.

3.2.1 Field experiences in diverse settings

Field experience in diverse settings plays a critical role in the preparation of pre-service mathematics teachers, particularly in fostering an understanding of equity and social justice in education. As classroom demographics continue to evolve, it becomes increasingly essential for future educators to engage with a variety of cultural, socioeconomic, and linguistic backgrounds. This exposure not only enriches their pedagogical skills but also equips them with the necessary tools to address the complex realities of their students' lives. Thus, a participant adds:

> *Field experiences such as participating in community-based projects in diverse communities and schools enable pre-service teachers to engage with learners from various cultural backgrounds. This exposure helps them understand the importance of culturally relevant pedagogy in mathematics education. By witnessing first-hand how different cultural perspectives influence learners' understanding of mathematical concepts, pre-service teachers can tailor their teaching strategies to be more inclusive and equitable. In my experience, these direct interactions help future teachers recognize the rich mathematical knowledge that students bring from their communities, fostering an environment where all voices are valued. (MTE 2 – University B).*

Similarly, another participant added:

> *Relationships are at the heart of effective teaching, especially in diverse settings. Field experiences such as school experience/school-based placements give pre-service teachers the chance to build relationships with students, families, and communities. This relational approach is fundamental for creating a supportive learning environment where learners feel valued and engaged. By working closely with communities, pre-service teachers can identify the unique needs of their students and*

design mathematics instruction that is not only relevant but also empowering. These experiences highlight the role of trust and connection in promoting equitable learning. (MTE 1 — University A)

The above responses illustrate how mathematics teacher educators perceive the impact of field experiences in diverse settings on the preparation of future teachers for equitable and social justice-oriented practices. Similarly, the head of department added:

…….as a head of the mathematics department, I believe that field experiences in the form of school-based placement, intercultural exchange programmes, and partnerships with NGOs are essential for fostering an understanding of social justice in education among our pre-service teachers. These experiences allow them to recognise the varied backgrounds and lived experiences of their would-be learners. By engaging with communities that face systemic inequalities, they are able to tailor their teaching activities to be more inclusive and culturally relevant. This not only enhances their teaching strategies but also empowers them by connecting mathematical concepts to real-world issues, illustrating how math can be a tool for social change. (HOD – University B)

These responses highlight the transformative potential of field experiences in creating a more equitable and socially just education system within mathematics departments. Thus, given that field experiences in diverse settings are crucial for pre-service teachers, especially in mathematics education, as they provide opportunities to understand and implement equitable and social justice practices in the classroom, a mathematics pre-service teacher iterates:

…… During my school experience, when I was collecting information for my community-based project at a school with diverse learners, I observed first-hand how different cultural backgrounds influence learners' engagement with mathematics. I learned that incorporating culturally relevant pedagogy can help bridge the gap between learners' lived experiences and the mathematics curriculum. This has taught me the importance of recognising each learner's unique perspective, which is essential for creating an equitable learning environment. (MST 12 – University A).

Similarly, another participant added that:

During my school experience, I worked closely with learners from various socioeconomic backgrounds. I found that building strong relationships based on trust and respect was vital in encouraging them to participate in math discussions. I realised that when learners feel valued and understood, they are more willing to share their thoughts and struggles. This has motivated me to prioritise relationship-building in my future classroom to foster an inclusive environment. (MST 5 – University B).

Also, another pre-service mathematics teacher added:

In a setting where community engagement was strong, I learned how to leverage local resources to make math more relevant to students' lives. For instance, using local businesses for real-world math problems helped learners see the value in what they were learning. This experience has *inspired me to seek out partnerships that can enhance my teaching and provide learners with practical applications of math, fostering equity in learning. (MST 8 – University A)*

The research findings illustrate how field experiences, such as community-based projects, school-based placements, intercultural exchange programmes, and partnerships with NGOs in diverse educational settings, can profoundly impact pre-service teachers' understanding of equity and social justice. These experiences ultimately shape their approach to teaching mathematics in a more inclusive and responsive manner. Additionally, the findings revealed that the teacher education programmes offered opportunities for field experiences in diverse educational settings and community organisations, which became crucial in helping pre-service teachers apply theoretical knowledge to real-world contexts.

3.2.2 Partnerships with Community Organizations

The partnership between mathematics teacher preparation programmes and community organisations is vital for equipping future educators with the knowledge, skills, and dispositions necessary to promote equity and social justice in their classrooms. By engaging with the community, pre-service teachers can develop a deeper understanding of the challenges faced by their learners, learn to advocate for systemic change and create meaningful connections between mathematics and their learners' lives. For instance, a participant stated that:

As a pre-service teacher, I've learned that engaging with community organisations allows me to understand the diverse backgrounds of my future learners. Partnering with local groups helps me integrate culturally relevant pedagogy into my math instruction. For example, when collaborating with a local cultural centre, I discovered how traditional games could be tied to mathematical concepts. This connection not only makes math more relatable to my learners but also honours their heritage, fostering a sense of belonging in the classroom. (MST 4 – University B)

Similarly, another participant stated:

Community engagement helps me connect mathematical concepts to real-world issues that affect my learners' lives. By working with non-profits focused on social justice, I have gained insights into local issues such as housing inequality and food deserts. Incorporating these topics into my math lessons through statistics allows my learners to see math as a tool for advocacy and change, not just a set of abstract concepts. (MST 1 – University B)

Also, a participant iterates that:

…..my community engagement experiences have encouraged a reflective practice essential to my growth as a pre-service teacher. Participating in community forums has helped me hear voices and perspectives that I might not encounter in a school setting. This reflection allows me to critically assess my own biases and assumptions about teaching math. I believe that being aware of these factors is crucial for creating an equitable classroom, where every learner feels valued and capable of achieving success in mathematics. (MST 10 – University B)

These above responses illustrate how partnerships and community engagement can significantly enrich pre-service teachers' understanding of equity and social justice, ultimately informing their teaching practices and fostering an inclusive classroom environment. Furthermore, a mathematics teacher educator added:

> *By partnering with local schools, we engage in co-planning and co-teaching initiatives that allow us to understand the unique challenges students face in different communities. This first-hand experience informs our approaches to teaching mathematics in ways that are culturally relevant and inclusive. We can draw on the diverse backgrounds of our students to make math more relatable and approachable, encouraging equitable participation. (MTE 1 – University B)*

Similarly, another participant posits that

> *………Engaging our students in community projects allows them to take ownership of their learning while also addressing real-world problems through mathematics. Our pre-service teachers participate in local math tutoring programs where they engage with students from diverse backgrounds. This experience not only enhances their pedagogical skills but also deepens their understanding of the social context in which their future students will learn. It's a way to empower our future educators to advocate for equitable practices in their own schools. (MTE 2 – University A)*

The above research findings illustrate the importance of collaboration and engagement with communities in preparing mathematics teachers to create equitable and socially just learning environments. This was further supported by a head of department who argued that:

> *……as head of the department, I engage with other teacher educators and community stakeholders to participate in professional development workshops focused on equity and social justice in education. These sessions often involve discussions on best practices, case studies, and collaborative learning opportunities. By continually improving our understanding of these vital issues, we can better prepare ourselves and our future teachers to foster environments where every learner feels empowered to succeed in mathematics, regardless of their background. (HOD – University A).*

The above research findings reflect a commitment to creating inclusive and equitable mathematics classrooms through active community engagement and partnerships. These partnerships often involved collaborative projects that addressed local mathematical literacy needs, such as tutoring programmes, after-school initiatives, and community workshops. Such engagements provided pre-service teachers with first-hand experience in applying equitable practices outside of the traditional classroom setting.

4. Discussion of Findings

The findings of this study elucidate the critical importance of integrating equity and social justice frameworks within teacher education programmes. As the landscape of education continues to evolve, it becomes increasingly essential for pre-service teachers to develop a comprehensive understanding of the diverse cultural, social, and economic contexts that influence student learning. Thus, central to this discussion is the notion that teacher education programmes must prioritise equity and social justice as foundational elements of their curricula (Freire, 1970; Ladson-Billings, 1995; Skovsmose, 2019; Ries et al., 2024). The integration of these frameworks is not merely an additive process but rather a transformative approach that shapes the very ethos of teacher preparation. Therefore, teacher educators who are equipped with an understanding of systemic inequities are better positioned to foster inclusive learning environments that recognise and celebrate the diverse identities of their students. This understanding is particularly vital in light of the persistent achievement gaps that disproportionately affect marginalised communities. Thus, by embedding equity and social justice into the fabric of teacher education, we can cultivate a new generation of educators who are not only aware of these disparities but are also committed to enacting change within their classrooms and beyond (Gutstein, 2003; Gay, 2015; Bell, 2016; Ladson-Billings, 2023).

Similarly, culturally relevant pedagogy emerges as a crucial component in the findings of the study. Ladson-Billings (1995) posits that culturally relevant teaching is not merely about incorporating diverse content into the curriculum; rather, it involves creating a learning environment that empowers students by validating their cultural identities. This chapter also reinforces this notion, revealing that pre-service teachers who engage with culturally relevant pedagogical practices are more adept at connecting with their students and fostering a sense of belonging. This connection is particularly important in diverse classrooms where students may feel marginalised or disconnected from the curriculum. By integrating culturally relevant pedagogy into teacher education programmes, teacher education programmes can equip pre-service teachers with the tools necessary to create inclusive and affirming learning spaces that honour the cultural backgrounds of all learners (Ladson-Billings, 2006; Gutstein & Peterson, 2005; Gutiérrez, 2011; Olawale, Mncube, & Harber, 2021).

Moreover, exposure to field experiences in diverse settings is essential for developing culturally responsive educators. The findings indicate that pre-service teachers who participate in practicum experiences within diverse communities demonstrate a greater understanding of the complexities of teaching in multicultural contexts. These experiences allow pre-service teachers to engage directly with the realities of their students' lives, fostering empathy and a deeper appreciation for the challenges faced by marginalised populations (Gutstein, 2003; Hodge & Lawson, 2018). Furthermore, these field experiences provide opportunities for pre-service teachers to practise culturally relevant pedagogy in real-world settings, enabling them to refine their skills and adapt their teaching strategies to meet the needs of diverse learners (Bell, 2016; Bartell, 2018; Bancroft & Nyirenda, 2020). As such, teacher education programmes must prioritise partnerships with schools and communities that reflect the diversity of the student population, ensuring that candidates are adequately prepared to navigate the complexities of contemporary classrooms.

The role of partnerships with community organisations cannot be overstated in the pursuit of equity and social justice in education. The study highlights the potential for collaboration between teacher education programmes and local organisations to enhance the preparation of future educators. These partnerships can provide valuable resources, support, and insights into the unique challenges faced by students and families in the community. By working alongside community organisations, pre-service teachers can gain a more nuanced understanding of the socio-economic factors that impact student learning and develop strategies to address these challenges within their classrooms (Gutiérrez, 2013; Ladson-Billings, 2021). Additionally, such collaborations can facilitate the development of culturally relevant curricula that reflect the needs and aspirations of the community, thereby fostering a sense of ownership and investment in the educational process.

Furthermore, the integration of equity and social justice frameworks into teacher education programmes necessitates a critical examination of the existing structures and practices within these programmes. Thus, the study findings suggest that traditional approaches to teacher preparation often perpetuate inequities, as they may fail to adequately address the diverse needs of all students. This calls for a reimagining of teacher education that prioritises social justice as a guiding principle. As such, teacher education programmes must critically assess their curricula, pedagogical practices, and assessment methods to ensure that they are aligned with the goals of equity and social justice. This may involve revising course content to include diverse perspectives, employing inclusive teaching strategies, and implementing assessment practices that recognise and value the different ways that students demonstrate learning (Gay, 2015; Civil, Hunter, & Crespo, 2019; Ladson-Billings, 2023). In addition to curricular changes, it is imperative that teacher education programmes engage in ongoing professional development for faculty and staff (Bancroft & Nyirenda, 2020; Ries et al., 2024; Olawale, 2024). Similarly, mathematics teacher educators within these programmes must be equipped with the knowledge and skills necessary to effectively teach about equity and social justice. This includes understanding the historical and contemporary contexts of systemic oppression, as well as the ways in which these issues manifest in educational settings. By fostering a culture of continuous learning and reflection among faculty, teacher education programmes can ensure that they are effectively preparing future educators to confront and challenge inequities in their classrooms.

5. Conclusions and Recommendations

This study explores how mathematics teacher education programs prepare pre-service mathematics teachers for equitable and socially just classrooms. It emphasises the critical importance of curriculum design, pedagogical approaches, practical experiences, and community engagement in preparing pre-service teachers for these classrooms.

Firstly, the incorporation of culturally relevant teaching practices emerged as a critical theme. Educators who adapt their pedagogical approaches to reflect the cultural backgrounds and lived experiences of their students are better equipped to engage learners meaningfully. This alignment not only enhances student motivation but also fosters an inclusive classroom atmosphere where all students feel valued and capable of succeeding in mathematics.

Moreover, the enhancement of field experiences was identified as a crucial component of mathematics teacher preparation. By providing pre-service teachers with opportunities to engage in diverse educational settings, programs can cultivate an understanding of the varied challenges and strengths present in different student populations. These experiences allow pre-service teachers to observe and implement strategies that promote equity in practice, thereby reinforcing their commitment to social justice in their teaching philosophy.

Community engagement also emerged as a significant theme in the findings. Mathematics teacher preparation programs that actively involve local communities in their training initiatives create a more holistic educational experience. Such engagement not only enriches the curriculum but also establishes vital connections between schools and the communities they serve. This synergy is essential for developing educators who are aware of the socio-economic factors influencing their students' learning and are motivated to advocate for systemic change within their educational contexts.

Based on the findings, the chapter concludes that the integration of equity and social justice principles within mathematics teacher education is not merely an additive approach; it is essential for preparing educators who can effectively address the needs of all students. By embedding culturally relevant pedagogy, promoting diverse field experiences, and fostering community partnerships, teacher education programs can cultivate a new generation of educators committed to social justice and equity in their classrooms. This holistic approach is vital for transforming educational practices and ensuring that all students have access to equitable learning opportunities.

The implications of this study call for systemic change in teacher education, advocating for a curriculum that prioritizes social equity and prepares student teachers to be agents of change in their communities. This may encompass modules on culturally sensitive pedagogy, the historical background of mathematics, and the socio-political ramifications of mathematical practices. Mathematics teacher education programs should foster collaborative learning spaces in which pre-service teachers can engage in discussions and reflections on their experiences related to equity and social justice in mathematics teaching. Lastly, these programs should enhance field experience opportunities by partnering with diverse schools that serve marginalised communities and employing reflective evaluation methodologies that necessitate pre-service educators to scrutinise their biases and the effects of their instruction on varied student demographics.

6. Declarations

Funding: This research did not receive any external funding.

Conflicts of Interest: The authors declare no conflict of interest.

References

Achinstein, B., & Athanases, S. Z. (2005). Focusing new teachers on diversity and equity: Toward a knowledge base for mentors. *Teaching and Teacher Education*, *21*(7), 843-862. https://doi.org/10.1016/j.tate.2005.05.017

Bancroft, S. F., & Nyirenda, E. M. (2020). Equity-focused K-12 science teacher professional development: A review of the literature 2001-2017. *Journal of Science Teacher Education*, *31*(2), 151-207. https://doi.org/10.1080/1046560X.2019.1685629

Bartell, T. G. (2018). *Toward equity and social justice in mathematics education*. New York: Springer. https://doi.org/10.1007/978-3-319-92907-1

Bell, L. A. (2016). Theoretical foundations for social justice education. In *Teaching for diversity and social justice*. New York: Routledge.

Bonner, E. P. (2009). Achieving success with African American learners: A framework for culturally responsive mathematics teaching. *Childhood Education*, *86*(1), 2-6. https://doi.org/10.1080/00094056.2009.10523100

Braun, V., & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology*, 3(2), 77–101. https://doi.org/10.1191/1478088706qp063oa

Civil, M., Hunter, R., & Crespo, S. (2019). Mathematics teachers committed to equity: A review of teaching practices. *International Handbook of Mathematics Teacher Education*, *1*(1), 243–273. https://doi.org/10.1163/9789004418875_010

Cohen, L., Manion, L., & Morrison, K. (2018). *Research methods in education* (8th edition). New York: Routledge. https://doi.org/10.4324/9781315456539

Datko, J. (2015). On research methodology: Semi-structured interview in language pedagogy research. *Journal of Language and Cultural Education*, *3*(2), 142–156.

Davis, E. A. (2022). Supporting preservice elementary teachers in teaching science for equity and justice: A practical framework. *Innovations in Science Teacher Education*, *7*(4), 1–26.

Freire, P. (1970). Cultural action and conscientisation. *Harvard Educational Review*, *40*(3), 452–477. https://doi.org/10.17763/haer.40.3.h76250x720j43175

Gay, G. (2015). The what, why, and how of culturally responsive teaching: International mandates, challenges, and opportunities. *Multicultural Education Review*, *7*(3), 123–139. https://doi.org/10.1080/2005615X.2015.1072079

Gay, G. (2018). *Culturally responsive teaching: Theory, research, and practice*. New York: Teachers College Press.

Grudnoff, L., Dixon, H., & Murray, J. (2021). The power of inquiry in equity-oriented teacher education. *Beijing International Review of Education*, 3(3), 368–386. https://doi.org/10.1163/25902539-03030004

Gutiérrez, R. (2013). Why (urban) mathematics teachers need political knowledge. *Journal of Urban Mathematics Education*, *6*(2), 1-13. https://doi.org/10.21423/jume-v6i2a223

Gutiérrez, R. (2011). Context matters: How should we conceptualise equity in mathematics education? In *Equity in discourse for mathematics education: Theories, practices, and policies*. Dordrecht: Springer Netherlands. https://doi.org/10.1007/978-94-007-2813-4_2

Gutstein, E. (2003). Teaching and learning mathematics for social justice in an urban, Latino school. *Journal for Research in Mathematics Education*, *34*(1), 37–73. https://doi.org/10.2307/30034699
Gutstein, E. (2007). "And that's just how it starts": Teaching mathematics and developing student agency. *Teachers College Record, 109*(2), 420–448. https://doi.org/10.1177/016146810710900203
Gutstein, E., & Peterson, B. (2005). Rethinking mathematics: Teaching social justice by the numbers. Wisconsin: Rethinking Schools.
Hodge, L. L., & Lawson, M. (2018). Strengthening partnerships through family math nights. *Mathematics Teaching in the Middle School, 23*(5), 284–287. https://doi.org/10.5951/mathteacmiddscho.23.5.0284
Ladson-Billings, G. (1995). But that's just good teaching! The case for culturally relevant pedagogy. Theory into Practice, 34(3), 159-165. https://doi.org/10.1080/00405849509543675
Ladson-Billings, G. (2006). "Yes, but how do we do it?": Practicing culturally relevant pedagogy. In *White Teachers, Diverse Classrooms: Creating inclusive schools, building on Students' Diversity, and providing true educational equity* (pp. 33-46). Amsterdam: Stylus. https://doi.org/10.4324/9781003448709-6
Ladson-Billings, G. (2014). Put up or shut up: The challenge of moving from critical theory to critical pedagogy (a formative assessment). In *Democratic Social Education.* Routledge.
Ladson-Billings, G. (2021). Culturally relevant pedagogy: Asking a different question. New York: Teachers College Press.
Ladson-Billings, G. (2023). "Yes, but how do we do it?": Practicing culturally relevant pedagogy. In White Teachers/Diverse Classrooms (pp. 33-46). Oxfordshire, UK: Routledge. https://doi.org/10.4324/9781003448709-6
Leonard, J. (2018). *Culturally specific pedagogy in the mathematics classroom: Strategies for teachers and students.* United Routledge. https://doi.org/10.4324/9781351255837
Liao, W., Wang, C., Zhou, J., Cui, Z., Sun, X., Bo, Y., & Dang, Q. (2022). Effects of equity-oriented teacher education on preservice teachers: A systematic review. *Teaching and Teacher Education, 119*(1), 1-16. https://doi.org/10.1016/j.tate.2022.103844
Marshall, S. (2023). But what does it look like in maths? A framework for culturally sustaining pedagogy in mathematics. *International Journal of Multicultural Education, 25(*1), 1-29. https://doi.org/10.18251/ijme.v25i1.3251
McGee, E. O., & Hostetler, A. L. (2014). Historicizing mathematics and mathematizing social studies for social justice: A call for integration. *Equity & Excellence in Education, 47*(2), 208-229. https://doi.org/10.1080/10665684.2014.900428
McLead, S. (2023, 3 April). Vygotsky's sociocultural theory of cognitive development. Retrieved 4 April 2023, from Simply Psychology: https://www.simplypsychology.org/vygotsky.html
National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematics success for all.* Reston, National Council of Teachers of Mathematics.
Oats, R. (2014). The responsiveness of social studies teacher training curriculum: Towards a democratic citizenship education in Botswana. Unpublished PhD thesis, University of South Africa.
Olawale, B. E. (2021). *Democratic citizenship in mathematics teachers' preparation in South African universities.* [Unpublished PhD thesis submitted to the University of Fort Hare] South Africa.

Olawale, B. E. (2022). Democratic citizenship in mathematics teacher preparation in South African universities: Contradictions in theory and practice. *International Journal of Pedagogy and Teacher Education*, 6(2), 126-139. https://doi.org/10.20961/ijpte.v6i2.61574

Olawale, B. E. (2024). Inclusive innovations: Promoting digital equity and inclusion through technological solutions. In X. Liu (Ed.), *Innovation and evolution in higher education* (pp. 1-16). London, U.K.: Intechopen.

Olawale, B. E., Mncube, V., & Harber, C. (2021). Critical social pedagogy in mathematics teacher education. *International Journal of Higher Education*, *10*(6), 93–104. https://doi.org/10.5430/ijhe.v10n6p93

Ormston, R., Spencer, L., Barnard, M., & Snape, D. (2014). The foundations of qualitative research. In *Qualitative research practice: A guide for social science students and researchers* (2nd ed., pp. 52-55).

Premier, J. A., & Miller, J. (2010). Preparing pre-service teachers for multicultural classrooms. *Australian Journal of Teacher Education*, 35(2), 35-48. https://doi.org/10.14221/ajte.2010v35n2.3

Ries, E., Holyoke, E. S., Dunham, H., Young, M. K., Mosley Wetzel, M., Garcia, C., ... & Trautman, K. (2024). Learning to expand our communities to support coaching for equity: A collaboration of seven university-based teacher preparation programmes. *International Journal of Mentoring and Coaching in Education*, 13(2), 178-194. https://doi.org/10.1108/IJMCE-10-2022-0087

Skepple, R. G. (2014). Preparing culturally responsive pre-service teachers for culturally diverse classrooms. *Kentucky Journal of Excellence in College Teaching and Learning*, 57-69.

Skovsmose, O. (1994). Towards a critical mathematics education. *Educational Studies in Mathematics*, 27(1), 35-57. https://doi.org/10.1007/BF01284527

Skovsmose, O. (2020). *Critical mathematics education*. Cham, Switzerland: Springer International Publishing. https://doi.org/10.1007/978-3-030-15789-0_34

Skovsmose, O. C. (2019). Crisis, critique, and mathematics. *Philosophy of Mathematics Education Journal*, *1*(1), 1-16.

Spielman, L. J. (2012). Equity in mathematics education: Unions and intersections of feminist and social justice literature. In *Towards equity in mathematics education: Gender, culture, and diversity* (pp. 39-56). https://doi.org/10.1007/978-3-642-27702-3_5

Tate, W. F. (1995). Returning to the root: A culturally relevant approach to mathematics pedagogy. *Theory into Practice*, *34*(3), 166–173. https://doi.org/10.1080/00405849509543676

Villegas, A. M., & Lucas, T. (2002). Preparing culturally responsive teachers: Rethinking the curriculum. *Journal of Teacher Education*, *53*(1), 20-32. https://doi.org/10.1177/0022487102053001003

Yin, R. (2014). *Case study research: Design and methods*. Thousand Oaks: Sage Publications.

CHAPTER TWO

Pre-Service Teachers' Self-Efficacy in Teaching Mathematics at Senior Primary Phase

Frans N. Haimbodi[1*]

Hesekiel K. Iilonga[2]

AFFILIATIONS

[1]Faculty of Education & Human Sciences, University of Namibia, Rundu, Namibia.
[2]Ministry of Education, Arts & Culture, Ongwediva, Namibia.

CORRESPONDENCE

Email: fhaimbodi@unam.na*

Reference Formats (APA 7th Edition)
Haimbodi, F. N., & Iilonga, H. K. (2025). Pre-service teachers' self-efficacy in teaching mathematics at senior primary phase. In B. E. Olawale (Ed.), *Building the Foundations: Effective Approaches in Mathematics Teacher Preparation* (pp. 22-29). ERRCD Forum. https://doi.org/10.38140/obp3-2025-02

Abstract: This chapter investigates the self-efficacy levels of pre-service mathematics teachers at the senior primary phase within a Namibian teacher education context. Employing a quantitative research approach with a descriptive design, the study examined the confidence levels of 27 randomly selected third- and fourth-year pre-service teachers from one campus of the University of Namibia. Data were collected using a closed-ended questionnaire with Likert scale items adapted from the Fennema-Sherman scales, focusing on self-efficacy attributes. The findings revealed that while most pre-service teachers expressed confidence in designing effective lesson plans, using technology, and managing classrooms, notable challenges persisted. These included limited access to teaching aids, learner misconceptions, and difficulties in time management. Self-efficacy was found to be significantly influenced by content knowledge, pedagogical strategies, classroom management skills, mentorship, and observational learning. The study highlights the importance of robust teacher training programmes that integrate technology, mentorship, and practical teaching experiences. Recommendations include expanding micro-teaching opportunities, providing access to teaching resources, and exploring the role of demographic factors in shaping teacher self-efficacy. The findings aim to inform teacher education programmes and contribute to the preparation of confident and competent mathematics educators.

Keywords: Self-efficacy, pre-service teachers, mathematics education, teacher training, teaching strategies.

1. Introduction

Teacher self-efficacy is a critical factor influencing educational outcomes, defined as teachers' beliefs in their ability to affect learner engagement and achievement (Lazarides et al., 2018). Mok and Moore (2019) highlight a positive association between teacher self-efficacy and teaching effectiveness, emphasising that teachers with higher self-efficacy are more effective educators. Rooted in Bandura's social cognitive theory, self-efficacy is shaped by how individuals think, feel, and behave, as well as by the support they receive (Bandura, 2019; Verma, 2022). Teachers with strong self-efficacy exhibit greater openness to innovative methods, better planning, and adaptability when facing challenges, resulting in improved teaching quality and learner outcomes (Li, 2023; Holzberger & Prestele, 2021).

Research underscores the multifaceted nature of self-efficacy. While high self-efficacy positively correlates with classroom management, learner motivation, and academic achievement, it can also mitigate teacher burnout and attrition (Bal-Taştan et al., 2018; Lazarides et al., 2020).

Teachers with high self-efficacy are more likely to adopt learner-centred approaches, use collaborative strategies, and integrate technology effectively (Mitchell, 2021). Conversely, low self-efficacy may hinder performance, particularly when external rewards are minimal (Tzur et al., 2016). Olawale and Hendricks (2024) explored mathematics teachers' perceptions of their teaching practice and self-efficacy, considering factors such as educational background, teaching phases, school type, and gender. The findings highlight that school type and educational background significantly influence teachers' performance and accomplishments. Despite strong subject matter knowledge, low self-efficacy in teaching practices can hinder teachers from achieving expected success and performance levels (Olawale & Hendricks, 2024).

Pre-service teachers, in particular, often experience fluctuating levels of self-efficacy influenced by prior teaching experience, mentorship, and contextual factors (Liu et al., 2021; Zhang et al., 2021). Effective teacher education programmes can bolster pre-service teachers' self-efficacy by fostering their confidence, knowledge, and instructional skills (Hendricks et al., 2024; Ruiz, 2024). Innovative teaching practices, such as the flipped classroom model, have been shown to strengthen self-efficacy among pre-service teachers (Ding et al., 2023). Furthermore, mentorship and positive field experiences play a crucial role in preparing pre-service teachers for the complexities of the teaching profession (Terfa et al., 2022).

The importance of teacher self-efficacy extends beyond individual educators. Collective teacher efficacy, defined as the shared belief of a group of teachers in their ability to influence learner outcomes, highlights the broader impact of teacher collaboration and school-wide practices on student achievement (Lazarides et al., 2020). A teacher's attitude toward their profession, coupled with their sense of self-efficacy, significantly affects their job satisfaction and professional growth, ultimately contributing to improved educational outcomes for learners (Kiralp & Bolkan, 2016; Segarra & Julià, 2022).

Given the pivotal role of teacher self-efficacy in shaping educational practices and outcomes, this study aims to explore the self-efficacy levels of mathematics pre-service teachers in Namibian primary schools. By examining the factors influencing their self-efficacy, the study seeks to inform teacher education programmes and contribute to the preparation of effective, confident educators capable of fostering meaningful learner engagement and achievement.

1.1 Problem statement

Pre-service teachers face significant challenges in adjusting to new school environments and applying their subject knowledge to effective pedagogical practices. These difficulties are further compounded by increasing student populations and chaotic school systems (McLennan et al., 2021). Low self-efficacy among pre-service teachers is often exacerbated by inadequate preparation, limited teaching experience, and negative feedback from peers or superiors (Mozahem et al., 2021). Additionally, individual attitudes and beliefs about education and learning play a crucial role in shaping self-efficacy (Hendricks et al., 2024). Self-efficacy directly impacts the effort, perseverance, and resilience of aspiring teachers, particularly when faced with challenging tasks such as solving complex mathematics problems. It is a critical component of the learning process, influencing the ability to overcome obstacles and achieve success. Although self-efficacy has been a subject of study for decades, there is a noticeable gap in research that determines the level of self-efficacy among pre-service teachers specifically within a Namibian context. This study aims to fill that gap by measuring the self-efficacy levels of mathematics pre-service teachers. Unveiling the self-efficacy levels of pre-service teachers could aid in developing the motivation, persistence, and skills needed to influence student learning outcomes and foster academic success positively.

1.2 Research questions

The following research question guided the study: What is the Self-Efficacy level of pre-service teachers in teaching mathematics at the Senior Primary school phase?

2. Methodology

This study employed a quantitative research approach with a descriptive design to measure the self-efficacy levels of pre-service mathematics teachers at the senior primary school phase. The focus was on determining the confidence levels of pre-service teachers in their ability to teach mathematics effectively. The target population comprised 97 third- and fourth-year students from one campus of the University of Namibia. This group was selected because they were in the final stages of their teacher education programme and had practical experience teaching mathematics in schools during their teaching practice. A random sampling method was used to select a sample of 27 mathematics pre-service teachers for the study. Data were collected using a closed-ended questionnaire containing Likert-scale items. The questionnaire items were adapted from the Fennema-Sherman Scales, specifically focusing on the self-efficacy attribute. It was administered online via Google Forms, allowing participants to complete and submit their responses electronically. Descriptive statistics were utilised to analyse the data and determine the self-efficacy confidence levels of the pre-service teachers in teaching mathematics.

3. Presentation of Results

This section presents the results of pre-service teachers' self-efficacy. Table 1 presents the demographics of the participants.

Table 1: Distribution of participants in accordance with their biographical variable

Gender		Academic year	
Male	Female	3rd	4th
13	14	4	23

According to Table 1, 13 (48.1%) of the participants were male and 14 (51.9%) were female. Four (14.8%) of the respondents were third-year students, while 23 (85.2%) were fourth-year students. This distribution indicates that most participants were nearing the end of their teacher preparation programmes and in their final year of study. Due to the more thorough education and hands-on experience they acquire in the later phases of their academic careers, fourth-year students are more likely to have greater classroom experience than third-year students, as indicated by their higher representation.

Table 2: Pre-service teachers' self-efficacy levels

Abilities	**Strong disagree**		**Disagree**		**Neutral**		**Agree**		**Strong agree**	
	N	%	N	%	N	%	N	%	N	%
I am confident in instructing learners and providing an alternate explanation when students are confused about mathematical topics.	3	11	-	-	6	22	11	41	7	26
I can design good lesson plans for teaching mathematics that are interesting and effective.	2	7	1	4	6	22	5	19	13	48
I am confident in my ability to respond to learners' inquiries on mathematical subjects.	3	11	-	-	5	19	6	22	16	59
I am confident in my ability to properly run a mathematics lesson.	2	7	1	4	7	26	7	26	10	37
I am skilled at teaching mathematics using technology.	-	-	-	-	8	30	10	37	9	33
I am confident in my ability to prevent disruptive behaviour in the classroom.	2	7	2	7	2	7	7	26	14	52

The findings from Table 2 indicate that 11% of the pre-service mathematics teachers lack confidence in their competence to instruct students and give them alternative explanations in mathematics topics when they find themselves confused, as evidenced by the small number of respondents who chose "strongly disagree." More respondents (22%) chose "neutral," indicating a reasonable level of confidence among some participants. However, the majority of about 41% chose "agree," indicating high trust in their communication and educational abilities. Of the participants, 26% chose "strongly agree," indicating that a sizable proportion of respondents felt very confident in teaching mathematical concepts. Overall, the statistics show that most pre-service teachers are confident in their competence to instruct and respond to the questions of senior primary students in mathematics.

Table 2 shows trends in the responses of pre-service teachers' capacity to create interesting and successful lesson plans for teaching mathematics. According to the results, the category "strongly disagree" garnered only 7% of the participants, indicating that only a small percentage of respondents strongly disagree with the statement. Out of all the categories, the response option “disagree” had the lowest frequency of 4%, suggesting that it was the least popular choice for this degree of disagreement. An average number of respondents (22%) chose the “neutral” option, suggesting that many pre-service teachers are undecided about the statement. This trend implies that a sizable portion of respondents might have ambivalent or neutral opinions about their own ability to create engaging and successful mathematics lesson plans. The “agree” category was chosen by 19%, suggesting that more respondents agreed with the statement. Finally, the “strongly agree” option had the most votes, about 48%, indicating that those who were extremely confident in their capacity to create such lesson plans agreed the most. With “strongly agree” being the most common response, these findings highlight a clear trend toward greater levels of agreement among respondents.

Pre-service teachers’ confidence levels are often high when it comes to answering students’ questions about mathematical topics. According to the results in Table 2, “Strongly disagree” was the least popular response, at 11%, suggesting that few pre-service teachers who participated are unconfident in this area. Interestingly, there were no recorded answers for “disagree,” indicating that no participant expressed a lack of confidence. An average of 19% and 22% of the participants chose “neutral” and “agree,” respectively. This indicates that some pre-service teachers have mixed feelings about their confidence. Since “strongly agree” received the most responses, from 59% of the participants, it is clear that the majority of pre-service teachers strongly believe they are capable of handling senior primary-level arithmetic problems.

The results in Table 2 show that, among those who are confident in their ability to conduct a mathematics lesson, 7% of the participants chose “strongly disagree,” while 4% indicated the “disagree” category. Both “neutral” and “agree” received 26% of the responses, while “strongly agree” had the highest percentage, at 37%. This trend suggests that having a strong belief in one’s ability to teach mathematics can boost confidence in delivering successful lessons. However, subject knowledge and prior teaching experience are likely additional factors that influence confidence in teaching mathematics at the senior elementary level. Therefore, a thorough evaluation should consider these influencing factors, even if self-belief plays a major role in teaching confidence.

The results from Table 2 show how pre-service teachers’ self-assessed proficiency in using technology to teach mathematics was distributed among five degrees of agreement. “Strongly disagree” and “disagree” were not chosen at all, indicating that none of the respondents were unconfident in their ability to use technology to teach mathematics. About 30% of the participants chose “neutral,” suggesting that a sizable percentage of respondents remain unsure or unconvinced of their competence in this area. Interestingly, 37% and 33% chose “agree” and “strongly agree,” respectively, indicating that many pre-service teachers have a favourable opinion of their use of technology in mathematics classes. Overall, these results point to a range of viewpoints, with many respondents indicating varying or neutral degrees of confidence in their abilities.

Based on the statement regarding confidence in preventing disruptive behaviours in the classroom, 7% of participants chose "strongly disagree," "disagree," and "neutral," respectively. A larger proportion, 26%, chose "agree," while 52% chose "strongly agree," indicating that most pre-service teachers have confidence in preventing disruptive behaviour in the classroom.

3.1 Discussion of findings

The findings of this study provide valuable insights into the self-efficacy levels of pre-service mathematics teachers at the University of Namibia, particularly regarding their confidence to instruct students, develop lesson plans, manage classroom behaviour, and use technology in teaching mathematics. These results align with existing literature, emphasising the critical role of teacher self-efficacy in shaping educational outcomes (Lazarides et al., 2018).

The results revealed that most participants exhibited high levels of confidence in their ability to instruct students and provide alternative explanations when learners encountered confusion. A combined 67% of respondents (41% "agree" and 26% "strongly agree") indicated confidence in their instructional capabilities. These findings align with Mok and Moore (2019), who highlight the positive correlation between teacher self-efficacy and teaching effectiveness. High self-efficacy levels equip teachers to engage learners effectively and adapt their teaching strategies to diverse learning needs (Li, 2023).

The participants displayed a strong belief in their capacity to create engaging and successful lesson plans for teaching mathematics, with 48% selecting "strongly agree." However, the neutral response rate of 22% suggests that a portion of respondents were uncertain about their skills in this area. This ambivalence might reflect a need for enhanced training in lesson planning during teacher education programmes (Ruiz, 2024). The strong confidence expressed by the majority supports the findings of Holzberger and Prestele (2021), who assert that teachers with high self-efficacy are better planners and more adaptable in their teaching approaches.

A notable 59% of participants expressed strong confidence ("strongly agree") in their ability to address learners' mathematical queries, while no participants selected "disagree." This result reflects the participants' preparedness to tackle classroom challenges, corroborating findings by Bal-Taştan et al. (2018) and Lazarides et al. (2020), which emphasise that high self-efficacy enhances classroom management and academic achievement.

The majority of participants demonstrated confidence in leading mathematics lessons, with 37% choosing "strongly agree" and an additional 26% selecting "agree." However, a significant proportion (30%) reported neutral or lower confidence levels. These findings suggest that while self-belief is crucial, other factors, such as subject knowledge and teaching experience, may also influence confidence levels, as noted by Zhang et al. (2021). Effective mentorship and hands-on teaching opportunities during teacher training can address this variability (Terfa et al., 2022).

The findings indicated a favourable view of technology integration in mathematics instruction, with 33% "strongly agreeing" and 37% "agreeing." However, the 30% neutral response rate suggests that some pre-service teachers remain uncertain about their proficiency in this area. As Mitchell (2021) and Ding et al. (2023) highlight, incorporating innovative practices such as technology use in teacher education programmes can enhance confidence and instructional quality.

The study found that most participants (78%, combining "agree" and "strongly agree") were confident in their ability to prevent disruptive behaviour in the classroom. This result aligns with Lazarides et al. (2020), who underscore the role of teacher self-efficacy in fostering effective classroom management. The findings also emphasise the importance of equipping pre-service teachers with strategies to maintain a positive learning environment, as noted by Kiralp and Bolkan (2016).

These findings highlight the generally high levels of self-efficacy among pre-service mathematics teachers at the University of Namibia, particularly in instructional competence, lesson planning, and classroom management. However, areas such as technology integration and lesson planning still show variability, suggesting a need for targeted interventions in teacher education programmes. Aligning with Bandura's (2019) social cognitive theory, these results affirm that fostering self-efficacy through mentorship, field experiences, and innovative teaching practices is essential to preparing effective, confident educators who can positively influence learner outcomes.

4. Conclusions and Recommendations

This study investigated the self-efficacy levels of pre-service mathematics teachers at the senior primary phase in Namibia, revealing both challenges and strengths in their preparation for teaching mathematics. The findings indicate that while pre-service teachers expressed confidence in key areas such as lesson planning, conducting mathematics lessons, and integrating technology into instruction, they also faced notable challenges. These included the limited availability of teaching resources, learner misconceptions, and time management difficulties.

The study underscores the multifaceted nature of self-efficacy, shaped by factors such as content knowledge, pedagogical strategies, classroom management skills, personal teaching experiences, and mentorship. These findings align with Bandura's social cognitive theory and existing research that highlights the importance of teacher self-efficacy in fostering educational outcomes (Lazarides et al., 2018; Mok & Moore, 2019). Despite these challenges, the participants' confidence in their teaching abilities suggests that targeted interventions can significantly enhance their self-efficacy and teaching effectiveness.

To enhance pre-service teachers' self-efficacy in teaching mathematics, teacher education programmes should provide comprehensive training in the use of teaching aids and technology through workshops, seminars, and hands-on practice. Structured mentorship programmes should be established, enabling experienced educators to guide pre-service teachers in lesson planning, classroom management, and addressing learner misconceptions. Opportunities for micro-teaching and reflective practice should be expanded, allowing pre-service teachers to build confidence and refine their instructional skills through practical experience. Access to adequate teaching resources, including study guides and digital tools, must be ensured to support effective teaching. Additionally, future research should explore how factors such as gender and academic background influence pre-service teachers' self-efficacy and examine the long-term impact of teacher education programmes on their professional development and teaching performance. These measures collectively aim to prepare confident, competent educators capable of fostering meaningful learner engagement and achievement in mathematics.

5. Declarations

Funding: This research did not receive any external funding.

Conflicts of Interest: The authors declare no conflict of interest.

References

Bal-Taştan, S., Davoudi, S. M. M., Masalimova, A. R., Bersanov, A. S., Kurbanov, R. A., Boiarchuk, A. V., & Pavlushin, A. A. (2018). The impacts of teacher efficacy and motivation on students' academic achievement in science education among secondary and high school students. *EURASIA Journal of Mathematics, Science and Technology Education*, *14*(6), 2353-2366. https://doi.org/10.29333/ejmste/89579

Bandura, A. (2019). Applying theory for human betterment. *Perspectives on Psychological Science*, *14*, 12–15. https://doi.org/10.1177/1745691618815165

Hendricks, W., Olawale, B. E., & Saddiq, K. (2024). An investigation of high school preservice teachers' self-efficacy in teaching mathematics. *Education Sciences*, *14*(11), 1262. https://doi.org/10.3390/educsci14111262

Holzberger, D., & Prestele, E. (2021). Teacher self-efficacy and self-reported cognitive activation and classroom management: A multilevel perspective on the role of school characteristics. *Learning and Instruction*, *76*, 101513. https://doi.org/10.1016/j.learninstruc.2021.101513

Kiralp, F. S. S., & Bolkan, A. (2016). Relationship between candidate teachers' attitude towards the teaching profession and their life satisfaction levels. *The Anthropologist*, *23*(1-2), 11-20.

Lazarides, R., & Warner, L. M. (2020). Teacher self-efficacy. *Oxford Research Encyclopedia of Education* 1e22. https://doi.org/10.1093/acrefore/9780190264093.013.890

Lazarides, R., Buchholz, J., & Rubach, C. (2018). Teacher enthusiasm and self-efficacy, student-perceived mastery goal orientation, and student motivation in mathematics classrooms. *Teaching and Teacher Education*, *69*, 1–10. https://doi.org/10.1016/j.learninstruc.2020.101346

Li, S. (2023). The effect of teacher self-efficacy, teacher resilience, and emotion regulation on teacher burnout: A mediation model. *Frontiers in Psychology*, *14*, 1185079. https://doi.org/10.3389/fpsyg.2023.1185079

Liu, Y., Liu, J., Xia, H., Zhang, X., Fontes-Garfias, C. R., Swanson, K. A., ... & Shi, P. Y. (2021). Neutralising activity of BNT162b2-elicited serum. *New England Journal of Medicine*, *384*(15), 1466–1468. https://doi.org/10.1056/NEJMc2102017

McLennan, S., Nussbaumer-Streit, B., Hemkens, L. G., & Briel, M. (2021). Barriers and facilitating factors for conducting systematic evidence assessments in academic clinical trials. *JAMA Network Open*, *4*(11), e2136577.

Mitchell, P. (2021). *Teacher technology self-efficacy and its impact on instructional technology integration*. Gardner-Webb University.

Mok, M. M. C., & Moore, P. J. (2019). Teachers & self-efficacy. *Educational Psychology*, *39*(1), 1–3. https://doi.org/10.1080/01443410.2019.1567070

Mozahem, N. A., Boulad, F. M., & Ghanem, C. M. (2021). Secondary school students and self-efficacy in mathematics: Gender and age differences. *International Journal of School & Educational Psychology*, *9*(sup1), S142-S152. https://doi.org/10.1080/21683603.2020.1763877

Olawale, B. E., & Hendricks, W. (2024). Mathematics teachers' self-efficacy beliefs and their relationship with teaching practices. *EURASIA Journal of Mathematics, Science and Technology Education*, *20*(1), em2392. https://doi.org/10.29333/ejmste/14123

Ruiz, S. K. (2024). *A Transcendental Phenomenological Study on Teacher Self-Efficacy: Exploring the Lived Experiences of Teachers with Provisional Licensure While Working in the Classroom* [Doctoral thesis, Liberty University]. Doctoral Dissertations and Projects. 5990. https://digitalcommons.liberty.edu/doctoral/5990

Segarra, J., & Julià, C. (2022). Mathematics teaching efficacy belief and attitude of pre-service teachers and academic achievement. *European Journal of Science and Mathematics Education*, *10*(1), 1-14. https://doi.org/10.30935/scimath/11381

Terfa, A., Olufemi, A., James, L., & Thanduxolo, M. (2022). Effect of mentorship on regular teachers' self-efficacy towards implementation of inclusive education at basic education level. *Specijalna Edukacija I Rehabilitacija*, *21*(4), 233-253.

Tzur, K. S., Ganzach, Y., & Pazy, A. (2016). On the positive and negative effects of self-efficacy on performance: Reward as a moderator. *The Journal of Psychology*, *29*(5), 362–377. https://doi.org/10.1080/08959285.2016.1192631

Verma, M. A., & Bhandari, M. (2022). An insight into self-efficacy and its impact on students' achievement: A review. *Journal of Positive School Psychology*, 2746-2752.

Zhang, C., Bengio, S., Hardt, M., Recht, B., & Vinyals, O. (2021). Understanding deep learning (still) requires rethinking generalisation. *Communications of the ACM*, *64*(3), 107-115. https://doi.org/10.1145/3446776

CHAPTER THREE

Visualising Mathematical Concepts through Dual Digital and Non-Digital Teaching Tools on Preservice Teachers' Pedagogical Content Knowledge

Abongile Ngwabe[1*]
Sakyiwaa Boateng[2]

AFFILIATIONS
[1&2]Faculty of Education, Walter Sisulu University, Mthatha, South Africa.

CORRESPONDENCE
Email: angwabe@wsu.ac.za*

Reference Formats (APA 7th Edition)
Ngwabe, A., & Boateng, S. (2025). Visualising mathematical concepts through dual digital and non-digital teaching tools on preservice teachers' pedagogical content knowledge. In B. E. Olawale (Ed.), *Building the Foundations: Effective Approaches in Mathematics Teacher Preparation* (pp. 31-56). ERRCD Forum. https://doi.org/10.38140/obp3-2025-03

Abstract: This chapter explores how dual digital and non-digital visualisation tools contribute to the formation of robust pedagogical content knowledge (PCK) in PSTs, enhancing their PCK and ability to teach mathematics effectively in diverse classroom settings. Grounded in the TPACK framework, the study examines the influence of these tools on PSTs' PCK. An interpretive qualitative approach was adopted, focusing on a cohort of 20 third-year PSTs from one class. Initially, their PCK was assessed using observational tools and focus group discussions during their microteaching sessions. Thereafter, design-based interventions were implemented during lectures, allowing PSTs to explore, develop, and integrate digital and non-digital tools in teaching Grade 10 mathematics topics (functions, measurements, and analytical geometry) over a semester. In the post-intervention phase, their PCK was re-evaluated as they integrated digital and non-digital tools into their microteaching. The same observational tools and focus group discussions were utilised to assess any changes in their PCK. Finally, semi-structured interviews were conducted to gather their reflections. Data were collected through observational tools, focus group discussions, and semi-structured interviews. The data were analysed using the TPACK framework as the analytical tool, intertwined with thematic analysis. The findings show that integrating digital and non-digital teaching tools to visualise mathematics concepts can significantly enhance PSTs' PCK and their ability to teach mathematics effectively. Therefore, this chapter recommends that mathematics teacher education programmes highlight the need for a balanced integration of diverse instructional tools to better prepare teachers for the challenges of contemporary mathematics education.

Keywords: Digital teaching tools, mathematics education, mathematics - 31 -aption- 31 -ing- 31 -o, non-digital teaching tools, pedagogical content knowledge.

1. Introduction

Mathematics teaching and learning remain critical areas of focus in education, with the persistent challenge of effectively communicating abstract concepts across educational levels. According toOlawale(2024), pre-service teacher (PST) education programmes significantly influence mathematics teachers' ability to instruct mathematical content in accordance with curriculum standards, augment their subject knowledge, and bolster their confidence. Nevertheless, the programmes exert negligible impact on their capacity to execute differentiated instruction within their instructional methodologies.For pre-service mathematics teachers, developing Pedagogical Content Knowledge (PCK), which integrates subject matter knowledge and pedagogical strategies, is essential for presenting complex ideas in ways that students can understand

(Shulman, 1986). Microteaching serves as a vital component of teacher education, offering pre-service teachers (PSTs) structured opportunities to practice and refine their teaching skills in a controlled environment (Fki, 2023; García-Esteban et al., 2016). Through cycles of teaching, feedback, and reflection, this approach enhances instructional techniques and fosters the development of PCK (Setyaningrahayu et al., 2019; Ramadhanti & Yanda, 2021).

Visualisation, as a pedagogical strategy that includes self-made visual representations, visual arts, and technology-based tools (An et al., 2023), further supports this development by bridging the gap between abstract mathematical theory and practical application, thereby improving PCK (Shulman, 1986). Digital tools, such as GeoGebra and augmented reality (AR), have been shown to facilitate the visualisation of complex mathematical concepts through dynamic and interactive features (Muslim, 2023). GeoGebra, for example, supports learning in calculus and geometry by providing dynamic representations that enhance conceptual clarity (Muslim, 2023). Similarly, AR enriches the learning experience by enabling students to manipulate and visualise mathematical objects in three dimensions, fostering deeper comprehension (Cirneanu & Moldoveanu, 2024). In contrast, non-digital tools, including manipulatives and visual aids, remain indispensable in resource-limited contexts, providing concrete experiences that aid in transitioning from tangible to abstract understanding (Urrutia et al., 2019).

While these tools offer unique benefits, there is limited research on their combined use in improving PSTs' PCK. Existing studies predominantly focus on either digital tools or traditional methods without examining their synergistic effects (Sangwin, 2021). Moreover, research tends to emphasise the impact of these tools on learners' understanding, leaving a significant gap in understanding their effects on PSTs' instructional capabilities, particularly during microteaching sessions. Furthermore, studies often overlook how PSTs' attitudes toward technology and pedagogical beliefs influence the integration of these tools into teaching practices (Voogt et al., 2012). Against this background, this chapter investigates how the combined use of digital and non-digital tools influences PSTs' PCK during microteaching sessions. Specifically, it explores how PSTs visualise and teach mathematical concepts before and after an intervention programme utilising tools like GeoGebra alongside manipulatives and visual aids. This research seeks to inform teacher education programmes about the optimal integration of these tools to enhance mathematics teaching and learning, equipping future teachers to navigate diverse classroom contexts effectively.

1.1 Research questions

This chapter, therefore, seeks to answer the following questions:

- In what ways do visualising mathematical concepts using both digital and non-digital tools influence preservice teachers' PCK during their microteaching sessions?
- What are preservice teachers' experiences of using dual digital and non-digital tool approaches during their microteaching sessions?

2. Literature

2.1 Pedagogical content knowledge in mathematics teacher education

Pedagogical Content Knowledge (PCK) is a vital construct in mathematics teacher education, enabling teachers to integrate mathematical content and pedagogical strategies effectively (Marshman & Porter, 2013). It plays a critical role in fostering student understanding and addressing misconceptions, yet, research highlights that pre-service teachers (PSTs) often struggle to develop robust PCK (Ekiz-Kiran et al., 2021; Wakhata et al., 2022). Studies emphasise that combining general pedagogical knowledge with mathematics-specific approaches enhances teaching efficacy and student outcomes (Charalambous et al., 2020; Sarama et al., 2021). This suggests that effective teacher training programmes should offer targeted experiences that cultivate both content knowledge and pedagogical knowledge, as these are essential for impactful instruction, as confirmed by Ball et al. (2008) and Tröbst et al. (2018). As noted by Olawale (2023), teachers' experience, academic qualifications, and pedagogical content knowledge are integral to teacher quality, which significantly influences learners' academic achievement by enhancing the teacher's capacity to engage students, deliver instruction effectively, and facilitate meaningful learning experiences. This highlights the need to explore how tools like visualisation technologies influence PSTs' PCK development.

2.2 Digital and non-digital tools in mathematics education

A wide range of concepts is utilised to delineate digitisation in relation to the investment, adoption, and application of advanced technologies in educational practices and research. These concepts include digital instruments, digital technologies, information technology (IT), information and communication technology (ICT), and educational technology. Typically, these terms are employed interchangeably, as a clear distinction between them is absent (Salavati, 2016). As noted by Griffin (2003), these technologies hold considerable promise for educational purposes, with the effectiveness and applicability of digital technology being dependent on the teacher's proficiency and passion. The educator ultimately plays a pivotal role in enhancing the learning environment, contingent upon their adept utilisation of technology to their advantage (Griffin, 2003). Tondeur et al. (2008) contend that teachers are more predisposed to adopt innovations that align with their individual philosophies and beliefs concerning teaching and learning.

The integration of digital and non-digital visualisation tools, such as GeoGebra, is critical in developing PSTs' PCK by blending content knowledge with effective teaching strategies (Shulman, 1986). GeoGebra, in particular, offers dynamic and interactive learning opportunities, enabling preservice teachers to explore and teach complex mathematical concepts effectively (Dockendorff & Solar, 2018; Bakar et al., 2020; Marange & Tatira, 2023). Non-digital tools, such as physical models and diagrams, complement this by fostering hands-on engagement and enhancing conceptual understanding (Samuel, 2019; Vladušić et al., 2020). Together, these tools encourage diverse instructional strategies tailored to learners' needs (Rice & Kitchel, 2015; Wooditch et al., 2018).

GeoGebra's integration into teacher training programmes significantly enhances teachers' ability to employ technology in the classroom (Buchori & Puspitasari, 2023). Research indicates that pre-service teachers (PSTs) who use GeoGebra improve their instructional skills and gain a better understanding of the intersection between pedagogy and technology (Marange & Tatira, 2023; Bueno et al., 2021). Additionally, GeoGebra supports inquiry-based learning, which fosters student self-efficacy and motivation, enabling more engaging, student-centred teaching approaches (Zakariya, 2022; Barçin & Yenmez, 2023). Studies have also shown its potential in addressing misconceptions in mathematics and enhancing PSTs' professional growth (Dağlı & Elif, 2021; Horzum & Ünlü, 2017; Putra et al., 2021). Beyond its impact on teaching, GeoGebra positively influences student learning by making abstract mathematical concepts accessible and enjoyable. Students demonstrate improved attitudes towards mathematics and greater conceptual understanding when GeoGebra is used in lessons (Uwurukundo et al., 2022; Muslim, 2023). The tool's ability to foster visualisation and simplify complex ideas supports deeper engagement with mathematical content, particularly in areas such as geometry and three-dimensional models (Dahal et al., 2022).

GeoGebra's transformative role in modern mathematics education is evident in its capacity to foster innovative instructional practices and deepen understanding among both teachers and students. For PSTs, it offers a robust platform to develop TPACK and equips them to create active, learner-centred environments (Kuzu, 2021; Nzaramyimana et al., 2021). These findings underscore GeoGebra's crucial role in preparing pre-service teachers for effective mathematics instruction in technology-rich educational contexts.

In contrast, non-digital tools such as manipulatives, visual aids, and traditional paper-and-pencil methods significantly enhance students' understanding of mathematical concepts. These tools facilitate a tactile and visual approach to learning, which can be particularly beneficial for students who struggle with abstract mathematical ideas. Traditional resources, such as textbooks and physical manipulatives, provide tangible experiences that can enhance understanding and retention of information (Clark-Wilson, 2020). For instance, the use of physical objects in mathematics education has been shown to improve students' conceptual understanding by allowing them to visualise and manipulate abstract concepts (Clark-Wilson, 2020). Research has indicated that the use of physical manipulatives can lead to a deeper conceptual understanding and improved problem-solving skills among students (Hussein & Khoiruzzadittaqwa, 2024; Ng & Tsang, 2021).

One of the primary advantages of non-digital tools is their ability to promote active learning. When students engage with physical objects, they can explore mathematical concepts in a hands-on manner, fostering engagement and motivation. For example, using blocks to teach addition and subtraction allows students to visualise the process, making it more concrete and understandable. This approach aligns with the principles of Realistic Mathematics Education (RME), which emphasises the importance of context and real-life applications in learning mathematics (Agustina et al., 2018; Sari & Mutmainah, 2018).

Moreover, non-digital tools encourage collaborative learning. In a classroom setting, students can work together to manipulate objects, discuss their strategies, and share their findings. This collaborative approach enhances mathematical understanding and develops critical social skills such as communication and teamwork (Ersozlu et al., 2022). Studies have indicated that cooperative learning strategies, which often utilise non-digital tools, can significantly improve students' mathematical performance and attitudes towards the subject (Shah, 2023; Bhagwonparsadh, 2024). The dual-tool approach enhances content knowledge and promotes a constructivist mindset, encouraging pre-service teachers to view mathematics as a dynamic and interconnected discipline rather than a series of isolated facts (Carbonneau et al., 2018). The integration of both tool types can create a balanced learning environment that caters to diverse learning styles and preferences.

2.3 Theoretical frameworks

2.3.1 Technological pedagogical content knowledge

TPACK is a framework that integrates three fundamental domains of knowledge: technological knowledge (TK), pedagogical knowledge (PK), and content knowledge (CK). This integration is essential for effective instruction, particularly as teachers grapple with the complexities of incorporating both digital and non-digital tools into their pedagogical methodologies. The framework was originally introduced by Mishra and Koehler in 2006, highlighting the interaction among these knowledge domains to improve teaching and learning outcomes (Koehler et al., 2013). Furthermore, the TPACK framework comprises various components that further elucidate the interactions between technology, pedagogy, and content. These include Technological Content Knowledge (TCK), Technological Pedagogical Knowledge (TPK), and Pedagogical Content Knowledge (PCK) (Saubern et al., 2020; Raihanah, 2024). Technological Content Knowledge (TCK) pertains to understanding how technology can be employed to effectively represent specific content. In contrast, Technological Pedagogical Knowledge (TPK) encompasses the understanding of how technology can be leveraged to enhance pedagogical approaches. Conversely, Pedagogical Content Knowledge (PCK) emphasises the adaptation of pedagogical strategies to facilitate the effective instruction of specific content areas (Stapf & Martin, 2019; Padmavathi, 2017). The interaction of these elements is crucial for PSTs as they acquire the skills necessary to navigate the intricacies of teaching mathematics within a technology-enhanced environment.

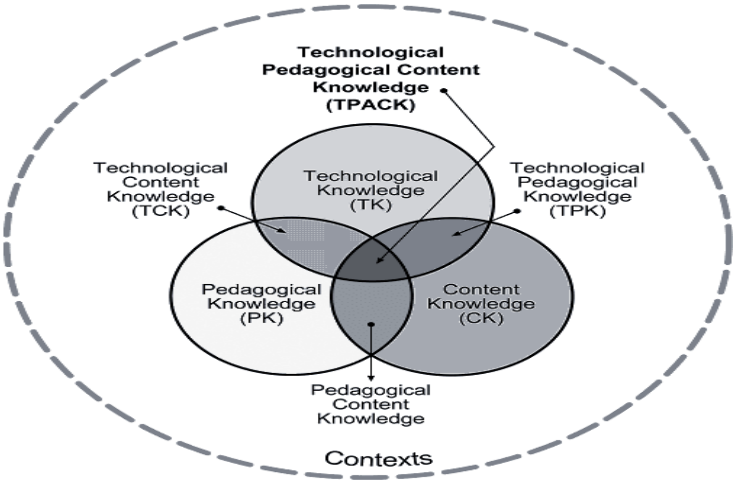


Figure 1: *The TPACK Framework* (adopted from Koehler & Mishra, 2009)

The TPACK framework, as shown in Figure 1 above, is particularly relevant in the education of pre-service teachers (PSTs), as these student teachers must cultivate competencies to integrate technology effectively into their pedagogical practices. Research indicates that engaging PSTs in TPACK-focused training can significantly improve their capacity to devise and execute technology-integrated teaching (Chai et al., 2020; Valtonen et al., 2020). For instance, studies have shown that PSTs participating in TPACK-based professional development programmes demonstrate increased confidence and competence in using technology to facilitate student learning (Admiraal et al., 2016; Chai et al., 2018). This is especially important in mathematics education, where visualising mathematical concepts through various teaching tools can lead to deeper understanding and engagement among students (Rohmitawati, 2018).

3. Materials and Methods

The study adopts an interpretivist qualitative approach, characterised by its focus on understanding the subjective experiences and perspectives of individuals within their specific contexts. Interpretivism highlights the significance of context and the meanings attributed by participants to their experiences, making it particularly suitable for examining the intricacies of PSTs' perceptions and practices regarding the utilisation of digital and non-digital tools in teaching mathematics (Mohajan, 2018). Participants were selected through purposeful sampling, a method frequently employed in qualitative research to identify individuals who possess specific characteristics relevant to the research question. Six PSTs from a cohort of twenty were sampled to take part in the study. Data collection was conducted through observations, focus group discussions, and semi-structured interviews, guided by the analytical framework.

Data collection occurred in three phases:

- Pre-Intervention Phase: PSTs' baseline PCK was assessed during microteaching sessions using observational tools and focus group discussions.

- Intervention Phase: A design-based intervention was conducted during lectures, exposing PSTs to digital tools like GeoGebra and opportunities to create non-digital solid objects. The intervention focused on teaching Grade 10 mathematics topics, including functions, measurements, and analytical geometry, over a semester.
- Post-Intervention Phase: PSTs' PCK was re-evaluated through microteaching sessions, utilising the same observational tools and focus group interviews. Lastly, we conducted semi-structured interviews to gather reflections on the tools' impact on their teaching practices and their ability to visualise mathematical concepts.

Observation data were analysed using a Likert scale checklist to assess PSTs' PCK, with scores converted into qualitative criteria based on established intervals (Widyoko, 2014; Sugiyono, 2013). The TPACK framework was applied to code and analyse data, categorising it into CK, PK, PCK, and TPACK. Emerging themes from the semi-structured interviews were integrated into the framework to enrich the analysis. This dual approach ensured a comprehensive understanding of how digital and non-digital tools influenced PSTs' instructional strategies and PCK.

The data analysis procedure is based on research instruments utilising a Likert scale presented as a checklist, as indicated in Table 1. Sugiyono (2019) asserts that Likert scales are employed to assess the attitudes, views, and perceptions of individuals or groups regarding social issues. The questionnaire employs the Likert scale presented as a checklist. The obtained data are then converted into qualitative criteria displayed in Table 2. The data gathered from the pre-and post-observations of the PSTs were analysed using the TPACK framework. The framework was also utilised to systematically analyse the data collected from observations by categorising it into the following domains: CK, PK, PCK, and TPACK.

Table 1: *Likert scale categories*

Interval	Criteria
1.00 < score < 1.75	Very low
1.75 < score < 2.50	Low
2.50 < score < 3.25	Good
3.25 < score < 4.00	Very good

Source: (Adopted from Widyoko, 2014)

Table 2: *The Percentage range values and qualitative criteria*

Value	Range	Qualitative Criteria
1	0 — 20	Very low
2	21 — 40	Low
3	41 — 60	Fair
4	61 — 80	Good
5	81 — 100	Very good

Source: (Adopted from Sugiyono, 2013)

Emerging themes from the interview data were identified and integrated into the analytical framework to comprehensively understand how digital and non-digital tools influenced preservice teachers' PCK. This approach ensured a nuanced analysis by linking the participants' observed and expressed experiences to the theoretical components of the TPACK framework.

This chapter establishes the study's trustworthiness through credibility, transferability, dependability, and confirmability. The study's credibility was augmented in multiple ways. We employed an effective approach involving the implementation of member checking (Lincoln & Guba, 1985). In this study, following the preliminary analysis of interviews and observations, the PSTs were solicited to reflect on the findings and verify the accurate representation of their experiences. This technique authenticates the data and enables participants to rectify any misconceptions, thus augmenting the study's credibility. We utilised various data collection methods, including interviews, lesson observations, focus group discussions, and video recordings of micro-teaching sessions. This facilitated the triangulation of results (Creswell & Poth, 2018). Observations of the micro-teaching sessions supplemented the interview data, enabling the researchers to discern how the PSTs implemented their knowledge of GeoGebra and other tools in practice. This comprehensive approach ensures that the findings are not derived from a singular perspective but instead represent a more holistic understanding of the participants' experiences.

To improve transferability, we included comprehensive descriptions of the research context, the training programme, the tools used (GeoGebra and non-digital instruments), and the instructional strategies implemented during micro-teaching (Merriam & Tisdell, 2016). This enabled other researchers to assess the applicability of the findings to their contexts.

The researchers upheld a detailed audit trail during the study to guarantee confirmability (Shenton, 2004). This comprises documentation of the research process, the decisions undertaken, and the modifications to the technique as the study advanced. By ensuring transparency in the research design and execution, the researchers enable others to replicate the techniques and evaluate the trustworthiness of the results. Lastly, we involved colleagues in mathematics education to evaluate the research methodology and offer insights on methodological coherence and data analysis (Creswell & Creswell, 2017). This provided an external viewpoint that aids in recognising potential biases or preconceptions that may have impacted the analysis. This collaborative method enhances the thoroughness of data analysis and bolsters the study's overall trustworthiness.

4. Presentation of Results

4.1 Biographic profile of pre-service teachers

The study involved six PSTs specialising in mathematics teaching in their third year. These participants, with an average age of 21 years, were part of a cohort of 21 PSTs, which included 6 males and 15 females from one class. For the sake of anonymity, they were assigned the following codes: PSTB, PSTD, PSTE, PSTI, PSTK, and PSTO. Each participant demonstrated a strong commitment to advancing their pedagogical and content knowledge in mathematics, aspiring to become effective teachers in diverse classroom settings. Their academic training included theoretical coursework and practical teaching components, equipping them with the necessary skills to integrate digital and non-digital teaching tools into their practice. This group was selected to represent various perspectives within the programme.

4.2 Pre-service teachers PCK on visualisation of mathematics concepts before and after the intervention

Interview transcripts and observation codes have been analysed to provide a narrative version of pre-service teachers' visualisation and teaching of mathematics concepts before and after the interventions.

4.2.1 Content Knowledge (CK)

CK is an understanding of the subject matter that does not consider the pedagogical aspects of teaching the subject (Chai et al., 2013). CK is essential as it shapes the distinctive cognitive approach to the discipline within each field of study. The analysis of CK competencies among PSTs prior to the interventions yielded an average score of 42, categorising their performance as fair. Table 3 indicates that PSTB, with a score of 54, and PSTO, with a score of 58, are classified within the satisfactory category. Nonetheless, PSTD (scoring 22), PSTE (scoring 39), PSTE (scoring 46), and PSTK (scoring 34) are all classified within the low category. The results suggest that the PSTs have not yet attained proficiency in the mathematical concepts necessary to effectively visualise and teach students (see Table 3). Moreover, the ratings for each component within the CK section for all PSTs are distinctly presented in Table 4. The data indicates that the performance of each component is classified as low, with an average score of 2.13 within the low category (refer to Table 4).

This was evident during the interview. When pre-service teachers were asked how they visualise and teach mathematics concepts, most indicated that they used traditional, low-tech methods to visualise mathematical concepts and found it difficult to explain some complex concepts in mathematics. Two pre-service teachers indicated the following:

> *I explained verbally and wrote on the board to visualize concepts." I wrote on the board to explain concepts and described visualizing a triangle to help illustrate the topic being taught. I also struggled to explain some concepts and hence I confused my students* (PSTO).
>
> *"I used the traditional method of teaching...I focused on the distance, gradient, and midpoint." I used only verbal explanations to teach the concept of calculating the distance between two points and the midpoint without any visualisation tool* (PSTK).

Freehand drawings, although a common substitute, were often imprecise and prone to causing misconceptions. PSTs recognised that this hindered their ability to accurately demonstrate mathematical associations. PSTK expressed his frustration:

> *The main challenge was that my drawings were freehand, not to scale, which could easily cause misconceptions. I could not demonstrate the parameters' impact visually, especially if they were negative.*

These challenges highlight the need for access to both digital and non-digital tools to improve clarity, accuracy, and engagement in mathematical teaching.

This chapter presents an analysis of content knowledge abilities among pre-service teachers post-intervention, revealing an average score of 79, categorised as good. Table 3 indicates that PSTB, with a score of 82, and PSTO, with a score of 86, are classified in the very good category. PSTD (score: 78), PSTE (score: 80), PSTI (score: 78), and PSTK (score: 72) are all classified within the good category. The results demonstrate that the PSTs have effectively mastered the mathematical content, facilitating their ability to visualise and teach mathematics to students (Table 3). Furthermore, Table 4 presents the scores for each component in the CK section post-intervention for all PSTs. The data indicate that each component's ability is classified as efficient, with an average score of 3.44 in the very good category (Table 4). Tables 3 and 4 provide an overview of the preparedness of PSTs in developing professional competencies pertinent to the mastery of the material intended for student instruction.

Table 3: *Pre-service teachers; CK abilities*

No	Pre-service teacher Code	CK Score before intervention	Qualitative Criteria	CK Score after intervention	Qualitative Criteria
1	PSTB	54	Fair	82	Very good
2	PSTD	22	Low	78	Good
3	PSTE	39	Low	80	Good
4	PSTI	46	Fair	78	Good
5	PSTK	34	Low	72	Good
6	PSTO	58	Fair	86	Very good
Average		42	Fair	79	Good

Table 4: *Pre-service teachers' average scores of CK abilities*

No	CK Component	Average score before intervention	The average score after intervention
CK1	Accuracy in explaining mathematical concepts using digital and non-digital visualisation tools.	2.00	3.4
CK2	Depth of understanding of the mathematical content demonstrated through their use of tools.	2.00	3.3
CK3	Sufficient knowledge of mathematics concepts	1.75	3.2
CK4	Use mathematical way of thinking	2.50	3.9
CK5	Use various ways and strategies of developing understanding of mathematics	2.40	3.4
	Average	2.13	3.44

The results of the data analysis indicate that the CK component of PSTs in preparing learning tools to assist students in visualising and learning mathematical concepts falls into the very good category. Therefore, it can be concluded that PSTs with strong content knowledge positively influence their competencies in other areas. PSTE correctly explained concepts related to 2D and 3D shapes using both digital and non-digital tools, demonstrating robust content knowledge in his explanation of the concepts.

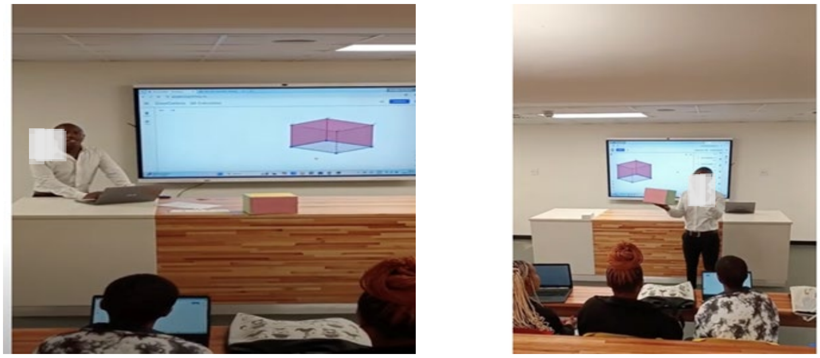


Figure 2: *Authors own illustrations of PSTE*

The field notes indicated that the digital tools available to PSTE were relevant to each mathematical concept he explained throughout his lesson. He seamlessly transitioned from discussing the properties of basic rectangular paper to a rectangular box and further performed dynamic visual demonstrations using GeoGebra software.

This observation was supported during the interview, where it was found that most PSTs (e.g., PSTE, PSTB) emphasised that using both digital and non-digital tools deepened their understanding of the content and enhanced their ability to teach effectively. This demonstrates that integrating these tools positively impacts both content mastery and pedagogical practices. Teachers such as PSTB, PSTO, and PSTK highlighted that tools like GeoGebra allowed for dynamic visualisation of abstract concepts, enriching their ability to connect mathematical ideas to the world as they explained algebraic functions and their graphical representations using both digital and non-digital methods.

The responses reveal significant improvements in the PSTs' content knowledge for teaching mathematics, emphasising the dynamic and interactive capabilities provided by digital tools like GeoGebra. Two pre-service teachers shared their joy by narrating:

> *I used points with coordinates to illustrate concepts dynamically, such as showing faces on a 3D net, which unfolded and closed dynamically* (PSTK).
>
> *With GeoGebra, I could click on a tool for the gradient, select the line, and immediately see the gradient displayed. I displayed triangles on the screen and used the formula for calculating distance directly within the GeoGebra window, which dynamically showed how the distance values changed* (PSTD).

Pre-service teachers noted that the interactive nature of the tools contributed to a deeper connection with the mathematical content. This finding aligns with a growing body of literature that emphasises the positive impact of digital tools on content knowledge (CK) development. This observation is supported by Rabi et al. (2021), who highlight that GeoGebra improves students' mathematical representation skills, enabling them to see and manipulate mathematical concepts in dynamic ways. Such interaction deepens the pre-service teachers' understanding of the content and equips them with the skills to present abstract mathematical ideas in more accessible and engaging ways. Moreover, these findings are consistent with research by Bakar et al. (2020), which found that using digital tools like GeoGebra enhanced pre-service teachers' ability to connect mathematical concepts with real-life applications. This capability to connect theory with practice is vital for effective teaching, as it encourages students to see the relevance of mathematics in their daily lives.

4.2.2 Pedagogical Knowledge (PK)

PK delineates the distinctive "general purpose" knowledge pertinent to pedagogy. It comprises a set of competencies that teachers must cultivate to effectively manage and organise instructional activities to achieve specified learning goals (Koehler et al., 2013). Analysis of PK abilities among the PSTs in visualising and teaching mathematical concepts using both digital and non-digital tools prior to the interventions revealed an average score of 28, placing them in the low category. Table 5 indicates that PSTB, with a score of 20, falls into the very low category. PSTD (score: 32), PSTE (score: 28), PSTI (score: 30), PSTK (score: 32), and PSTO (score: 26) are categorised as low. The results suggest that PSTs possess limited pedagogical knowledge necessary for effective student instruction (Table 5). In addition, Table 6 presents a clear overview of the scores for each component in the PK subsection across all PSTs. The data indicates that the performance of each component is classified as low, with an average score of 2.14 within this category (Table 6).

As indicated by the results, the low scores across all components of PK suggest that PSTs are inadequately prepared to deliver subject content effectively to their students. This was also evident during the interviews. Some PSTs indicated that they relied solely on traditional methods, such as freehand drawings, and expressed lower confidence in their ability to explain complex mathematical ideas effectively. They identified the absence of appropriate visualisation tools as a significant barrier. One PST lamented during the interview:

> *I was not that confident; I could not visually explain complex concepts. My confidence was a bit low because I lacked visualisation tools. I think this lack of tools might have impacted the overall effectiveness of my lesson* (PSTD).

The analysis of PK abilities among the PSTs post-intervention yielded an average score of 75, categorising it as good. Table 6 indicates that PSTB (score of 68), PSTO (score of 70), PSTD (score of 80), PSTE (score of 78), PSTI (score of 76), and PSTK (score of 75) are also categorised as good. The results demonstrate that the PSTs effectively mastered teaching following the intervention (see Table 5). Additionally, Table 6 presents a clear overview of the scores for each component of the PK aspect following the intervention for all PSTs. The data indicate that each component's ability is classified as efficient, with an average score of 3.44 falling within the very good category (see Table 6).

Table 5: Pre-service teachers' PK ability

No	Pre-service teacher Code	PK Score before intervention	Qualitative Criteria	PK Score after intervention	Qualitative Criteria
1	PSTB	20	Very less	68	Good
2	PSTD	32	Less	80	Good
3	PSTE	28	Less	78	Good
4	PSTI	30	Less	76	Good
5	PSTK	32	Less	75	Good
6	PSTO	26	Less	70	Good
Average		28	Less	75	Good

Table 6: Pre-service teachers' average scores for PK ability

No	CK Component	Average score before intervention	The average score after intervention
PK1	Ability to implement effective teaching strategies that integrate visualisation tools.	2.00	3.25
PK2	Use of formative assessment strategies through the integration of visualisation tools.	1.75	3.50
PK3	I know how to organise and maintain classroom management.	2.50	3.72
PK4	I am familiar with common student understandings and misconceptions.	1.25	3.00
PK5	I can adapt my teaching based-upon what students currently understand or do not understand.	2.00	3.25

This finding was evident during the interviews. PSTs indicated that the intervention programme had improved their efficiency and accuracy in concept representation, enhanced their pedagogical knowledge, and simplified the demonstration of complex mathematical ideas. This reduced the effort required for explanations while improving the accuracy of visualisations. This marked a significant improvement over their previous reliance on freehand drawings or static visuals. PSTD shared her joy:

> *I could immediately see the gradient displayed with GeoGebra, which was a big improvement. Extending and reducing a triangle's vertices, I could dynamically show how the distance values changed as the points were moved.*

PSTs diversified their instructional approaches by combining technology tools like GeoGebra with interactive elements like Mentimeter. This broadened their pedagogical repertoire and allowed for more engaging lessons. PSTB concurred with PSTD:

> *I used GeoGebra software to visualise concepts and an interactive tool, Mentimeter, at the start to engage learners. In addition, I also incorporate physical non-digital objects as well in my instructions.*

The intervention programme significantly transformed the PSTs' approaches to teaching mathematics. Integrating GeoGebra and other digital tools enabled dynamic and precise visualisations, improved student engagement, and enhanced the clarity of explanations. The shift from static or traditional methods to interactive and technology-based approaches marked a critical development in their teaching practices. The literature supports this finding. Research has consistently demonstrated that the use of GeoGebra in mathematics education facilitates a deeper understanding of mathematical concepts. For instance, Putra et al. (2021) highlight that prospective elementary teachers who integrated GeoGebra into their training were able to construct their mathematical knowledge of three-dimensional shapes, thereby enhancing their understanding of complex mathematical theories and practices. This aligns with findings from Barçin and Yenmez (2023), who noted that GeoGebra software aids in developing mathematical language and self-efficacy among students, suggesting that teachers trained in such technologies can better facilitate communication in mathematics classrooms.

4.2.3 Pedagogical Content Knowledge (PCK)

PCK refers to the integration of content knowledge with teaching approaches. It encompasses more than mere content expertise or familiarity with general pedagogical principles; it involves comprehending the specific interactions between content and pedagogy (Koehler et al., 2013). The analysis of PCK abilities among PSTs in visualising and teaching mathematical concepts, both digital and non-digital, prior to the interventions yielded an average score of 27, categorising it as low. Table 7 indicates that PSTB, with a score of 17, and PSTO, with a score of 19, are classified in the very low category. Nonetheless, PSTD (score of 28), PSTE (score of 24), PSTI (score of 35), and PSTK (score of 38) are all classified within the low category. Furthermore, Table 8 presents a clear overview of the scores for each component in the PCK aspects across all PSTs. The data indicates that the performance of each component is classified as very low, with an average score of 1.85 falling within the low category (Table 8).

The low scores across all components of PCK, as indicated by the results, suggest that PSTs are inadequately prepared to effectively deliver subject content to their students. This finding highlights the urgent need for targeted intervention programmes that focus on building PCK. This conclusion was evident during the interviews, where PSTs indicated that they relied on verbal explanations to communicate their ideas to students. Two PSTs lamented:

> *I relied solely on the traditional method of teaching without incorporating any visual aids or technological tools and that made me struggle to teach the concept. When teaching Analytical Geometry, I focused on explaining concepts like distance, gradient, and midpoint through verbal explanations and writing on the board. I didn't explore using diagrams, graphs, or software to help students visualise these concepts, which might have limited their understanding* (PSTI).

PSTO concurred with PSTI:

> *"In my lesson on Analytical Geometry, I focused on calculating the distance between two points and finding the midpoint of a line. I didn't use any technology or interactive tools to demonstrate these concepts. My approach was primarily verbal, and I didn't provide any visual representations or practical examples that could help students relate to the material more effectively.*

This finding indicates that PSTs' reliance on traditional, verbal-only teaching methods suggests insufficient preparation for modern classrooms where technology integration is essential. This may limit their ability to meet diverse learner needs and leverage digital tools to enhance engagement and comprehension.

The analysis of PCK abilities among the PSTs following the interventions yielded an average score of 77, categorising it as good. Table 7 indicates that PSTB (score of 69), PSTO (score of 68), PSTE (score of 72), and PSTI (score of 78) are all classified within the good category. Both PSTK, with a score of 83, and PSTD, with a score of 89, are classified within the good category. The results demonstrate that the PSTs effectively mastered the material, enabling them to visualise and teach mathematics using technology proficiently (Table 8). Additionally, Table 7 presents a clear overview of the scores for each component in the PCK aspects following the intervention for all PSTs. The data indicates that each component's ability is classified as efficient, with an average score of 3.40 falling within the very good category (Table 7). Tables 7 and 8 provide an overview of the PSTs' preparedness to visualise and instruct mathematics concepts, demonstrating robust PCK competencies pertinent to the mastery of the material intended for students.

Table 7: *Pre-service teachers' PCK ability*

No	Pre-service teacher Code	PCK Score before intervention	Qualitative Criteria	PCK Score after intervention	Qualitative Criteria
1	PSTB	17		69	Good
2	PSTD	28		89	Very good
3	PSTE	24		72	Good
4	PSTI	35		78	Good
5	PSTK	38		83	Very Good
6	PSTO	19		68	Good
Average		27		77	Good

Table 8*: Pre-service teachers' average score for PCK ability*

No	CK Component	Average score before intervention	Average score after intervention
PCK1	Ability to explain complex mathematical ideas using both digital and non-digital visualisation tools to enhance learners' understanding.	2	3.45
PCK2	Adaptability in using visual tools to address common learner misconceptions.	2.20	3.25
PCK3	Ability to select effective teaching approaches to guide student thinking and learning in mathematics.	1.34	3.50

The data from the field notes concurred with the data from the observation schedules. This finding is also consistent with the results from the interviews. PSTs indicated that the intervention enhanced their PCK, enabling them to explain complex mathematical concepts using both digital and non-digital visualization tools to improve learners' understanding.

PSTB narrated:

> *I used GeoGebra to illustrate how algebraic changes affect graphical outputs and employ physical models to reinforce the same concept. The GeoGebra and 2D/3D shapes visually and dynamically helped me to explain complex concepts such as the total surface area. Then again, I managed also to recognize if my learner struggles, then I would switch to a different visualization tool to clarify and target any misconceptions. For instance, when learners struggled to understand the surfaces of a rectangular prism, I would shift from using a non-digital box to a GeoGebra window, which dynamically unpacked/unfolded the box into 6 faces.*

This finding suggests that the intervention programme has significantly impacted the development of PSTs' PCK. It is consistent with studies by Dağlı and Elif (2021), which found that GeoGebra helps PSTs better identify and address common misconceptions in mathematics. Misconceptions are a frequent barrier to student learning in mathematics, and the ability to diagnose and correct them requires a deep understanding of both the content being taught and the pedagogical methods that can make that content more accessible. By using GeoGebra, PSTs are not only enhancing their own understanding of mathematical concepts but also developing strategies to help students overcome difficulties in learning these concepts. Additionally, the findings from Tröbst et al. (2019) reinforce the idea that instruction focused on PCK improves content knowledge and pedagogical strategies. This suggests that the intervention aimed at enhancing PCK through GeoGebra is effective not only in developing technical proficiency with the tool but also in deepening PSTs' overall understanding of how to teach mathematics.

4.2.4 Technological Pedagogical and Content Knowledge (TPACK)

TPACK is a framework that delineates the knowledge required by teachers to enhance pedagogical practices and conceptual understanding through the integration of technology within the learning environment (Koehler & Mishra, 2009). The analysis of TPACKabilities among Pre-Service Teachers (PSTs) in visualising and teaching mathematical concepts, both digital and non-digital, prior to the interventions yielded an average score of 25, categorising it as low. Table 9 indicates that PSTB, with a score of 38, is classified in the low category. PSTD (21), PSTE (24), PSTI (20), PSTK (28), and PSTO (18) are categorised as very low scores. Furthermore, Table 9 presents a clear overview of the scores for each component within the TPACK framework for all PSTs. The data indicates that the performance of each component is classified as low, with an average score of 1.61, falling within the very low category (Table 10).

The analysis of TPACK abilities among PSTs post-intervention yielded an average score of 75, categorising the results as good. Table 9 indicates that PSTB (score: 84), PSTO (score: 69), PSTD (score: 72), PSTE (score: 70), PSTI (score: 74), and PSTK (score: 78) are all classified within the good category. The results indicate that the PSTs effectively mastered the material, enabling them to utilise technological tools for visualising and teaching mathematical concepts.

Additionally, Table 10 presents a clear overview of the scores for each component in the TPACK aspect following the intervention for all PSTs. The data indicates that each component's ability is classified as good, with an average score of 3.24, falling within the very good category (Table 10). Tables 9 and 10 present an overview of the preparedness of PSTs in utilising technology to visualise and teach mathematical concepts.

***Table 9:** Pre-service teachers' TPACK ability*

No	Pre-service teacher Code	TPACK Score before intervention	Qualitative Criteria	TPACK Score after intervention	Qualitative Criteria
1	PSTB	38		84	Very good
2	PSTD	21		72	Good
3	PSTE	24		70	Good
4	PSTI	20		74	Good
5	PSTK	28		78	Good
6	PSTO	18		69	Good
Average		25		75	Good

***Table 10:** Pre-service teacher's average score for TPACK ability*

No	TPACK Component	Average score before intervention	Average score after intervention
TPACK1	Mastery in integrating technology and pedagogy to enhance content delivery and learner engagement.	1,48	3.20
TPACK2	Ability to teach lessons that appropriately combine algebraic functions, technologies and teaching approaches	1.38	3.50
TPACK3	Ability to teach lessons that appropriately combine analytical geometry, technologies and teaching approaches	1.27	3.25
TPACK4	Ability to teach lessons that appropriately combine measurements, technologies and teaching approaches	2.32	3.00

ACK4 ility to teach lessons that appropriately com2 0
asurements, technologies and teaching approaches

This finding was also evident during the interviews. PSTs indicated that they selected technologies for use in their microteaching classrooms that enhance what they teach, how they teach, and what students learn. The findings resonate with studies showing that preservice teachers who participate in TPACK-based professional development programmes demonstrate increased confidence and competence in using technology to support student learning (Admiraal et al., 2016; Chai et al., 2018). This is particularly important in mathematics education, where visualising mathematical concepts through various teaching tools can lead to deeper understanding and engagement among students (Rohmitawati, 2018). This finding further aligns with the study by Bueno et al. (2021), which discussed how an online course incorporating

GeoGebra facilitated the development of TPACK among mathematics teachers, emphasising the importance of blending technology with pedagogical strategies. This integration prepares PSTs to use digital tools effectively, enhancing their teaching efficacy.

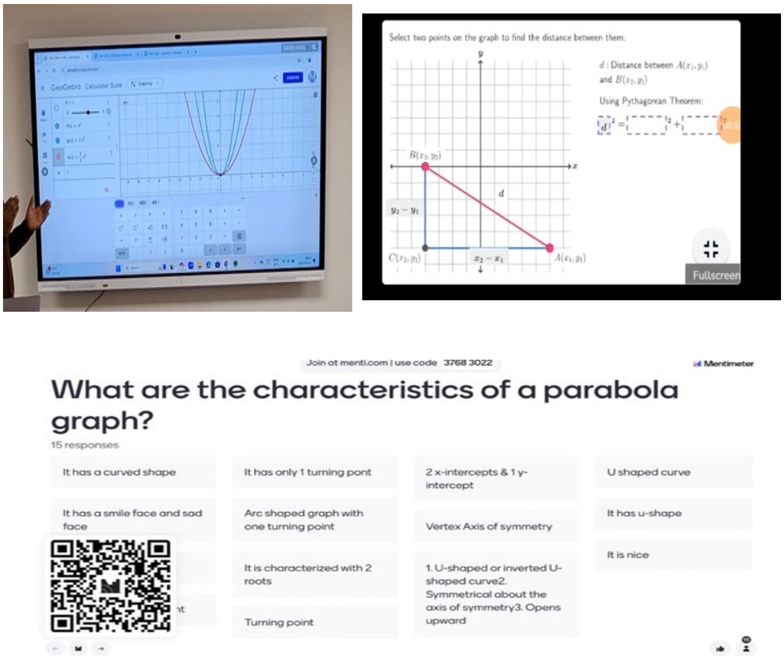


Figure 3: Pre-service use of digital tool in teaching (Author's own illustration)

Figure 3 shows that PSTs employed technologies such as GeoGebra and Mentimeter to enhance learners' participation and conceptual understanding, thereby influencing student learning outcomes. Studies have demonstrated that when PSTs effectively utilise GeoGebra in their teaching, their students exhibit improved understanding and engagement in mathematics (Putra et al., 2021). This reciprocal relationship between teacher preparation and student learning underscores the importance of equipping PSTs with the necessary tools and knowledge to foster positive learning experiences.

4.3 Pre-service teachers' experiences using dual digital and non-digital tools approaches during their microteaching

PSTs reflected on their experiences integrating digital and non-digital tools during microteaching sessions after the intervention. Their responses highlight a mix of positive outcomes, personal growth, and challenges encountered in balancing these approaches. Several participants (PSTE, PSTB, PSTD) described their overall experience as transformative, emphasising increased confidence and an appreciation for the importance of tool integration in teaching practice. Some specifically mentioned a shift in perspective, viewing the dual approach as essential for their future teaching. GeoGebra was repeatedly highlighted as a powerful digital tool for enhancing teaching and learning. PSTs noted its ability to provide precise, dynamic visuals, which helped explain abstract concepts such as distance and midpoint. Although digital tools were highly effective, some participants (PSTI, PSTO) emphasised the continued importance of non-digital tools for accessibility and foundational teaching, particularly for students who struggle with digital technologies.

Some PSTs highlighted how the dual approach boosted their teaching confidence, allowing them to engage students dynamically while maintaining clarity in explanations. This combination was seen as making their lessons more effective and adaptable. PSTD had this to say:

> *I employed both digital and non-digital tools during my microteaching. Both tools helped me reach diverse learners and encouraged interaction, as students could better relate to the practical, visual aspects. Using GeoGebra encouraged more questions from my students, as they could see changes immediately, making them curious and interactive and encouraging them to participate actively.*

This finding indicates that the approach benefits PSTs during teaching practice. Several studies align with these findings, emphasising the transformative impact of such an approach on PSTs' confidence and pedagogical effectiveness. Using digital tools like GeoGebra, which facilitates dynamic visualisation of mathematical concepts, enhances PSTs' teaching confidence. Research by Dintarini et al. (2024) supports this, noting that GeoGebra enables PSTs to present abstract concepts interactively, making lessons more engaging and student-centred. Similarly, Mentimeter allows real-time student feedback and interaction, creating a two-way communication channel that fosters engagement and responsiveness (Kohnke & Moorhouse, 2021). When combined with non-digital tools, such as physical shapes for tactile learning, PSTs reported feeling more in control and capable of delivering clear and comprehensive explanations.

Nevertheless, PSTs' responses revealed technical and practical challenges when integrating digital and non-digital tools into their microteaching sessions. These challenges were related to technical difficulties, time constraints, tool familiarity, and accuracy in teaching methods. Some PSTs faced connectivity issues with GeoGebra, which initially hindered their teaching ability. Once resolved, however, they felt more confident in using digital tools. Others also mentioned challenges related to students' familiarity with GeoGebra, which required additional effort to ensure all students could effectively engage with the tool. PSTs lamented:

> *My biggest challenge was ensuring that all students could interact with the digital tools, as not all students were familiar with using GeoGebra at first. That took most of my time to navigate through with the students.*

Some PSTs found that switching between tools (digital and non-digital) required careful time management and planning, making it difficult to focus on one method fully. Moreover, the time needed to become comfortable with GeoGebra limited what they felt they could achieve with the tool. PSTO shared her frustration:

The main challenge was the time and effort to switch between tools that required planning. Also, becoming comfortable with GeoGebra took time, so I felt limited in what I could achieve without more practice.

Some PSTs expressed challenges related to their lack of proficiency in using GeoGebra, as they had limited exposure to the tool during the microteaching preparation phase. PSTI narrated:

I was not really skilled in GeoGebra, probably because of my short time exposure to the tool.

The challenges faced by PSTs when integrating digital and non-digital tools were primarily related to technical issues (e.g., connectivity and tool familiarity), the need for careful planning to balance both methods, and the lack of accuracy in traditional tools, such as freehand drawings.

5. Conclusions and Recommendations

In concluding this chapter, it is essential to reflect on the main arguments presented throughout the text, the key findings regarding the enhancement of pedagogical content knowledge (PCK) among preservice teachers (PSTs), and the broader implications of these findings for mathematics education. The chapter establishes that integrating digital and non-digital tools in mathematics education significantly enhances PSTs' PCK. This dual approach allows PSTs to experience a richer pedagogical repertoire, equipping them with diverse strategies to address various learning styles and needs in future classrooms. The findings indicate that PSTs who engage with various teaching tools develop a more profound understanding of mathematical concepts and their instructional implications, which is critical for effective teaching (Tröbst et al., 2019). Using manipulatives and visual aids, alongside digital resources, fosters a more comprehensive grasp of mathematical principles, enabling PSTs to visualise and convey complex ideas more effectively. Moreover, the chapter highlights the importance of mathematics teaching modules in shaping PSTs' beliefs, attitudes, and self-efficacy regarding mathematics instruction. The dual-tool approach enhances content knowledge and promotes a constructivist mindset, encouraging PSTs to view mathematics as a dynamic and interconnected discipline rather than a series of isolated facts (Carbonneau et al., 2018). This shift in perspective is crucial for developing future teachers who can inspire and engage their students in meaningful mathematical teaching and learning.

The findings also underscore the role of collaborative experiences in the development of PCK. Engaging preservice teachers in group activities that utilise both digital and non-digital tools fosters a community of practice where they can share insights, strategies, and challenges. This collaborative learning environment enhances their ability to reflect on their teaching practices and adapt their approaches based on peer feedback and shared experiences (Ersozlu et al., 2022). Such interactions are vital for building a supportive network that can sustain preservice teachers throughout their professional journeys. Furthermore, the chapter addresses the broader implications of these findings for teacher education programmes. In order to keep up with the changing nature of education, teacher preparation programmes need to incorporate hands-on experience with both digital and traditional teaching resources to prepare PSTs to meet the demands of contemporary classrooms and equip them with the skills necessary to foster critical thinking and problem-solving abilities in their students, as advised by Mouza et al. (2017).

In conclusion, the chapter has demonstrated that using both digital and non-digital teaching tools significantly enhances pre-service teachers' (PSTs) Pedagogical Content Knowledge (PCK), ultimately leading to improved mathematics instruction. Integrating these tools fosters a deeper understanding of mathematical concepts, promotes positive attitudes towards teaching mathematics, and encourages collaborative learning experiences. As teacher education programs continue to evolve, it is essential to prioritise the development of PSTs' PCK through the strategic use of diverse teaching tools, ensuring they are equipped to inspire and educate future generations of learners.

This chapter highlights the transformative potential of integrating digital tools, such as GeoGebra and Mentimeter, alongside non-digital tools like physical objects, to enhance pre-service teachers' PCK in mathematics. The findings emphasise the need for teacher education programs to prioritise blended pedagogical approaches that combine technology with hands-on materials. This integration equips pre-service teachers with diverse strategies to effectively explain abstract mathematical concepts and address various student learning styles.

The successful use of tools like GeoGebra facilitates dynamic visualisations, promoting a deeper conceptual understanding of mathematical ideas. In contrast, non-digital tools allow for tangible manipulation, which is especially beneficial for foundational learners. Intervention training programs that stress the complementary use of both types of tools can help pre-service teachers design engaging lessons, accommodate diverse learners, and build confidence in adopting innovative teaching practices.

Furthermore, this chapter underlines the importance of incorporating targeted professional development workshops into teacher education curricula. These workshops should provide hands-on experience with both digital and non-digital tools, ensuring that pre-service teachers can seamlessly integrate them into their lesson plans. Such training enhances PCK and prepares future teachers to create inclusive, interactive, and resource-rich mathematics classrooms. This, in turn, contributes to improved learning outcomes and addresses the demands of 21st-century education.

6. Declarations

Funding: This research did not receive any external funding.

Acknowledgements: We extend our heartfelt gratitude to the pre-service teachers who enthusiastically participated in the research intervention during their microteaching sessions.

Conflicts of Interest: The authors declare no conflict of interest.

References

Admiraal, W., Vugt, F. v., Kranenburg, F., Köster, B., Smit, B., Weijers, S., & Lockhorst, D. (2016). Preparing pre-service teachers to integrate technology into K–12 instruction: Evaluation of a technology-infused approach. *Technology, Pedagogy and Education, 26*(1), 105–120. https://doi.org/10.1080/1475939x.2016.1163283

Agustina, L., Utomo, B., & Chikita, L. (2018). Improving mathematical ability and student learning outcomes through realistic mathematic education (RME) approach. *International Journal of Engineering & Technology, 7*(2.10), 55. https://doi.org/10.14419/ijet.v7i2.10.10954

Bakar, A. R., et al. (2020). Enhancing pre-service teachers' mathematical understanding through GeoGebra. *International Journal of Mathematical Education in Science and Technology, 51*(3), 391–407.

Ball, D., Thames, M., & Phelps, G. (2008). Content knowledge for teaching. *Journal of Teacher Education, 59*(5), 389–407. https://doi.org/10.1177/0022487108324554

Barçın, N., & Yenmez, A. A. (2023). GeoGebra software on the mathematical language developments and self-efficacy perceptions of students. *International e-Journal of Educational* Studies, *7*(15), 682–704. https://doi.org/10.31458/iejes.1340349

Bhagwonparsadh, Y. (2024). The effects of educators' mathematics pedagogical content knowledge on the mentoring of grade 12 students using problem-based learning. *Journal of Culture and Values in Education, 7*(1), 99–117. https://doi.org/10.46303/jcve.2024.7

Buchori, A., & Puspitasari, G. D. (2023). Development of mathematics learning media assisted by the MathcityMap to improve students' critical thinking skills. *Journal of Higher Education Theory and* Practice, *23*(10).

Bueno, R. W. d. S., Lieban, D., & Ballejo, C. C. (2021). Mathematics teachers' TPACK development based on an online course with GeoGebra. *Open Education Studies, 3*(1), 110–119. https://doi.org/10.1515/edu-2020-0143

Carbonneau, K. J., Xue, Z., & Ardasheva, Y. (2018). Preservice educators' perceptions of manipulatives: The moderating role of mathematics teaching self-efficacy. *School Science and Mathematics, 118*(7), 300–309. https://doi.org/10.1111/ssm.12298

Chai, C. S., Koh, J. H. L., & Tsai, C. C. (2013). A review of technological pedagogical content knowledge. *Journal of Educational Technology & Society, 16*(2), 31–51.

Chai, C., Koh, J., & Teo, Y. (2018). Enhancing and - 52 -aption- 52 -i teachers' design beliefs and efficacy of technological pedagogical content knowledge for 21st-century quality learning. *Journal of Educational Computing Research, 57*(2), 360–384. https://doi.org/10.1177/0735633117752453

Chai, C., Rahmawati, Y., & Jong, M. (2020). Indonesian science, mathematics, and engineering pre-service teachers' experiences in STEM-TPACK design-based learning. *Sustainability, 12*(21), 9050. https://doi.org/10.3390/su12219050

Charalambous, C. Y., Hill, H. C., Chin, M. J., & McGinn, D. (2020). Mathematical content knowledge and knowledge for teaching: Exploring their distinguishability and contribution to student learning. *Journal of Mathematics Teacher Education, 23*(6), 579–613.

Clark-Wilson, A. (2020). Mathematics education and technology (pp. 1126–1134). https://doi.org/10.1007/978-3-030-10576-1_96

Cirneanu, A. L., & Moldoveanu, C. E. (2024). Use of digital technology in integrated mathematics education. *Applied System* Innovation, *7*(4), 66.

Creswell, J. W., & Creswell, J. D. (2017). *Research design: Qualitative, quantitative, and mixed methods approaches*. Sage Publications.

Creswell, J. W., & Poth, C. N. (2018). *Qualitative inquiry and research design: Choosing among five approaches*. Sage Publications.

Dağlı, H., & Elif, D. (2021). The relationship between preschool teachers' pedagogical content knowledge in mathematics, children's math ability and liking. *Research in Pedagogy, 11*(2), 359–376. https://doi.org/10.5937/istrped2102359d

Dahal, N., Pant, B. P., Shrestha, I. M., & Manandhar, N. K. (2022). Use of GeoGebra in teaching and learning geometric transformation in school mathematics. *International Journal of Interactive Mobile Technologies (iJIM), 16*(08), 65–78. https://doi.org/10.3991/ijim.v16i08.29575

Dintarini, M., Budiarto, M. T., & Fuad, Y. (2024). From concept to assessment: Developing a GeoGebra assisted mathematics test to measure semiotic-spatial thinking using abstraction in context framework. In *Proceedings of the 2024 8th International Conference on Education and Multimedia Technology* (pp. 162–170).

Dockendorff, M., & Solar, H. (2018). ICT integration in mathematics initial teacher training and its impact on visualization: The case of GeoGebra. *International Journal of Mathematical Education in Science and Technology, 49*, 66–84. https://doi.org/10.1080/0020739X.2017.1341060

Ekiz-Kiran, B., Boz, Y., & Oztay, E. S. (2021). Development of pre-service teachers' pedagogical content knowledge through a PCK-based school experience course. *Chemistry Education Research and Practice, 22*(2), 415–430.

Ersozlu, Z., Blake, D., Uşak, M., & Hawken, S. (2022). Addressing pre-service teacher's reasons for mathematics and test anxiety. *European Journal of Educational Research, 11*(3), 1715–1728. https://doi.org/10.12973/eu-jer.11.3.1715

Fki, N. (2023). Towards more effective feedback strategies to enhance microteaching for pre-service teachers at Iseah Mahdia. *Studies in Applied Linguistics and TESOL, 23*(2). https://doi.org/10.52214/salt.v23i2.12090

García-Esteban, S., Laborda, J. G., & Llamas, M. R. (2016). ICTs, ESPs and ZPD through microlessons in teacher education. In *New Perspectives on Teaching and Working with Languages in the Digital Era* (pp. 105–113). https://doi.org/10.14705/rpnet.2016.tislid2014.426

Griffin, J. D. (2003). Technology in the teaching of neuroscience: Enhanced student learning. *Advances in Physiology Education, 27*(3), 146–155.

Horzum, T., & Ünlü, M. (2017). Pre-service mathematics teachers' views about GeoGebra and its use. *Acta Didactica Napocensia, 10*(3), 77–90. https://doi.org/10.24193/I.10.3.8

Hussein, S., & Khoiruzzadittaqwa, M. (2024). The effectiveness of the generative learning model: Student understanding and engagement in mathematics learning. *AB-JME: Al-Bahjah Journal of Mathematics* Education, *2*(1), 148–162. https://doi.org/10.61553/abjme.v2i1.152

Koehler, M., & Mishra, P. (2009). What is technological pedagogical content knowledge (TPACK)? *Contemporary Issues in Technology and Teacher* Education, *9*(1), 60–70.

Koehler, M., Mishra, P., & Cain, W. (2013). What is technological pedagogical content knowledge (TPACK)? *Journal of Education, 193*(3), 13–19. https://doi.org/10.1177/002205741319300303

Kohnke, L., & Moorhouse, B. L. (2021). Adopting HyFlex in higher education in response to COVID-19: Students' perspectives. *Open Learning: The Journal of Open, Distance and E-Learning, 36*(3), 231–244. https://doi.org/10.1080/02680513.2021.1906641

Kuzu, Ç. I. (2021). Views of mathematics teacher candidates on the use of GeoGebra in probability teaching. *Asian Journal of Contemporary Education, 5*(1), 45-56. https://doi.org/10.18488/journal.137.2021.51.45.56

Lincoln, Y. S., & Guba, E. G. (1985). *Naturalistic inquiry*. Sage Publications.

Marange, I. Y., & Tatira, B. (2023). Teaching Euclidean geometry with GeoGebra: Perceptions for in-service mathematics teachers. *Eurasia Journal of Mathematics, Science and Technology Education, 19*(12), em2367. https://doi.org/10.29333/ejmste/13861

Marshman, M., & Porter, G. (2013). Pre-service teachers' pedagogical content knowledge: Implications for teaching. *Mathematics Education Research Group of Australasia.*

Merriam, S. B., & Tisdell, E. J. (2016). *Qualitative research: A guide to design and implementation.* Jossey-Bass.

Mohajan, H. (2018). Qualitative research methodology in social sciences and related subjects. *Journal of Economic Development Environment and People, 7*(1), 23. https://doi.org/10.26458/jedep.v7i1.571

Mouza, C., Huí, Y., Pan, Y., Ozden, S. Y., & Pollock, L. (2017). Resetting educational technology coursework for pre-service teachers: A computational thinking approach to the development of technological pedagogical content knowledge (TPACK). *Australasian Journal of Educational Technology, 33*(3). https://doi.org/10.14742/ajet.3521

Muslim, N. (2023). A systematic review of GeoGebra in mathematics education. *International Journal of Academic Research in Progressive Education and Development, 12*(3). https://doi.org/10.6007/ijarped/v12-i3/19133

Ng, O., & Tsang, W. (2021). Constructionist learning in school mathematics: Implications for education in the fourth industrial revolution. *ECNU Review of Education, 6*(2), 328-339. https://doi.org/10.1177/2096531120978414

Nzaramyimana, E., Mukandayambaje, E., Iyamuremye, L., Hakizumuremyi, V., & Ukobizaba, F. (2021). Effectiveness of GeoGebra towards students' active learning, performance and interest to learn mathematics. *International Journal of Mathematics and Computer Research, 9*(10). https://doi.org/10.47191/ijmcr/v9i10.05

Padmavathi, M. (2017). Preparing teachers for technology-based teaching-learning using TPACK. *I-Manager's Journal on School Educational Technology, 12*(3), 1. https://doi.org/10.26634/jsch.12.3.10384

Putra, Z., Hermita, N., Alim, J., Dahnilsyah, D., & Hidayat, R. (2021). GeoGebra integration in elementary initial teacher training: The case of 3-D shapes. *International Journal of Interactive Mobile Technologies (IJIM), 15*(19), 21. https://doi.org/10.3991/ijim.v15i19.23773

Olawale, B. E. (2023). Teacher quality and learner achievement in South African schools. *IntechOpen*. https://doi.org/10.5772/intechopen.113774

Olawale, B. E. (2024). Impact of pre-service teacher education programme on mathematics student teachers' teaching practices during school experiences. *Education Sciences, 14*(7), 762. https://doi.org/10.3390/educsci14070762

Rabi, F., Feng-qi, M., Aziz, M., Ullah, M. I., & Abduraxmanovna, N. H. (2021). The impact of the use of GeoGebra on students' mathematical representation skills and attitude. *European Journal of Education Studies, 8*(12). https://doi.org/10.46827/ejes.v8i12.4007

Raihanah, D. (2024). Analysis of technological pedagogical content knowledge (TPACK) ability for prospective chemistry teacher students and chemistry teachers: A literature review. *Jurnal Pijar Mipa, 19*(1), 67-74. https://doi.org/10.29303/jpm.v19i1.6395

Ramadhanti, D., & Yanda, D. (2021). The strategy of observing, imitation, and modification in the preparation of lesson plan in microteaching practice. *Edukatif Jurnal Ilmu Pendidikan, 3*(6), 5202-5212. https://doi.org/10.31004/edukatif.v3i6.1664

Rice, A., & Kitchel, T. (2015). Preservice agricultural education teachers' experiences in and anticipation of content knowledge preparation. *Journal of Agricultural Education, 56*(3), 90–104. https://doi.org/10.5032/jae.2015.03090
Rohmitawati, R. (2018). The implementation of the TPACK (technology, pedagogy, and content knowledge) framework on Indonesian online mathematics teachers training. *Southeast Asian Mathematics Education Journal, 8*(1), 61-68. https://doi.org/10.46517/seamej.v8i1.64
Salavati, S. (2016). *Use of digital technologies in education: The complexity of teachers' everyday practice* [Doctoral dissertation, Linnaeus University Press].
Samuel, J. (2019). Developing a theoretical framework for visualization-based pedagogical content knowledge (V-PCK) based on middle school teachers' views and uses of visualisations as an instructional tool. *Dissertations*. 1669. https://aquila.usm.edu/dissertations/1669
Sangwin, C. (2023). Running an online mathematics examination with STACK. *International Journal of Emerging Technologies in Learning (iJET), 18*(03), 192–200. https://doi.org/10.3991/ijet.v18i03.35789
Sarama, J., Clements, D. H., & Guss, S. S. (2021). Longitudinal evaluation of a scale-up model for professional development in early mathematics. In *Early Childhood Teachers' Professional Competence in Mathematics* (pp. 163–186). Routledge. https://doi.org/10.4324/9781003172529-10
Sari, W., & Mutmainah, S. (2018). Increased motivation and results of student math learning through realistic mathematics education. *Journal of Mathematics Education, 3*(1). https://doi.org/10.31327/jomedu.v3i1.479
Saubern, R., Henderson, M., Heinrich, E., & Redmond, P. (2020). TPACK – time to reboot? *Australasian Journal of Educational Technology, 36*(3), 1-9. https://doi.org/10.14742/ajet.6378
Setyaningrahayu, I., Widhi, P., & Murtisari, E. (2019). The use of video-based reflection to facilitate pre-service English teachers' self-reflection. *Indonesian Journal of English Language Teaching, 14*(2), 147-165. https://doi.org/10.25170/ijelt.v14i2.1443
Shah, Y. (2023). Analytical study of cooperative learning in mathematics. *Patan Pragya, 12*(02), 120-126. https://doi.org/10.3126/pragya.v12i02.64211
Shenton, A. K. (2004). Strategies for ensuring trustworthiness in qualitative research projects. *Education for Information, 22*(2), 63-75.
Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*, 4-14.
Stapf, K., & Martin, B. (2019). TPACK + mathematics: A review of current TPACK literature. *International Journal on Integrating Technology in Education, 8*(3), 13-20. https://doi.org/10.5121/ijite.2019.8302
Sugiyono. (2019). *Metode penelitian kuantitatif* (Cetakan ke-2). Alfabeta.
Sugiyono. (2013). *Metode penelitian - 55 -aption- 55 -in: Pendekatan kuantitatif, kualitatif, dan R&D*. Bandung: Alfabeta.
Tondeur, J., Hermans, R., van Braak, J., & Valcke, M. (2008). Exploring the link between teachers' educational beliefs and profiles and different types of computer use in the classroom. *Computers in Human Behaviour, 24*, 2541-2553.
Tröbst, S., Kleickmann, T., Heinze, A., Bernholt, A., Rink, R., & Kunter, M. (2018). Teacher knowledge experiment: Testing mechanisms underlying the formation of preservice elementary school teachers' pedagogical content knowledge concerning fractions and

fractional arithmetic. *Journal of Educational Psychology, 110*(8), 1049-1065. https://doi.org/10.1037/edu0000260
Urrutia, F. Z., Loyola, C. C., & Marín, M. H. (2019). A tangible user interface to facilitate learning of trigonometry. *International Journal of Emerging Technologies in Learning (iJET), 14*(23), 152. https://doi.org/10.3991/ijet.v14i23.11433
Uwurukundo, M., Maniraho, J., & Tusiime, M. (2022). Enhancing students' attitudes in learning 3-D geometry using GeoGebra. *International Journal of Learning Teaching and Educational Research, 21*(6), 286-303. https://doi.org/10.26803/ijlter.21.6.17
Valtonen, T., Leppänen, U., Hyypiä, M., Sointu, E., Smits, A., & Tondeur, J. (2020). Fresh perspectives on TPACK: Pre-service teachers' own appraisal of their challenging and confident TPACK areas. *Education and Information Technologies, 25*(4), 2823-2842. https://doi.org/10.1007/s10639-019-10092-4
Vladušić, R., Bucat, R., & Ožić, M. (2020). Evidence of the development of pedagogical content knowledge related to chemical bonding during a course for preservice chemistry teachers. *Center for Educational Policy Studies Journal, 10*(1), 59-81. https://doi.org/10.26529/cepsj.783
Voogt, J. (2012). Are teachers ready to teach in the knowledge society? Considerations based on empirical findings. *Jahrbuch Medienpädagogik, 9*, 17-28.
Wakhata, R., Mutarutinya, V., & Balimuttajjo, S. (2022). Pedagogical content knowledge of mathematics teachers: A focus on identifying and correcting sources of students' misconceptions in linear programming. *The International Journal of Pedagogy and Curriculum, 29*(2), 23.
Widyoko, E. P. (2014). *Teknik penyusunan - 56 -aption- 56 -in penelitian*. Yogyakarta: Pustaka Pelajar.
Wooditch, A., Rice, A., Peake, J., & Rubenstein, E. (2018). The development of preservice agriculture teachers' pedagogical content knowledge through a greenhouse for teachers course. *Journal of Agricultural Education, 59*(3), 1-14.
Zakariya, Y. (2022). Improving students' mathematics self-efficacy: A systematic review of intervention studies. *Frontiers in Psychology, 13*. https://doi.org/10.3389/fpsyg.2022.986622

CHAPTER FOUR

Exploring Shifts in Pre-Service Teachers' Perceptions of Mathematicians: From First Year to The End of Second Year

Zaheera Jina Asvat[1*]

AFFILIATIONS

[1]Department of Mathematics Education, University of the Witwatersrand, Johannesburg, South Africa.

CORRESPONDENCE

Email: zaheera.jina@wits.ac.za*

Reference Formats (APA 7th Edition)

Jina Asvat, Z. (2025). Exploring shifts in pre-service teachers' perceptions of mathematicians: from first year to the end of second year. In B. E. Olawale (Ed.), *Building the Foundations: Effective Approaches in Mathematics Teacher Preparation* (pp. 57-71). ERRCD Forum. https://doi.org/10.38140/obp3-2025-04

Abstract: This study examines how pre-service mathematics teachers perceive mathematicians' roles and practices, emphasising the shifts in their epistemic and social understandings during their teacher education programme. Using Legitimation Code Theory (LCT) and variation theory, data were collected from 60 pre-service teachers at a South African university, comparing their views from their first to second year. Initial perceptions often conflated mathematicians with educators, focusing on societal roles and pedagogical functions. However, by their second year, after engaging in a mathematics methodology course rooted in variation theory, participants demonstrated an increased appreciation for mathematicians' epistemic practices, including problem-solving, abstraction, and theoretical application. Despite these advancements, many participants continued to frame their understanding through a pedagogical lens, with limited attention to the social dimensions of mathematical work, such as collaboration and interdisciplinarity. These findings underscore the need for teacher education programmes to balance epistemic and social dimensions, enabling future educators to view mathematics as both an intellectual discipline and a collaborative, dynamic field. This study contributes to understanding how theoretical frameworks can inform and transform pre-service teachers' conceptions of mathematicians' roles, bridging the gap between classroom teaching and authentic mathematical practices.

Keywords: Legitimation Code Theory, mathematical practices, mathematicians' perceptions, pre-service teachers, variation theory.

1. Introduction

Pre-service mathematics teachers (PSTs) face the challenge of mastering the complexities of teaching mathematics within a four-year programme. Although they have spent years observing mathematics teachers, they have not observed mathematicians engaged in authentic mathematical practices. Moreover, PSTs are only formally introduced to theoretical perspectives on teaching and learning mathematics during their second year of teacher education. This delay, coupled with the well-documented gap between theory and practice in teacher education, often results in novice teachers abandoning theoretical insights. Instead, they conform to established school norms or resist integrating theory into their teaching practices (Mårtensson & Ekdahl, 2021). Past school experiences heavily shape teaching decisions (Lortie, 1975; Sugrue, 1997). Without encountering practical challenges during training, PSTs may perceive theories as irrelevant, reverting to familiar methods of teaching (Mårtensson & Ekdahl, 2021; Olawale, 2024). While traditional instruction from mathematicians may not directly support research-based, student-centred teaching (Wagner, Speer & Rossa, 2007), robust mathematical knowledge acquired through university training is considered essential for successful teaching (Leikin, Zazkis & Meller, 2018; Hofman & Even, 2018). Therefore, mathematicians' work must expand PSTs' knowledge beyond the school curriculum, exposing them to deeper mathematical thinking. Understanding mathematicians' practices helps PSTs integrate these approaches into their teaching. Mathematicians solve challenging problems—those that are "perplexing or difficult" (Schoenfeld, 2016). Pólya (1954, in Schoenfeld, 2016) describes problem-solving as engaging with unfamiliar tasks where solutions are not immediately known. In contrast, emphasising procedural drills can inhibit students' ability to develop broader problem-solving skills. This practice risks portraying mathematics as a set of rigid facts and procedures (Boaler et al., 2022). Students often internalise the misconception that mathematics lacks creativity or reasoning, reducing it to rote memorisation. When PSTs adopt such classroom routines, they risk perpetuating this limited view of mathematics, conflating the roles of mathematics teachers and mathematicians.

1.2 Problem statement and research questions

PSTs must develop both mathematical content knowledge and mathematical practices to address these challenges. Content knowledge encompasses concepts, skills, and applications, while practices involve disciplinary habits such as argumentation, precision, and problem-solving (Lai & Ahrens, 2022). However, applying theoretical learning to practice remains a significant challenge (Korthagen, 2010; Sugrue, 1997). Treating theory as a "solution" to bridge the gap between research and practice is insufficient. Instead, Lampert (2010) argues that theory should deepen reasoning about practice, integrating with practical knowledge to generate new insights. This study examines how integrating theoretical frameworks with lesson planning in a mathematics education course transforms PSTs' understanding of their roles as mathematicians. At the outset, first-year PSTs primarily identified personal attributes in their teacher identities.

The following are the research questions that guided the study:

- How do pre-service mathematics teachers' perceptions of mathematicians change throughout their teacher education programme?
- What changes occur in pre-service mathematics teachers' understanding of the epistemic dimensions of a mathematician's work?
- How do pre-service mathematics teachers' perspectives on the social and relational aspects of mathematicians evolve over time?

Through iterative processes such as lesson planning, critical analysis, and revisions rooted in variation theory (Marton, 2015) and mathematical proficiency theory, second-year PSTs developed a deeper appreciation for specialised knowledge. By the conclusion of the course, their epistemic understanding had notably advanced, highlighting the critical role of linking mathematical theory to teaching practice (Beswick, 2011).

2. **Literature Review**

Mathematics, unlike fields such as Biology, History, or English, presents a unique challenge in education due to the inherent nature of the subject. Taylor (2018) argues that the limited technical scope of the school mathematics curriculum is a direct result of the complexity and abstractness of mathematics. While other disciplines engage learners with sophisticated questions and creative works that interest professionals, school mathematics often falls short of providing such experiences. This disconnection represents a fundamental difference between mathematics and other fields, where learners can directly relate to and engage with professional-level inquiries.

The discrepancy between learners' and mathematicians' perceptions of mathematics is well documented. Cirillo and Herbel-Eisenmann (2011) explore how classroom practices shape learners' views of mathematicians, revealing that teachers' mathematical backgrounds and portrayals significantly influence these perceptions. They emphasise the need for preservice teachers to reflect on their beliefs about mathematicians' roles and behaviours to present a more accurate and inspiring image of the profession. Latterell and Wilson (2012) reinforce this perspective by noting that mathematicians create new theorems and apply existing results to practical problems, a viewpoint rarely conveyed in traditional mathematics education. Building on this, researchers (Boaler, 2016; Schoenfeld, 2016) highlight the contrast between learners' view of mathematics as a collection of calculations, procedures, and rules and mathematicians' view of it as the study of problems, emphasising creativity and elegance. Together, these studies suggest the need for classroom practices that cultivate a deeper understanding and appreciation of the true nature of mathematics.

Most learners' mathematical experiences occur in the classroom, making these experiences critical in forming their images of mathematics and mathematicians (Schoenfeld, 1985). For example, Rock and Shaw (2000) found that learners primarily believed mathematicians solved "hard problems that other people don't know," reflecting a limited understanding of the profession's scope. Similarly, Picker and Berry (2000) identified various stereotypes in learners' drawings of mathematicians, such as the mathematician as coercive or foolish, indicating a need for more accurate representations in education. Studies by Burton (2004) and Nardi (2008) delve into the perspectives of mathematicians on mathematics education, revealing insights that could bridge the gap between university mathematics and school teaching.

2.1 Collaboration between mathematicians and educators

Blanton and Stylianou (2009) advocate for collaboration between mathematicians and mathematics educators to develop effective teaching strategies, a sentiment echoed by Wagner et al. (2007), who found traditional instructional practices insufficient for supporting learner-centred teaching. Research by Hagenkötter et al. (2022) investigates the role of professional mathematicians in training secondary school mathematics teachers. The study highlights the need for integrating meta-mathematical skills into core coursework and fostering collaboration between mathematicians and educators to improve teacher education. Overcoming barriers to collaboration is crucial for bridging the gap between university-level mathematics and school teaching.

Hagenkötter et al. (2022) found that learners commonly believe mathematical scientists primarily engage in advanced calculations, reflecting their classroom experiences. This limited perception overlooks critical activities such as reasoning, modelling, and problem-solving. While many studies explore learner conceptions of mathematicians, few implement interventions explicitly aimed at demonstrating the diverse and creative nature of mathematical work, leaving a gap in effectively addressing these misconceptions.

Pre-service teachers, like other social actors, become "knowers" by adopting a new perspective that allows them to identify and understand what defines "authentic reality" (Bernstein, 2000, p. 164 in Langsford & Rusznyak, 2024). This perspective, as Maton (2014) explains, is shaped through exposure to new concepts, relevant experiences, and interactions with key individuals. For pre-service teachers aspiring to become mathematicians, developing this perspective is a more intricate process that extends beyond simply "noticing and understanding" the "intentions and actions of teachers" (Langsford & Rusznyak, 2024, p. 5). It requires immersion in the intellectual work of mathematicians, which is often abstract, complex, and not typically visible in traditional classroom settings. This process demands that pre-service teachers go beyond reflecting on "the study of teachers' practices through live or recorded lessons" and begin to "practice the theory in real situations, as they conduct lessons based on theoretical concepts" (Mårtensson & Ekdahl, 2021).

The variation theory of learning emphasises structuring critical aspects in patterns of variation and invariance, allowing students to discern them rather than being explicitly told (Pang & Marton, 2003; Marton, 2015). Watson and Mason (2006) suggest that this approach helps teachers design tasks that encourage students to identify these critical aspects. This approach is closely linked to problem-solving, as the curriculum aims to foster learners' critical and creative thinking skills through tasks that reflect the practices of mathematicians. For pre-service teachers, engaging with variation theory cultivates a "mathematical gaze," helping them develop a nuanced understanding of mathematical practices. This enhances their ability to think about and teach mathematics in ways that go beyond basic pedagogical strategies (Maton, 2014).

While it is well-documented that pre-service teachers' ability to interpret teachers' intentions and actions is crucial for pedagogical learning and understanding effective teaching choices (Langsford & Rusznyak, 2024), this alone is insufficient for pre-service teachers to become true mathematics educators. Developing a "mathematical gaze" through variation theory is essential for their growth into effective mathematics teachers, enabling them to engage deeply with the complexities of mathematical practices.

3. Methodology

This study employed a qualitative research design, specifically a basic interpretive approach, as described by Merriam (2002). This approach seeks to understand how participants create meaning from their experiences and to identify common themes or patterns within the data. The analytical framework used was Legitimation Code Theory (LCT), which focuses on codes and legitimation to analyse curriculum, pedagogy, and assessment practices. The study population comprised 60 pre-service teachers (PSTs) enrolled in a Bachelor of Education (BEd) programme, majoring in mathematics, at a South African university. These PSTs were in their first and second years of study. Participants were selected based on their enrolment in the programme and their willingness to participate in the research. A purposive sampling technique was used to select the participants. This approach was appropriate as it allowed the researcher to focus on PSTs who were directly engaged in the mathematics education courses relevant to the study's objectives. Data were collected through written responses to the prompt, "What do you think it will be like to work as a qualified mathematician?" The responses were gathered at two points: at the beginning of the first year and at the end of the second year. These responses provided insights into the participants' evolving conceptions of mathematicians. Additionally, lesson plans and reflections uploaded to the university's learning management system were analysed to understand how PSTs applied theoretical concepts to their teaching practices.

Legitimation Code Theory (LCT) offers sophisticated tools for analysing curriculum, pedagogy, and assessment practices, focusing on codes and legitimation. Codes signify the principles that guide a practice's legitimacy, but implicit codes may exclude certain practices or perspectives. LCT reveals these underlying codes, enhancing comprehension of their influence on educational practices. Two crucial codes within LCT—knowledge and knower codes—are founded on the notion that "practices and beliefs are about or oriented towards something and by someone" (Maton, 2014, p. 29). This enables an analytical distinction between two sets of relations.: epistemic and social. Epistemic relations pertain to "what can be legitimately described as knowledge," while social relations concern "who can claim to be a legitimate knower" (p. 29). These relations indicate "what counts" in each practice (Luckett & Hunma, 2014, p. 183).

Epistemic relations in the work of mathematicians encompass practices such as investigation, problem-solving, critical thinking, generalisation, and justification, with PSTs leveraging this knowledge to tackle complex problems (author, 2024). Social relations, on the other hand, relate to teachers' perceived legitimacy as knowers within the work of mathematicians. These relations involve traits associated with analytical, creative, and critical thinking, as well as persistence, mathematical proficiency, collaboration, adaptability, confidence, curiosity, and reflection. Enhanced social relations correlate with PSTs demonstrating expertise and confidence in the work of mathematicians. According to Maton (2014), knowers can be differentiated based on both their identity (kinds of knowers) and their method of knowing (ways of knowing). When legitimacy in becoming mathematicians derives from how students know, it mandates specific gazes:

- Cultivated gaze — recognising what is valued in the work of a mathematician.
- Social gaze — emphasises the type of knower based on their social position, resulting in a social gaze. Born gaze — a specific type of knower and a particular way of knowing.
- Trained gaze — practices that require neither a specialised knower nor a specific way of knowing cultivate knowers with a trained gaze.

If the work of a mathematician forms part of the training for mathematics teachers, then the conceptions that PSTs hold should reveal a trained gaze. This study offers insights into this issue by examining the roles of both knowledge and knower codes in facilitating epistemological access to the work of mathematicians within the BEd programme at one university in South Africa. Table 1 outlines the translation device utilised to analyse the PSTs' conceptions.

Table 1: *Translation device to analyse the Epistemic and Social relations of the PSTs conceptions*

KNOWLEDGE	ER + Knowledge of the work of mathematicians	ELITE
SR – No Attributes of a mathematician		SR + Some Attributes of a mathematician
RELATIVIST	ER – No Knowledge of the work of mathematicians	KNOWER

Adapted from Jina Asvat (2024)

According to Maton and Chen (2020), four specialisation codes illustrate the interplay between epistemic relations (ER) and social relations (SR):

Élite Code: Characterised by strong ER and SR, this code emphasises both the knowledge of mathematicians' work and the attributes of a mathematician as a knower. It represents the ideal balance for understanding mathematical practices.

Knowledge Code: Defined by stronger ER and weaker SR, this code highlights knowledge of mathematicians' work over personal attributes. Pre-service teachers (PSTs) focus more on expertise than on the social identity of mathematicians.

Knower Code: Marked by weaker ER and stronger SR, this code prioritises the social attributes and personal characteristics of mathematicians over their knowledge and work.

Relativist Code: With both weak ER and SR, this code reflects general or non-specific responses, showing little emphasis on either the knowledge or the personal attributes of mathematicians.

Through coding, the strengths of epistemic and social relations were assigned, allowing for a comparison of specialisation among first- and second-year PST data. Extracts from the interviews were then utilised to delineate overarching patterns.

Ethical clearance was obtained from the University of the Witwatersrand Human Research Ethics Committee (Education) under approval number H23/11/09. The research was classified as minimal risk. Informed consent was obtained from all participants to ensure their voluntary participation and the confidentiality of their responses. Data were anonymised to protect participants' identities. The dataset is restricted to one university and includes the written conceptions provided by PSTs at the start of their first year and at the end of their second year. It is acknowledged that they may have observed additional aspects that were not included in their writings. Nonetheless, their submitted responses can be regarded as the most significant elements of their understanding of the work of mathematicians, which they deemed important enough to express in writing. Additionally, interviews with the PSTs were not conducted, so further probing was not possible.

4. Presentation of Results

The students' perceptions of a mathematician's work in the first year of the BEd programme, in March 2023, revealed that their views were heavily skewed towards ER- (42%) and SR- (39%), with much less emphasis on ER+ (13%) and SR+ (7%). However, in the second year of the BEd programme, after completing the teaching mathematics methods course, a shift in their perceptions occurred. The data show a more balanced distribution, with ER- at 27%, ER+ at 23%, and SR+ at 11%. Notably, SR- remained consistent across both years at 39%. These changes represent a shift from weaker to stronger forms of epistemic and social relations in students' descriptions of what they imagine the work of a qualified mathematician to be like.

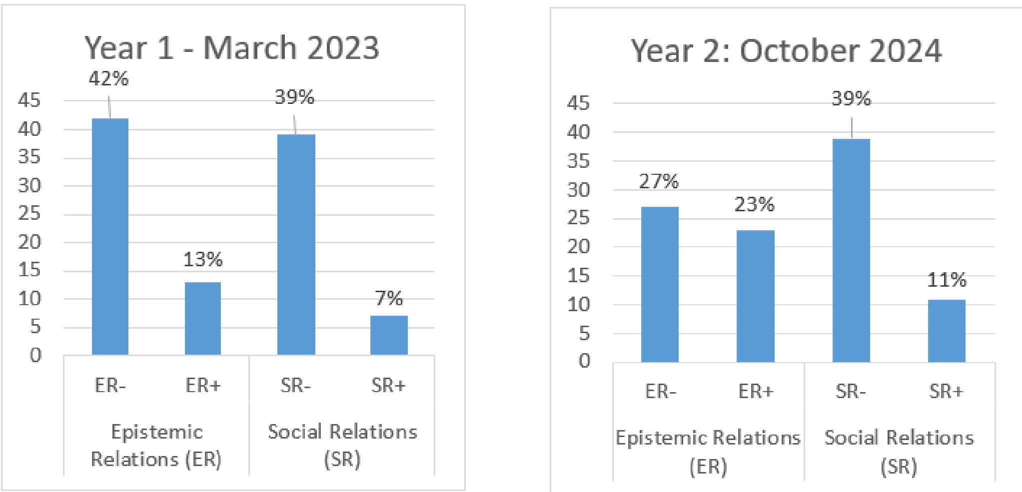


Figure 1 Shifts in the epistemic and social relations of the conceptions of the work of mathematicians

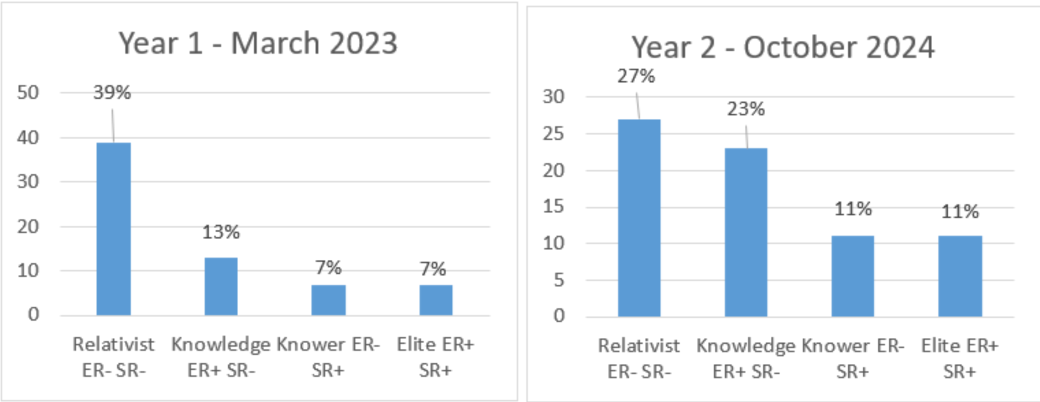


Figure 2 Shifts in the specialisation codes of the conceptions of the work of mathematicians

The graphs reveal a shift in pre-service teachers' (PSTs) perceptions of mathematicians from Year 1 (March 2023) to Year 2 (October 2024). Initially, the Relativist Code (ER- SR-) dominated at 39%, reflecting general, non-specific views on mathematicians, but this decreased to 27% by Year 2. Meanwhile, the Knowledge Code (ER+ SR-), which emphasises expertise over personal attributes, increased from 13% to 23%, indicating a growing focus on mathematicians' work and content mastery. The Knower Code (ER- SR+), prioritising personal and social attributes, rose slightly from 7% to 11%, suggesting increased recognition of identity and relatability. Similarly, the Elite Code (ER+ SR+), which balances expertise and personal traits, grew from 7% to 11%, pointing to a developing appreciation for holistic representations of mathematicians.

This study explored how pre-service mathematics teachers' (PSTs) perceptions of mathematicians evolved during their Bachelor of Education (BEd) programme, particularly between the first and second years. The findings reveal a significant shift in PSTs' understanding of mathematicians, reflecting changes in both epistemic relations (ER) and social relations (SR) as framed by Maton's Legitimation Code Theory (LCT).

4.1 A relativist lens on mathematicians

In Year 1, the dominant relativist perspective (ER- SR-) is reflected in statements that lack specificity about the work of mathematicians. For instance:

> *"I think it would be quite interesting and tedious at the same time."*
>
> *"It will be a good experience."*
>
> *"Obviously, it will be nice."*

These comments suggest a generalised view, often devoid of a concrete understanding of mathematicians' roles.

By Year 2, while ER- SR- responses have decreased from 42% to 27%, similar sentiments persist in comments such as:

"It will be a dream come true... because mathematics is a challenging subject in my community."

"I think it will be fun and sometimes challenging because mathematics is not an easy subject for the majority of learners in schools."

The decrease in relativist responses indicates a gradual move toward more nuanced perspectives but highlights the initial lack of specific knowledge about mathematicians' work.

4.2 Increasing recognition of Epistemic Relations

The rise in ER+ responses (from 13% in Year 1 to 23% in Year 2) illustrates the growing acknowledgement of mathematicians' specialised knowledge and contributions.

This shift is evident in Year 1 comments like:

"It will constantly train me to approach concepts differently and creatively, while still using the correct principles of mathematics."

"I should be able to know how to solve problems and attend to learners when they ask questions."

By Year 2, this theme becomes more pronounced with statements such as:

"Working as a qualified mathematician involves conducting research, solving complex problems, and applying mathematical principles in various fields such as science, engineering, finance, and technology."

"Theories learned can be applied to real-world problems, contributing to fields like artificial intelligence and finance."

"It will involve problem-solving, engaging in complex issues requiring deep analytical thinking and creativity."

These responses showcase students' evolving awareness of mathematics' intellectual rigour and its real-world applications.

4.3 Persistent pedagogical framing

Throughout both years, many responses frame mathematicians within a teaching context, reflecting pre-service teachers' roles. In Year 1, this is evident in statements such as:

"It will be fascinating because I will be able to teach my students ways to achieve their mathematics dreams."

"It will be interesting to teach children how to solve problems."

"I think it would be a fun experience, as once I know what I am doing, I become excited to help others understand as well."

By Year 2, similar views persist:

"I'll be helping learners understand and enjoy math. My day will be filled with sharing my passion for numbers and problem-solving."

"You would create lessons that make math interesting and relatable, showing students how it applies to their everyday lives."

"I believe teaching what you know will exactly help the student to understand."

These responses highlight a strong pedagogical framing, often centred on learners' needs and engagement.

4.4 Limited emphasis on Social Relations

Social relations (SR+) see a modest increase from 7% in Year 1 to 11% in Year 2, reflecting a slight shift toward considering mathematicians' personal and relational attributes. In Year 1, these are observed in comments such as:

> *"It would be an inspiration to others that being a mathematician is possible."*
>
> *"I love mathematics, but having to use it every day might not be something I enjoy."*

In Year 2, the focus expands slightly:

> *"I believe the experience will be fulfilling as it allows mathematicians to share their passion for mathematics with students and inspire the next generation."*
>
> *"Being a qualified mathematician will mean a big achievement, and people around me will surely get motivated."*
>
> *"It will involve collaboration with other professionals, which fosters teamwork and creativity."*

While social relations remain secondary to epistemic considerations, these responses reflect an increasing recognition of mathematicians' relational and societal impacts.

4.5 Evolution toward a balanced understanding

The growth in Elite Code responses (ER+ SR+), from 7% in Year 1 to 11% in Year 2, marks a shift toward a holistic understanding of mathematicians. This balance is reflected in Year 1 statements such as:

> *"I think it will be mentally challenging and fulfilling at the same time, as it constantly trains me to approach concepts differently."*

By Year 2, such balanced views are more evident:

> *"Being a qualified mathematician requires strong analytical skills, curiosity, and passion for continuous discovery."*
>
> *"The work would involve conducting research and applying knowledge while collaborating with others to solve complex challenges."*
>
> *"It involves using mathematical theories to solve real-world problems, with opportunities to explore and innovate in diverse fields."*
>
> *"It would be rewarding to see learners grow and develop their math skills over time, while also engaging in lifelong learning."*

These comments demonstrate the students' increasing appreciation for both the intellectual and relational aspects of mathematics.

5. Discussion of Findings

This study sought to explore shifts in pre-service mathematics teachers' (PSTs) perceptions of mathematicians from their first year to the end of their second year in a Bachelor of Education

(BEd) programme. The findings reveal nuanced changes in how PSTs understand the epistemic and social dimensions of a mathematician's work. Below, the findings are discussed in relation to the research questions, supported by previous studies, and presented in prose form.

Research Question 1: How do pre-service mathematics teachers' perceptions of mathematicians change throughout their teacher education programme?

The study found that PSTs' perceptions evolved significantly over the course of the programme. In their first year, PSTs predominantly exhibited a relativist perspective (ER- SR-), characterised by general and non-specific views of mathematicians. For instance, many described mathematicians' work as "showing young people different perspectives in mathematics" or "helping students learn maths better." These perceptions reflect a societal framing of mathematicians as educators rather than practitioners of mathematics, aligning with Cirillo and Herbel-Eisenmann's (2011) findings that classroom experiences shape learners' views of mathematicians, emphasising teaching roles over authentic mathematical practices.

By the second year, PSTs' descriptions reflected a growing recognition of mathematicians' epistemic practices (ER+). The study reveals that PSTs began articulating deeper insights into mathematicians' roles, such as engaging in "abstract thinking" and "applying mathematical theories to solve real-world problems." This shift aligns with Boaler's (2016) argument that mathematics education should extend beyond procedural drills to highlight creativity and problem-solving.

Pre-service teachers, like other social actors, become "knowers" by adopting a new perspective that allows them to identify and understand what defines "authentic reality" (Bernstein, 2000, p. 164 in Langsford & Rusznyak, 2024). As Maton (2014) explains, this perspective is shaped through exposure to new concepts, relevant experiences, and interactions with key individuals. However, many PSTs continued to view mathematicians primarily through a pedagogical lens, indicating an enduring influence of their teaching-focused experiences, consistent with Lortie's (1975) concept of the "apprenticeship of observation."

These findings align with prior research highlighting the challenges pre-service teachers (PSTs) face in bridging the theoretical and practical aspects of mathematics education (Mårtensson & Ekdahl, 2021). For instance, Lampert (2010) emphasises the importance of integrating theoretical knowledge with practical experiences to deepen reasoning about teaching practices. This study supports such approaches, as the integration of variation theory proved instrumental in reshaping PSTs' epistemic perceptions. By engaging with variation theory, PSTs began to develop a "mathematical gaze" that allowed them to identify critical aspects of mathematical practices and consider teaching mathematics in more sophisticated ways (Maton, 2014; Pang & Marton, 2003). However, the persistent framing of mathematicians' work through a pedagogical lens reflects Korthagen's (2010) observation that, in the absence of robust theoretical integration, PSTs often revert to familiar teaching narratives, as also noted by Mårtensson and Ekdahl (2021).

Research Question 2: What changes occur in pre-service mathematics teachers' understanding of the epistemic dimensions of a mathematician's work?

The study reveals that PSTs developed a stronger epistemic gaze, shifting from weaker to stronger epistemic relations (ER+). Initially, only 13% of PSTs demonstrated knowledge of the epistemic practices of mathematicians, such as abstraction, proof, and problem-solving. By the end of their second year, this increased to 23%, reflecting a growing appreciation for the intellectual rigour and theoretical aspects of mathematics.

This transformation was attributed to the mathematics methodology course, which used variation theory to structure learning around critical aspects of mathematical practices (Pang & Martone, 2003). For example, one PST described mathematicians as "innovators who develop new principles to address complex problems," illustrating a deeper engagement with the epistemic dimensions of mathematics. Such responses resonate with Schoenfeld's (2016) argument that problem-solving and abstract thinking are central to mathematicians' work and should be emphasised in mathematics education.

Developing a mathematical gaze requires immersion in the intellectual work of mathematicians, which is often abstract, complex, and not typically visible in traditional classroom settings (Langsford & Rusznyak, 2024). This process demands that PSTs go beyond reflecting on "the study of teachers' practices through live or recorded lessons" to "practise the theory in real situations" (Mårtensson & Ekdahl, 2021). Variation theory facilitated this by emphasising patterns of variation and invariance, enabling PSTs to discern critical aspects without explicit instruction (Pang & Marton, 2003; Marton, 2015). This method mirrors Watson and Mason's (2006) suggestion for task design that highlights problem-solving and fosters critical and creative thinking.

Research Question 3: How do pre-service mathematics teachers' perspectives on the social and relational aspects of mathematicians evolve over time?

The study found limited progress in pre-service teachers' (PSTs) recognition of the social and relational aspects of mathematicians' work (SR+). While some PSTs acknowledged collaboration and interdisciplinarity, their understanding often lacked depth. For example, responses such as "collaborating with professionals in various fields" reflect a superficial appreciation of teamwork and fail to fully capture the dynamic and relational nature of mathematical work, as emphasised by Burton (2004) and Hagenkötter et al. (2022).

By the end of their second year, only 11% of PSTs demonstrated a balanced understanding (ER+ SR+), compared to 7% in their first year. This marginal improvement highlights the persistent gap in PSTs' appreciation of the social dimensions of mathematics, underscoring the need for teacher education programmes to explicitly address this aspect. Fostering a holistic view of the discipline requires integrating opportunities for PSTs to engage with the collaborative and interdisciplinary practices that define authentic mathematical work. Additionally, this limited focus on social dimensions aligns with Taylor's (2018) critique of the narrow scope of school mathematics curricula, which often prioritise procedural knowledge at the expense of relational and collaborative skills. The findings reinforce the importance of exposing PSTs to the broader contexts of mathematics, encouraging them to move beyond individualistic or teaching-focused narratives. Blanton and Stylianou (2009) similarly stressed the value of collaboration between mathematicians and educators to bridge the gap between university-level mathematics and school teaching. By doing so, teacher education programmes can equip PSTs with a more comprehensive understanding of mathematicians' roles, fostering both epistemic and social engagement.

6. Conclusion

This study underscores the evolving perceptions of pre-service mathematics teachers (PSTs) regarding the work of mathematicians, revealing significant shifts in their epistemic understanding while highlighting enduring challenges in integrating social and relational dimensions. PSTs progressed from a relativist perspective to a more nuanced epistemic gaze, recognising the abstract and applied practices of mathematicians. However, their views remained strongly influenced by prior teaching experiences, reflecting persistent pedagogical biases and limited engagement with the collaborative and interdisciplinary aspects of mathematical work. The integration of variation theory in the curriculum played a pivotal role in fostering a deeper understanding of critical mathematical concepts and practices. By enabling PSTs to discern key aspects of mathematicians' intellectual work, this approach contributed to the development of a "mathematical gaze." Nonetheless, the findings highlight a need for teacher education programmes to place greater emphasis on the social and creative dimensions of mathematics, encouraging PSTs to view mathematicians not only as problem-solvers but also as collaborators engaged in dynamic and relational practices. Balancing the epistemic and social dimensions of mathematics education is essential for bridging the gap between theory and practice. Teacher education programmes must expose PSTs to authentic mathematical practices, fostering a comprehensive understanding of the discipline. By doing so, future educators will be better equipped to inspire a more dynamic, creative, and holistic appreciation of mathematics, advancing both their teaching strategies and the discipline as a whole.

7. Declarations

Funding: This research was sponsored by the University Capacity Development Grant Teaching Development and Research Grant for Staff.

Conflicts of Interest: The author declares no conflict of interest.

Ethical Considerations: Ethical clearance for this study was obtained from the University of the Witwatersrand, Human Research Ethics Committee (Education), Approved, Risk Level: Minimal (H23/11/09).

Data Availability: The datasets generated and/or analysed during the current study are available from the author upon reasonable request.

References

Blanton, M. L., & Stylianou, D. A. (2009). Interpreting a community of practice perspective in discipline-specific professional development in higher education. *Innovative Higher Education, 34*(2), 79–92. https://doi.org/10.1007/s10755-008-9081-6

Boaler, J., Brown, K., LaMar, T., Leshin, M., & Selbach-Allen, M. (2022). Infusing mindset through mathematical problem solving and collaboration: Studying the impact of a short college intervention. *Education Sciences, 12*(10), Article 694. https://doi.org/10.3390/educsci12100694

Bernstein, B. (2000). *Pedagogy, symbolic control, and identity: Theory, research, critique*. Rowman & Littlefield Publishers.

Burton, L. (2004). *Mathematicians as enquirers: Learning about learning mathematics*. Springer. https://doi.org/10.1007/978-94-017-0495-5

Beswick, K. (2012). Teachers' beliefs about school mathematics and mathematicians' mathematics and their relationship to practice. *Educational Studies in Mathematics, 79*(1), 127–147. https://doi.org/10.1007/s10649-011-9333-4

Cirillo, M., & Herbel-Eisenmann, B. A. (2011). "Mathematicians would say it this way": An investigation of teachers' framings of mathematicians. *School Science and Mathematics, 111*(2), 68–78. https://doi.org/10.1111/j.1949-8594.2010.00066.x

Hagenkötter, R., Nachtigall, V., Rolka, K., & Rummel, N. (2022). Exploring students' and mathematics teachers' conceptions about the work of mathematical scientists and possible relations to mathematics teaching. In *Proceedings of the 16th International Conference of the Learning Sciences (ICLS 2022)* (pp. 155–162). International Society of the Learning Sciences.

Hoffmann, A., & Even, R. (2018, July). What do mathematicians wish to teach teachers in secondary school about mathematics. In *Proceedings of the 42nd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 99–106). PME.

Asvat, Z. J. (2024). Examining South African teachers' approach to mathematics problem-solving through legitimation code theory. In *EDULEARN24 Proceedings* (pp. 8935–8943). IATED.

Korthagen, F. A. J. (2010). How teacher education can make a difference. *Journal of Education for Teaching, 36*(4), 407–423.

Lai, Y., & Ahrens, S. (2024). Some mathematicians perceived and envisioned instructional relationships in secondary teaching and teaching secondary teachers. *Journal of Mathematics Teacher Education, 27*(3), 379–410. https://doi.org/10.1007/s10857-023-09592-x

Lampert, M. (2010). Learning teaching in, from, and for practice: What do we mean? *Journal of Teacher Education, 61*(1–2), 21–34. https://doi.org/10.1177/0022487109347321

Langsford, D., & Rusznyak, L. (2024). Observing complexity in teachers' choices: Preparing preservice teachers for work-integrated learning. *Education as Change, 28*(1), Article 1.

Leikin, R., Zazkis, R., & Meller, M. (2018). Research mathematicians as teacher educators: Focusing on mathematics for secondary mathematics teachers. *Journal of Mathematics Teacher Education, 21*(5), 451–473. https://doi.org/10.1007/s10857-017-9366-9

Luckett, K., & Hunma, A. (2014). Making gazes explicit: Facilitating epistemic access in the humanities. *Higher Education, 67*(2), 183–198. https://doi.org/10.1007/s10734-013-9655-2

Lortie, D. C. (1975). *Schoolteacher: A sociological study*. University of Chicago Press.

Marton, F. (2015). *Necessary conditions of learning*. Routledge. https://doi.org/10.4324/9781315717632

Mårtensson, P., & Ekdahl, A. L. (2021). Variation theory and teaching experiences as tools to generate knowledge about teaching and learning mathematics—The case of pre-service teachers. *Nordisk Matematikkdidaktikk, 26*(3–4), 91–112.

Maton, K. (2014). *Knowledge and knowers: Towards a realist sociology of education*. Routledge. https://doi.org/10.4324/9780203078648

Merriam, S. B. (2002). *Qualitative research in practice: Examples for discussion and analysis*. Jossey-Bass.

Nardi, E. (2008). *Amongst mathematicians: Teaching and learning mathematics at university level*. Springer. https://doi.org/10.1007/978-0-387-72822-7

Olawale, B. E. (2024). Impact of pre-service teacher education programme on mathematics student teachers' teaching practices during school experiences. *Education* Sciences, *14*(7), 762. https://doi.org/10.3390/educsci14070762

Pang, M. F., & Marton, F. (2003). Beyond "lesson study": Comparing two ways of facilitating the grasp of some economic concepts. *Instructional Science, 31*(3), 175–194. https://doi.org/10.1023/A:1023280619631

Picker, S. H., & Berry, J. S. (2000). Investigating pupils' images of mathematicians. *Educational Studies in Mathematics, 43*(1), 65–94. https://doi.org/10.1023/A:1017523230758

Rock, D., & Shaw, J. M. (2000). Exploring children's thinking about mathematicians and their work. *Teaching Children Mathematics, 6*(9), 550–555.

Schoenfeld, A. H. (2016). Research in mathematics education. *Review of Research in Education, 40*(1), 497–528. https://doi.org/10.3102/0091732X16658650

Sugrue, C. (1997). *Complexities of teaching: Child-centred perspectives*. Falmer Press.

Taylor, P. (2018). Teach the mathematics of mathematicians. *Education Sciences, 8*(2), Article 56. https://doi.org/10.3390/educsci8020056

Wagner, J. F., Speer, N. M., & Rossa, B. (2007). Beyond mathematical content knowledge: A mathematician's knowledge needed for teaching an inquiry-oriented differential equations course. *The Journal of Mathematical Behavior, 26*(3), 247–266. https://doi.org/10.1016/j.jmathb.2007.09.002

Watson, A., & Mason, J. (2006). *Mathematics as a constructive activity: Learners generating examples*. Routledge.

Place-based Mathematics Education: An Education for Sustainable Development Pedagogy to Enhance Mathematics Teacher Training

Kevin Lance Teise[1*]

AFFILIATIONS

[1]Faculty of Education, Sol Plaatje University, Kimberley, South Africa.

CORRESPONDENCE

Email: kevin.teise@spu.ac.za*

Reference Formats (APA 7th Edition)

Teise, K. L. (2025). Place-based mathematics education: An education for sustainable development pedagogy to enhance mathematics teacher training. In B. E. Olawale (Ed.), *Building the Foundations: Effective Approaches in Mathematics Teacher Preparation* (pp. 72–87). ERRCD Forum. https://doi.org/10.38140/obp3-2025-05

Abstract: Mathematics education in South Africa is experiencing a crisis that disproportionately affects the majority of learners, enhancing inequality and inequity and promoting unsustainable development. Part of the problem lies in the quality and training of Mathematics teachers, as well as the pedagogy they follow. South African education, particularly Mathematics education, should contribute to the transformation of society into an equitable and sustainable one. Hence, the orientation of South African education towards Education for Sustainable Development (ESD). It is believed that ESD will contribute to equity, equality, and sustainability. One way to implement ESD is through Place-Based Education (PBE). PBE capitalises on place, community, experiences, location, and geography to create unique, authentic, meaningful, and personalised learning opportunities. Consequently, PBE expands the mathematics classroom to include the surrounding community. This empowers Mathematics teachers to make the subject relevant to learners by incorporating their local context into their pedagogy. This approach to Mathematics teaching and teacher training promotes equity and the realisation of sustainable development, as it addresses the educational needs of the majority of South Africa's learners within Mathematics education. This desktop chapter scrutinises literature on the topic to highlight the value and relevance of ESD and PBE in Mathematics teacher training, and to situate PBE as a pedagogy within Mathematics education and teacher training. I provide an overview of Mathematics teacher training in South Africa; thereafter, I discuss ESD and its connection to Mathematics education; and finally, I reflect on PBE and its relevance and value to Mathematics teacher training.

Keywords: Place-based education, education for sustainable development, mathematics education.

1. Introduction

Place-based Education (PBE) is education that occurs when learners, teachers, and the broader community utilise the social, cultural, and natural environment they inhabit as an inquiry-based learning laboratory (Reed & Klassen, 2020). Thus, PBE expands teaching and learning into the surrounding environment, creating authentic learning opportunities beyond the walls of the school. In this way, the learner's place becomes an auxiliary text that they are taught to read, synthesise, explore, and elaborate upon in their understanding and mastery of Mathematics (Buck et al., 2016). Andersson and Wagner (2021) concur that learners benefit from PBE because it enriches their (Western) Mathematical knowledge. In research on geometry, Farid et al. (2024) found that PBE enables learners to connect their Mathematics learning to their local spaces and to draw on these spaces as sources for their geometry projects. Similarly, Leonard et al. (2013) maintain that PBE engages learners emotionally and increases their engagement, which, in turn, positively impacts their retention and performance. PBE thus assumes that education, in general, and Mathematics education specifically, is not without a 'place'; rather, it is embedded in and informed by a unique social, environmental, and cultural context that guides teaching and learning and has the potential to improve the quality of Mathematics education.

Improving the quality of education is pivotal for realising the Sustainable Development Goals (SDGs) and for Education for Sustainable Development (ESD). ESD is "holistic and transformational education [that] addresses learning content and outcomes, pedagogy, and the learning environment" (UNESCO, 2014, p. 12). ESD is, therefore, an educational approach that promotes the integration of sustainability and the SDGs across all subjects in schools and institutions of learning. In this regard, Mathematics teacher education programmes and Mathematics education should be oriented towards sustainability and ESD. However, Li and Tsai (2022) find it difficult to understand how such an orientation could be achieved, while Renert (2011, p. 20) argues that sustainability and Mathematics education remain largely unconnected. Nevertheless, establishing links between sustainability, Mathematics, and Mathematics teacher training is important because of the potential of Mathematics education to address the needs of the 21st century (Mula & Tilbury, 2023).

Addressing 21st-century needs and achieving sustainable development (SD) requires high-quality education. Hence, Sustainable Development Goal 4 (SDG 4) focuses specifically on improving the quality of education by ensuring "inclusive and equitable quality education and promoting lifelong learning opportunities for all" (UNESCO, 2014, p. 12). In this regard, SDG 4 sets certain targets that governments and countries must reach to ensure a sustainable future. For example, Target 4.1 requires that governments ensure that by 2030, "… all girls and boys complete free, equitable and quality primary and secondary education" (UNESCO, 2014, p. 12). Similarly, Target 4.5 requires governments to "eliminate … ensure equal access to all levels of education and vocational training for the vulnerable, including persons with disabilities, indigenous peoples, and children in vulnerable situations" by 2030 (UNESCO, 2014, p. 12). The implication of this goal and its associated targets is that South African (SA) education will have to take definitive steps to improve the quality of education, paying particular attention to the eradication of inequalities and inequities and ensuring that Mathematics education is inclusive of the needs of all learners.

The quality of SA education, particularly in Mathematics, is worrisome (cf. Schirmer & Visser, 2023). This concern stems from national Mathematics results and international test scores, such as those from the Trends in International Mathematics and Science Study (TIMSS) (2023) and the Southern and Eastern Africa Consortium for Monitoring Educational Quality (SACMEQ), which paint a grim picture of the state of Mathematics education in the country. This situation is attributed to various factors, such as the inadequate or weak knowledge of Mathematics teachers (Gobede & Mosvold, 2022) and the quality of Mathematics teacher training (Taylor, 2021).

Moreover, Mathematics and Mathematics education are also variously accused of advancing gender stereotypes (Holmes et al., 2022) and of perpetuating oppressive norms while maintaining oppressive structures and an exclusionary culture (Harper & Kudaisi, 2023). In addition, Helliwell and Ng (2022, p. 128) claim that the discourse on Science, Technology, Engineering, and Mathematics (STEM) education has largely focused on shaping "a future that works for all by putting people first,…[emphasising] that all…new technologies are first and foremost tools made by people for people." This not only suggests that Mathematics and Mathematics education perpetuate inequality, but it is also part of a discourse that promotes anthropocentric values and a human-centred orientation. This jeopardises sustainable development and makes Mathematics and Mathematics education part of the problem of unsustainable development and social inequalities.

Against this background, calls have emerged for a new and ecologically based Mathematics education that is oriented towards the needs of people, their cultures, and the planet. This approach fosters an awareness that our current global reality is characterised by "unsustainability" rather than "sustainability" (Biccard, 2019). It is in this context that teacher training programmes can contribute to sustainable Development (SD) and Education for Sustainable Development (ESD) by sensitising student teachers to place-based education (PBE).

This chapter advocates for Place-based Education as a pedagogy to equip student teachers with the necessary skills, knowledge, and values to enhance Mathematics education, thereby contributing to the alignment of Mathematics education with ESD and the realisation of SD. I will provide a brief overview of Mathematics teacher training in South Africa, followed by a discussion of ESD and its connection to Mathematics education. Subsequently, I will reflect on PBE and its relevance and value to Mathematics teacher training and education. I will conclude the chapter with some thoughts on the implications of PBE for Mathematics teacher training and education.

1.2 Why a refocus of mathematics teacher education

Mathematics is generally perceived to be difficult, abstract, static, and disconnected from our daily lives (Holmes et al., 2022). It is furthermore conceived as teacher- and content-oriented and it focuses on "teaching to the test" resulting in Mathematics teaching being strongly academically oriented (Li & Tsai, 2022). These perceptions are fuelled by a conviction that not all learners are supposed to take Mathematics at school because not all have the capabilities to master it. Subsequent to this, many learners see no value in the subject, they lose interest in it, and they opt out of it the moment they have the chance. However, opting out of Mathematics is deeply problematic for society (Li & Schoenfeld, 2019), especially when Mathematics and Mathematics education forms part of the development trajectory of a country such as the case is with South Africa.

The place of Mathematics and Mathematics education in the development of South Africa is evident in the status awarded to Mathematics and Mathematics Literacy within the national basic education curriculum promulgated in 2012. Here these two subjects are compulsory, and all learners are expected to take either the former or the latter during their basic education. The significance of these subjects in societal development is acknowledged by Jojo (2020) who opines that Mathematics and Mathematics education can empower people, and it can contribute significantly to the eradication of poverty, whilst it also advances gender equality. South Africa's National Development Plan (NDP) (2012) also emphasises that the basic education system should improve the outputs in Mathematics education, and universities are expected to deliver more trained graduates in Mathematics and to, deliver more and better qualified Mathematics teachers (NDP, 2012). Mathematics is therefore centred within the national development project.

However, the realisation of this vision is hampered by the crisis Mathematics education finds itself in, with Mathematics teacher readiness and training frequently appearing as a potential factor contributing towards the crisis. Jojo, (2020) blames curriculum implementation and the readiness of Mathematics teachers, whilst (Pournara et al., 2015) blames the poor Mathematical knowledge of teachers for the crisis. A study conducted by SACMEQ found that only 41% of Grade 6 Mathematics teachers in South Africa had "good proficiency in mathematics" (Schirmer & Visser, 2023), whilst Spaull (in Mlachila & Moeketsi, 2019), found that four out of five teachers in public primary schools lack the content knowledge and pedagogical skills to teach Mathematics. Verster and Sayed (2020) also blame Mathematics learners' poor and underperformance on a shortage of confident and competent qualified Mathematics teachers. In terms of teacher training, Taylor, (2021) found the preparation of final year Bed Mathematics teachers so poor that they are described as "patently incompetent to teach maths to primary school learners". It is because of this that the United Nations Educational, Scientific and Cultural Organisation (UNESCO) (2015, p. 192) claims that "millions of South African children are not acquiring basic arithmetic skills and are therefore most likely to drop out of basic primary education".

These statistics points to rather stark educational inequalities in the quality of Mathematics education in a system where historically black schools make up most of the schools in South Africa. When it therefore comes to Mathematics education, black learners are still disproportionally disadvantaged and therefore set up for failure by an education system that is supposed to promote equity and take them out of poverty. There is therefore a pressing need to rethink and re-envision Mathematics education and to consider a) the use of a range of resources to support the development of Mathematical concepts (Naidoo, 2022), and b) the use of suitable pedagogies based on the learner's language, culture and living environments (Mapungubwe Institute for Strategic Reflection [MISTRA], 2020).

In this chapter I focus on the learners' place as one such a resource and a pedagogy. However, before I reflect on Place-Based Education (PBE), let me first locate it within the broader discourse around SD, the SDGs and ESD. Such a location is relevant because of the close connection of PBE with SD and ESD and because of the contribution it could make in realising quality education as is advanced by SDG 4.

2. Methodology

This chapter is based on an extensive literature review. In this desktop chapter primary and secondary literature were used to formulate and build out the argument that is presented. I used the identified keywords, and I searched for relevant and recent sources on platforms such as Google Scholar, EBSCO Academic Search Complete, ERIC, JSTOR. All sources were subjected to an intensive reading to identify relevant information. Policy documents used in the chapter were analysed to identify sections that speaks to the argument of the chapter. All sources were carefully read, and relevant information were coded. To ensure the trustworthiness of this chapter, all sources were appropriately cited and referenced, and care was taken to engage with all sources in an objective and unbiased manner.

3. Place-based Education

3.1 Place-based education: Its origin

PBE is not an entirely new concept to teaching and learning. Rather, it has a long history within education that can be traced back to Dewey's ideas on teaching and learning. In this regard, Dewey believed that learning occurred most naturally when focused on the intersection of people, their local environments, and an authentic purpose (Yemini et al., 2023). However, PBE owes its more recent popularity to the rise of neoliberalism in the 1990s and the impact of its associated ideologies on education policies. Neoliberalism is a political ideology that is focused on the economy, and is - 76 -aption- 76 -ing- 76 -o by an emphasis on market-driven policies, privatisation, individualisation, and the commodification of education (Villarin et al., 2024). This ideology impacts every sphere of our existence, and in education, it is most visible in standardised tests, performance-based funding, accountability regimes, efficiency, and competition, which variously disadvantage the poor and the marginalised and promote inequality. PBE thus serves as a counter-educational force against neoliberal educational reforms that propagated a disregard for culture in education and which promoted the decentring of cultural connections, community, and environmental stewardship (Yemini et al., 2023). This neoliberal turn resulted in schools increasingly becoming detached from the communities and people they serve and from the animals and plants that they share space with. As such, neoliberalism projected the learners' 'place' as bearing no significance on teaching and learning and thus as something of no pedagogical value and significance. With neoliberalism advancing a detachment from culture and from the 'place' of the learner, a form of education was established that "distracts our attention from and distorts our response to the actual contexts of our own lives (places)" (Gruenewald, 2003, p. 621). Such distractions and distortions are of course detrimental to effective and quality teaching and learning. Hence the focus on a reconnection and a reattachment with the 'place' through PBE.

3.2 What is 'place' in place-based education all about?

The concept of 'place' is the focal point of PBE. One cannot, therefore, come to deeper insights into PBE without a proper understanding of the concept of 'place'. On face value, one would be tempted to narrowly and superficially define the concept 'place'. However, within PBE 'place' is more than just an address, a plot, a piece of land, a home or a GPS location we inhabit, co-inhabit or can physically visit. Rather, 'place' is "a spiritual place, a place of being and understanding" where "interactions with places give rise to and define cultures and community" (Rubel & Nicol, 2020, p. 175). For Coughlin and Kirch (2010, p. 915) 'place' also represents "a lived entity that results from a dialogical transaction between a community and its material environment at a particular moment in cultural-historical time and which hence shapes and is shaped by the identity of the people". As such, 'place' is where living and non-living organisms exist and co-exist and where they form and are formed by the place they occupy at a given moment in time. Similar conceptions of 'place' are experienced by indigenous people and communities, who have an intimate connection with 'place' as a way of thinking about relations between and with humans, and with nature, and for whom "place is a way of knowing, experiencing, and relating with the world" (Rubel & Nicol, 2020, p. 175).

Its richness in terms of histories, emotions, stories, culture, aesthetics, and social problems (Buck, Cook & Carter, 2016), therefore makes place a valuable pedagogical asset that can enhance Mathematics teaching and learning, by presenting 'place' as a valuable educational resource and an alternative text to explore, read and synthesise. Gruenewald (2003, p. 620) attests to the pedagogical value of 'place' by stating that,

Place matters to educators, students, and citizens in tangible ways that include providing teachers and students with "firsthand experience" to link local contexts to learning environments in order to understand sociopolitical processes and shape what happens in the local community.

Similar sentiments are expressed by Rubel and Nicol (2020, p. 175) who view 'place' as conscious with memory that leads to its pedagogical potential as the first teacher with context specific "lessons to share". In this way place holds value and advantage for Mathematics teacher training, and it ought to be used in the service of Mathematics teaching and learning. Using it could assist in bridging divides between Mathematics teaching and learning as happens in the classroom setting and between the learner's real-life experiences, context and space. Thus, embedding Mathematics in the learners' place, and allowing the teaching and learning thereof to transcend the rather limiting boundaries of the classroom, could make the rather abstract and difficult Mathematics subject content more relevant and meaningful to the learner.

3.3 Place-based education: A definition

As a "transformative philosophy of education" (Carter-Guyette, 2019, p. 2), PBE is an umbrella term for pedagogical practices that prioritise experiential, community-based, and contextual or ecological learning to cultivate greater connectivity to local contexts, cultures, and environments (Yemini, Engel & Simon, 2023). PBE also supposes that education is grounded somewhere and to something for effective learning to take place (Leonard, et al., 2013). More specifically, PBE is also regarded as "an immersive learning experience that places students in local heritage, cultures, landscapes, opportunities, and experiences" (Ashari et al., 2018). These contexts, sites and spaces as well as the landscape and experiences are then used to leverage and to serve as foundation for the study of language arts, Mathematics, social studies, science and other subjects across the curriculum (Nadelson, 2014).

The use of the local context as a source of teaching and learning expands learning beyond the limiting space of the classroom, the school and places that are attached to the school — to also include spaces and sites that are effectively detached from the school, but who provide equally powerful sources and texts for effective and quality teaching and learning. In this way it situates teaching and learning within a different but local context and it provides local relevance and contextual motivation for learning.

With PBE embedded in the local context of the learner, local and family knowledges and place-based knowledge bridge the gap between the curriculum, new content and the learner's existing knowledge and community and family sources he/she has access to. This ensures that advantage is taken of geography, culture, and location to create authentic, meaningful and engaging - 78 -aption- 78 -ing- 78 - learning opportunities (Ashari et al., 2018). With teaching and learning taking place in spaces outside the classroom, the school now becomes an "open and inviting [place] in the community and the community welcomes student learning occurring in many dimensions" (Bandari & Khoshnevis, 2021). Thus, as education that takes place outdoors, PBE allows students to be a part of their outdoor surroundings. In this way it "provid[es] meaningful contextual experiences-in both natural and constructed environments-that complement and expand classroom instruction, which tends to be dominated by print and electronic media" (Kilinç & Evans, 2013, p. 263).

3.4 Locating place-based education within education for sustainable development

Mathematics and Mathematics education are rather incompatible and ESD in Mathematics education is complex (Li & Tsai, 2022). The teaching of Mathematics also continues to pose challenges for many Mathematics teachers. Subsequent to this there has, in general, been a lack of engagement with issues of sustainability within Mathematics education (Boylan & Coles, 2017). This is despite the expectation that education would be orientated towards ESD, and the SDGs highlights the importance of incorporating sustainability into education at all levels. Goal 4.7 of SDG 4 which deals with quality education for all, by implication also promotes ESD in

Mathematics teacher training by establishes the importance of ensuring learners acquire the relevant knowledge, skills and values that promote sustainable development (UNESCO, 2017).

As an orientation towards education, ESD provides a different lens to view and transform learning objectives, content, methods, contexts, and assessment (Mula & Tilbury, 2023). It also uses emancipatory learning approaches, such as systems thinking, critically reflective thinking, participatory learning, and interdisciplinary learning, but also more disruptive pedagogical strategies, like place-based learning (Mula & Tilbury, 2023). Such approaches 'disrupts' conventional education and Mathematics education by incorporating and sensitising learners about social justice in and through Mathematics education, and to develop in learners the competences required to address complex sustainability issues such as climate change. Harper (2017) concurs that criticality in Mathematics education leverage learners' cultural, linguistic, and community knowledge, resources, and experiences. This supports Mathematics learning and critical consciousness development and it guides learners to interrogate and address social injustices in and through Mathematics.

To integrate ESD into Mathematics education, a need exists to reimagine Mathematics as a general human concern, and to see it as an opportunity to learn critical thinking, problem-solving, and contextual understanding (Nicol et al., 2020). For example, learners should be - 79 -aption- 79 -in about the social and ecological implications of Mathematics education, particularly in relation to the creation of social inequalities and the climate crisis (Helliwell & Ng, 2022). Framed within such a new paradigm, Mathematics teacher education can thus push back the boundaries of the traditional philosophy of Mathematics and accommodate the power of Mathematical applications in guiding human lives and the social world (Li & Tsai, 2022) as well as sustainable development.

PBE thus provides a new approach to Mathematics teacher training and education that will foster new relationships between humans, Mathematics and the planet and reinforces the interconnectedness between these aspects. More so, PBE in Mathematics teacher education can address the quality of Mathematics education and address societal inequalities. Mathematics teachers can only teach what they know, and they will base their teaching primarily on the methods and pedagogies they were exposed to during teacher training. To expect that teachers must apply PBE requires that student teachers be trained in the use of PBE.

3.5 Principles of place-based education

Principles are generally accepted rules that govern a phenomenon or behaviour. In a similar fashion, PBE is also governed by generally accepted rules that not only distinguish PBE from other approaches to education, but that also gives PBE its unique character and governs its approach to education. What we know about PBE is that it pivots on 'place' and that it is "primarily concerned with connecting place with self and community" (Mcinerley et al., 2011, p. 5). As such, it promotes active learner-centred inquiry using the surrounding community – or place — as a classroom. In this regard, place represents more than the physical, but it includes experiences, the natural and the spiritual.

What makes PBE further unique is that it is "inherently multidisciplinary and experiential" (Mcinerley et al., 2011, p. 5), and it focuses on active learning modalities that is applied to the local context, in which learners become proactive researchers (Yemini, et al., 2023). Within this context, learning takes place from experiences in and from experiencing the local context, in which learners are taught to apply a problem-solving approach towards local issues and challenges. The multidisciplinary nature of PBE makes it possible for learners to supplement their exiting Mathematical knowledge with knowledge from diverse disciplines when trying to respond to community- or place-related challenges. The benefit of this is that learners come to understand that Mathematics does not exist in a silo, but that it is in a relationship with other subjects and disciplines. For the multidisciplinarity of PBE to come to full effectiveness, require schools, their environments and communities must become more 'permeable' (Mannion & Adey, 2011, p. 38). This permeability allows the learners' place to become an integral text and for the community to become an indispensable teaching and learning resource.

Furthermore, PBE is also context specific, and it makes use of authentic content, authentic assessment, teacher facilitation, cooperative learning, reflection, and incorporation of adult skills (Bandari & Khoshnevis, 2021). This context specificity focuses PBE on, and makes it applicable to, one particular location. PBE approaches of one context can therefore not effectively be applied to another context. Context specificity largely stems from the focus of PBE on the community, the natural environment, culture, etc. All these aspects are unique, and they experience unique challenges which require unique responses.

Buck et al. (2016) also identify the following principles that give PBE its unique character. These principles are: (a) taking advantage of local possibilities for teaching and learning; curriculum development; (b) students as knowledge producers; (c) learning from community members as well as teachers; and (d) fostering a regard and appreciation for home community.

Smith & Sobel, 2010 (in Carter-Guyette, 2019, p. 12) present a list of principles which according to them informs and drives PBE. According to them, PBE:

- is learning that takes place on-site (the school yard, in local community);
- is inherently experimental, including participatory action or service learning;
- comprises a curriculum that is multigenerational and multicultural
- interacting with community resources;
- is focused on local themes, systems, and content;
- is learning that is personally relevant to the learner;
- is learning experiences that contribute to the community's vitality and environmental quality and support the community role in fostering global environmental quality;
- is learning that is supported by strong and varied partnerships with local organisations/agencies/businesses/government;

- is learning that is interdisciplinary;
- is learning experiences that are tailored to the local audience;
- is learning that is grounded in and supports the development of a love for one's place; and
- is local learning that serves as the foundation for understanding and participating appropriately in regional and global issues.

These principles not only give PBE its unique character, but it also define PBE, and it makes it possible for PBE to realise its aims and objectives in a unique and a distinctive way

3.6 Aims and objectives of place-based education

In principle, PBE aims to amplify 'place' by locating learning within the local context or the community. This it aims to do, firstly by overcoming the often hegemonic and ubiquitous educational agendas developed elsewhere in favour of an approach that uses the local as a starting point (Lyle, 2015), and secondly by partnering teachers, students and community members in the process of teaching and learning using the local context (Coughlin & Kirch, 2010). In this way, learners and teachers become participants in meaning-making and knowledge production and so contribute in various ways to their place or their community (Starrett et al., 2020). These aims correspond with what is expected from Mathematics education in SA. According to the Curriculum and Assessment Policy Statement, Grades R-12 (CAPS) (2011, p. 8) teaching and learning of Mathematics aims to develop:

a) a critical awareness of how mathematical relationships are used in social, environmental, cultural and economic relations;
b) confidence and competence to deal with any mathematical situation without being hindered by a fear of mathematics
c) a spirit of curiosity and a love for mathematics
d) an appreciation for the beauty and elegance of mathematics
e) recognition that mathematics is a creative part of human activity
f) deep conceptual understanding in order to make sense of mathematics
g) acquisition of specific knowledge and skills necessary for:

- the application of mathematics to physical, social and mathematical problems
- the study of related subject matter (e.g. other subjects)

PBE also aims at creating better citizens (Carter-Guyette, 2019). These citizens will not only have developed learning skills, but they will also be able to use these skills to solve problems, to address inequality, to challenge inequity and to create a better world for all. This is also evident from the CAPS (DBE, 2011, p. 5) which expect that also Mathematics learners be equipped to:

a) identify and solve problems and make decisions using critical and creative thinking;
b) work effectively as individuals and with others as members of a team;
c) organise and manage themselves and their activities responsibly and effectively;

d) collect, analyse, organise and critically evaluate information;
e) communicate effectively using visual, symbolic and/or language skills in various modes;
f) use science and technology effectively and critically showing responsibility towards the environment and the health of others; and
g) demonstrate an understanding of the world as a set of related systems by recognising that problem solving contexts do not exist in isolation.

Within the context of South Africa, the development of such skills in learners are important to strengthen the democratic project of the country. However, the development of these skills can more effectively take place when they are integrated in Mathematics teacher training programmes.

4. Integrating Place-based Education in Mathematics Teacher Training

The aim of this chapter is to advocate for Place-based Education as a pedagogy to equip student teachers with the necessary skills, knowledge, and values to enhance and improve Mathematics education and so contribute towards the orientation of Mathematics education towards ESD and the realisation of SD. To become the favoured teaching and learning approach for Mathematics teachers, it is advisable that PBE be integrated in all mathematics teacher training programs. It is only when the skills, dispositions and attributes of PBE are developed and to a large extent - 82 -aption- 82 -ing- 82 - in Mathematics student teachers that PBE will spontaneously be used in the teaching of Mathematics at school level. In response to the NDP and education policies, higher education in SA is oriented towards SD. It is therefore expected that SA higher education institutions will contribute towards the - 82 -aption- 82 -ing of the SDGs and SD. Leal-Filho et al. (2019) report that SA higher education institutions have, over the last 30 years, been incorporating sustainable development within their systems, processes, curricula, teaching, research, and local communities. As a pedagogy of ESD, the expectation is, therefore, that teacher training programmes will also implement PBE in their preparation of future teachers. So how should Mathematics teacher training programmes be oriented towards PBE in order to make possible the development of an affinity for PBE?

In terms of the curriculum for teacher training, it is important that principles of ESD be reflected. Curricula, teaching practices, and content should be selected and designed in response to the SD requirements of the NDP and education policies such as CAPS (2011), which is informed by the principles of social transformation, active and critical learning, human rights, inclusivity, environmental and social justice, and valuing indigenous knowledge systems. It is therefore important that Mathematics teacher training be focused on the transformation of society, on sustainable living and on equity and equality. For Chabbott and Sinclair (2020) this implies that education students have access to appropriate textbooks, flexible teaching pedagogy, technology, the Internet and data, and guidance to support home-based learning. More so, ESD is premised on the assumption that the realisation of SD and the SDGs will best be achieved through interdisciplinary teaching and learning approaches. As such, the preparation of

Mathematics teachers should explicitly reflect interdisciplinarity, and teacher students should be demonstrated how, and be encouraged to find ways of working with others and across disciples.

Starrett et al. (2020) opine that PBE frequently uses constructivist or contextual approaches and often involves social or environmental justice issues. These approaches make use of teaching and learning methods such as project-based or inquiry-based learning OR service learning, environmental education, experiential learning and workplace education respectively. As such, Mathematics teachers should be exposed to ways of incorporating and using these approaches in the teaching of the various components and concepts of Mathematics.

Secondly, Reed and Klassen (2020) hold that in PBE everyone is a learner. PBE embraces the concept of the life-long learner, which supposes that learning is a continuous process and that teachers should continue to learn. The notion that everyone is a learner calls for a paradigm shift in the way Mathematics teachers perceive themselves. Amongst other, this requires that Mathematics teachers acknowledge they do not have all the answers and that they are not the holders of ultimate Mathematical and educational truths. Rather they need to develop the ability to acknowledge that they can also learn from others – that is from learners, from community members, from nature and broadly from the place they and the learners occupy. In that way they no longer have to teach only the little they know, and in traditional ways, but they become learners alongside the young ones they are supposed to teach, and they also become experimental and explorative.

Thirdly, during their training, Mathematics teachers should also be exposed to and encouraged to use multiple non-traditional texts for and approaches to teaching (Lowenstein et al., 2018). Using non-traditional Mathematical approaches during teaching will require from student teachers to 'teach against the grain'. Teaching against the grain can be unsettling and disruptive. However, the complex issues that 'place' presents, requires that Mathematics teachers use unconventional teaching approaches, and that they 'unlearn' some of the things that keep them in their 'comfort zones'. More so, it also requires that they draw from and integrate different perspectives and knowledges from diverse sources and disciplines.

Fourthly, the use of PBE in teacher training programmes requires that student teachers be skilled and trained in ways to locate and to situate mathematical problems within the learners' local area and so establish a strong educational grounding (Buck et al., 2016). PBE uses the learners' local community as an auxiliary text and source of teaching. Mathematics teachers should therefore be trained in ways that will make it easy for them to teach their learners how to read, how to synthesise, and how to explore their place, with the aim to enhance Mathematics teaching and learning.

This requires that Mathematics teachers understand the value and significance of the unique local history, geography, culture, and community, and appreciate it as valuable resources for enhancing, and being enhanced by, learners' learning of Mathematics (Showalter, 2012).

Mathematics student teachers must, therefore, have a thorough understanding and deep-rooted knowledge of the place learners occupy and hail from. More so, they need to validate and affirm it as part of the learners' identities and as such as a source of teaching and learning

5. Conclusions and Recommendations

The aim of this chapter was to make out a case for the use of Place-based Education (PBE) as an Education for Sustainable Development approach to promote equity and equality and to address the Mathematics crisis in South African education. I argued that the teaching of Mathematics, by merely transferring content from a textbook to a passive learner, is not effective. I belief that to be effective, Mathematics teaching should be connected to the real-life experiences and the real-life problems of the learner. In this regard, I proposed PBE, which is embedded in and informed by the learners' place and which fosters an appreciation of place as a carrier of relevant and valuable knowledge and teaching and learning opportunities.

The implication of which is that Mathematics teacher training programmes be designed in ways to develops in Mathematics student teachers the skills and knowledge to a) validate the educational value of the learners' place; b) know how to unlock and use the funds of knowledge and the multiple teaching and learning opportunities created by place; c) create different and out-of-the-classroom learning experiences for learners; .d) develop a constructivist orientation towards learning and learning; e) appreciate various teaching and learning approaches and ways of knowing. More so, Mathematics student teachers should also be taught to be open and flexible and to tap into the diverse learning opportunities a learners' place offers whilst celebrating the learners' place as a new and exciting source of teaching as well as a new source of knowledge. This does not imply that the classroom becomes redundant. Rather the classroom provides information about the place, and the place provides relevant, real-life examples and opportunities for Mathematics teaching and learning.

Within the South African context, using the learners' place will not only enhance teaching and learning, but will also change the story of Mathematics in the country. Rather, it will also address inequalities, advance inclusion, speak to the - 84 -aption- 84 -ing- 84 -on of education, and so promote sustainable development.

6. Declarations

Funding: This research did not receive any external funding

Conflicts of Interest: The author declares no conflict of interest

References

Ashari, A., Pujiank, S., Ibrohim, I., Suwono, H., & Lukiati, B. (2018). Integrating place-based education in biology learning through inquiry to improve students' cognitive ability and scientific attitudes in Sumbawa. *Advances in Social Science, Education and Humanities Research, 218,* Proceedings of the 1st Annual International Conference on Mathematics, Science, and Education (ICoMSE 2017), Indonesia. https://doi.org/10.2991/icomse-17.2018.27

Bandari, Z. R., & Khoshnevis, I. (2021). Comparing the role of Project-Based Teaching Method vs. Place-Based Teaching Method on motivation of Iranian EFL learners. *Turkish Journal of Computer and Mathematics Education, 12*(13) 4586-4596. https://doi.org/10.17762/turcomat.v12i13.9632

Biccard, P. (2019). The professional development of primary school mathematics teachers through a design — based research methodology. *Pythagoras — Journal of the Association for Mathematics Education of South Africa, 40*(1), 1-10. http://dx.doi.org/10.4102/pythagoras.v40i1.515

Boylan, M., & Coles, A. T. (2017). Is another mathematics education possible? An introduction to a special issue on mathematics education and the living world. *Philosophy of Mathematics Education Journal*, *32*.

Buck, G. A., Cook, K., & Carter, I. W. (2016). Attempting to make place-based pedagogy on environmental sustainability integral to teaching and learning in middle school: an instrumental case study. *Electronic Journal of Science Education, 20*(2), 32–47.

Carter-Guyette, M. (2019). *Place-Based Learning as an effective methodology for developing self-efficacy skills in students with learning disabilities* [Unpublished thesis from the University of New England].

Chabbott C., & Sinclair, M. (2020). SDG 4 and the COVID-19 emergency: Textbooks, tutoring, and teachers. *Prospects*, *49*, 51–57. https://doi.org/10.1007/s11125-020-09485-y

Coughlin, C. A., & Kirch, S. A. (2010). Place-based education: a transformative activist stance. *Cultural Studies of Science Education*, *5*, 911–921. http://dx.doi.org/10.1007/s11422-010-9290-6

Departments of Basic Education and Higher Education and Training (2011). *Integrated Strategic Planning Framework for Teacher Education and Development in South Africa, 2011-2025*. Pretoria; Government Printers.

Gruenewald, D. A. (2003). Foundations of place: A multidisciplinary framework for place-conscious education. *American Educational Research Journal, 40(3)*, 619-654. https://doi.org/10.3102/00028312040003619

Harper, F. K., & Kudaisi, Q. J. (2023). Geometry, groceries, and gardens: Learning mathematics and social justice through a nested, equity-directed instructional approach. *Journal of Mathematical Behavior,* 71. https://doi.org/10.1016/j.jmathb.2023.101069

Helliwell, T., & Ng, O. L. (2022). Imagining possibilities: innovating mathematics (teacher) education for sustainable futures. *Research in Mathematics Education, 24*(2), 128-149, https://doi.org/10.1080/14794802.2022.2079553

Holmes, K., Berger, N., Mackenzie, E., Attard, C., Johnson, P., Fitzmaurice, O., O'Meara, N., & Ryan, V. (2022). Editorial: The impact of place-based contextualised curriculum on student engagement and motivation in STEM education. *Frontiers in Education*, *6*, 1-3. https://doi.org/10.3389/feduc.2021.826656

Jojo, Z. (2020). Mathematics education system in South Africa. *IntechOpen.* http://dx.doi.org/10.5772/intechopen.85325

Kilinç, E., & Evans, R. (2013). History of place-based education in the social studies field. *Adıyaman University International Journal of Social Science.* http://dx.doi.org/10.14520/adyusbd.29875

Leal-Filho, W., Vargas, V. R., Salvia, A. L., Brandli, L. L., Pallant, E., Klavins, M., Ray, S., Moggi, S., Maruna, M., Conticelli, E., & Ayanore, M. A. (2019). The role of higher education institutions in sustainability initiatives at the local level. *Journal of Cleaner Production*, *233*, 1004-1015. http://dx.doi.org/10.1016/j.jclepro.2019.06.059

Leonard, J., Russell, N. M., Hobbs, R. M., & Buchanan, H. (2013). Using GIS to teach placed-based mathematics in rural classrooms. *The Rural Educator*, *34*(3), 10–17. https://doi.org/10.35608/ruraled.v34i3.395

Li, H-C., & Tsai, T-L. (2022). Education for sustainable development in mathematics education: What could it look like? *International Journal of Mathematical Education in Science and Technology*, *53*(9), 2532-2542. https://doi.org/10.1080/0020739X.2021.1941361

Li, Y., & Schoenfeld, A.H. (2019). Problematizing teaching and learning mathematics as "given" in STEM education. *International Journal of STEM Education, 6*(44). https://doi.org/10.1186/s40594-019-0197-9

Lowenstein, E; Grewal, I.K; Erkaeva, N; Nielsen, R., & Voelker, L. (2018). Place-based teacher education: a model whose time has come. *Issues in Teacher Education, 27*(2), 36-52.

Lozano, R., Id, M., Merrill, M., Sammalisto, K., Ceulemans, K., & Lozano, F. (2017). Connecting competences and pedagogical approaches for sustainable development in higher education: a literature review and framework proposal. *Sustainability*. *9*(10). http://dx.doi.org/10.3390/su9101889

Lyle, E. (2015). Pedagogies of place/geographies of experience. *Pathways: The Ontario Journal of Outdoor Education, 27*(3), 11-15.

Mannion, G., & Adey, C. (2011). Place-based education is an intergenerational practice. *Children, Youth and Environments*, *21*(1), 35–58.

McInerney, P., Smyth, J., & Down, B. (2011). 'Coming to a *place* near you?' The politics and possibilities of a critical pedagogy of place-based education. *Asia-Pacific Journal of Teacher Education*, *39*(1), 3-16, https://doi.org/10.1080/1359866X.2010.540894

MISTRA Mapungubwe Institute for Strategic Reflection. (2020). *Recommendations on the Pedagogy for Mathematics Education in South Africa*. Johannesburg, South Africa.

Mlachila, M., & Moeletsi, T. (2019). Struggling to make the grade: A review of the causes and consequences of the weak outcomes of South Africa's education system. *International Monetary Fund (IMF) Working Paper. Working Paper No. 2019/047*.

Mulà, I., & Tilbury, D. (2023). Teacher education for sustainability: Current practice and outstanding challenges. *AIEM -Avances de investigación en educación matemática, 23,* 5-18. https://doi.org/10.35763/aiem23.5414

Nadelson, L.; Seifert, A. L., & McKinney, M. (2014). Place based STEM: leveraging local resources to engage K-12 teachers in teaching integrated STEM and for addressing the local stem pipeline. *121st ASEE Annual Conference & Exposition, American Society for Engineering Education,* Indianapolis, 15-18 June, 2014. http://dx.doi.org/10.18260/1-2—22916

Pournara, C., Hodgen, J., Adler, J., & Pillay, V. (2015). Can improving teachers' knowledge of mathematics lead to gains in learners' attainment in Mathematics? *South African Journal of Education*. *35*(3), 1-10. http://dx.doi.org/10.15700/saje.v35n3a1083

Reed, M., & (Weber) Klassen, C. (2020). Place-based education: people, places, and spaces for interdisciplinary literacy learning. *The Reading Professor*. *43*(1).

Renert, M. (2011). Mathematics for life: sustainable mathematics education, *For the Learning of Mathematics*. *31*(1), 20-26. http://dx.doi.org/10.2307/41319547

Rubel. L. H., & Nicol, C. (2020). The power of place: spatializing critical mathematics education. *Mathematical Thinking and Learning*, 22(3), 173-194. https://doi.org/10.1080/10986065.2020.1709938

Schirmer, S., & Visser, R. (2023). *The silent crisis: South Africa's failing education system*. The Centre for Development and Enterprise. Johannesburg, South Africa

Showalter, D. (2012). *Place-Based Mathematics: A Conflated Pedagogy?* (Working Paper No. 43). Appalachian Collaborative Center for Learning, Assessment, and Instruction in Mathematics, Ohio University.

Starrett; A. Irvin., M. J., Lotter, C., & Yow, J. A. (2020). Understanding the relationship of science and mathematics place-based workforce development on adolescents' motivation and rural aspirations. *American Educational Research Journal*, *59*(6), 1090–1121. https://doi.org/10.3102/00028312221099009

Steffensen, L., & Kasari, G. (2023). Integrating societal issues with mathematical modelling in pre-service teacher education. *Education Sciences, 13*(721), 1-21. https://doi.org/10.3390/educsci13070721

Stevenson, R. B. (2008). A critical pedagogy of place and the critical place(s) of pedagogy. *Environmental Education Research*, *14*(3), 353-360. https://doi.org/10.1080/13504620802190727

Taylor, N., (2021). 'The dream of Sisyphus: Mathematics education in South Africa'. *South African Journal of Childhood Education, 11*(1). https://doi.org/10.4102/sajce.v11i1.911

UNESCO. (2015). *Education for All (EFA) Global Monitoring Report: Education for All 2000 2015: Achievements and Challenges*. UNESCO. Paris, France

UNESCO. (2017). *Education for Sustainable Development Goals: Learning Objectives*, 1st ed. UNESCO. Paris, France.

Verster, J., & Sayed, Y. (2020). Constraints influencing the efficacy of a PGCE mathematics program: A case study. *Alberta Journal of Educational Research, 66*(2), 192–206. http://dx.doi.org/10.55016/ojs/ajer.v66i2.67915

Villarin, S. C.; Kilag, O, Perez, M. Garrido, R., & Escarda, L. (2024). Tracing the Impact of Neoliberalism on Educational Administration Practices. *Excellencia: International Multi-disciplinary Journal of Education, 2*(1), 144-154. https://doi.org/10.5281/zenodo.10552507

Yemini, M., Engel, L., & Simon, A. B. (2023). Place-based education – a systematic review of literature, *Educational Review*, 1–21. https://doi.org/10.1080/00131911.2023.2177260

CHAPTER SIX

Education for Sustainable Development and Its Implication for the Preparation of Pre-service Mathematics Teachers

Babawande E. Olawale[1*]
Winston Hendricks[2]
Khalid Saddiq[3]

AFFILIATIONS
[1]Faculty of Education, Walter Sisulu University, Mthatha, South Africa.
[2]Faculty of Education, University of Fort Hare, Alice, South Africa.
[3]Mathematics Education, Faculty of Education, University of Tasmania, Australia.

CORRESPONDENCE
Email: bolawale@wsu.ac.za*

Reference Formats (APA 7th Edition)
Olawale, B. E., Hendricks, W., & Saddiq, K. (2025). Education for sustainable development and its implication for the preparation of pre-service mathematics teachers. In B. E. Olawale (Ed.), *Building the Foundations: Effective Approaches in Mathematics Teacher Preparation* (pp. 88-104). ERRCD Forum.
https://doi.org/10.38140/obp3-2025-06

Abstract: This chapter explores the integration of Education for Sustainable Development (ESD) into mathematics teacher education programmes. It highlights the fundamental principles of ESD and examines the potential benefits and challenges of incorporating these principles into mathematics education. The chapter methodically synthesises key themes related to ESD principles, such as an interdisciplinary approach, critical thinking and problem-solving, active participation, values and ethics, sustainable consumption, and holistic understanding. Furthermore, it identifies various pedagogical strategies that can facilitate the integration of ESD into mathematics teacher education programmes and discusses their significance. The chapter argues that integrating ESD into mathematics teacher education can enhance mathematics teacher educators' ability to foster critical thinking and problem-solving skills among pre-service mathematics teachers. It also reveals that while there are significant opportunities for enriching the mathematics curriculum through ESD, challenges such as limited resources, lack of training, and resistance to change within educational institutions persist. The chapter emphasises the need for collaborative efforts among educators, policymakers, and institutions to overcome these obstacles. Additionally, it contributes to the existing body of knowledge by providing valuable insights for mathematics teacher educators and stakeholders interested in promoting sustainable practices within mathematics curricula. Thus, by addressing both the challenges and opportunities associated with ESD integration, this chapter serves as a foundational resource for future research and practice in the field.

Keywords: Challenges and opportunities, education for sustainable development, integration, mathematics teachers, teacher preparation.

1. Introduction

Education for Sustainable Development (ESD) has become an essential paradigm for incorporating sustainability into educational practices across several disciplines. The United Nations Educational, Scientific and Cultural Organization (UNESCO, 2018) refers to ESD as a comprehensive educational approach that enables individuals to make informed decisions and undertake responsible activities for environmental sustainability, economic viability, and social equity. ESD is a developing framework aimed at providing individuals with the knowledge, skills, attitudes, and values essential for fostering a sustainable future. It underscores the interconnection of environmental, social, and economic facets of sustainability and seeks to cultivate critical thinking, problem-solving, and proactive strategies to address current global issues (Sharma, 2023; Assefa, 2024). The framework prioritises critical thinking, problem-solving, and interactive learning, urging learners to comprehend intricate interrelations within social, economic, and environmental systems (Taylor, 2014; Rieckmann, 2018; Agbedahin, 2019). Thus, as the globe confronts challenges including climate change, biodiversity decline, and social injustice, ESD has gained prominence in educational discussions and practices.

In mathematics education, ESD seeks to contextualise mathematical concepts within real-world sustainability issues, rendering mathematics relevant and practical to students' lives and future professions. However, research on the impact of ESD in mathematics teacher preparation is still in its nascent stages, highlighting the need for empirical studies that examine best practices and effective models. In mathematics education, ESD aims to prepare pre-service mathematics teachers with the knowledge, skills, and attitudes essential for promoting a sustainable future through their pedagogical practices.Therefore, the integration of ESD into mathematics education stems from the understanding that mathematics serves not just as a collection of abstract ideas but also as a means to tackle real-world challenges, such as environmental sustainability, social equity, and economic viability (UNESCO, 2014). For instance, studies (Renert, 2011; Li & Tsai, 2022) contend that mathematics educators should be prepared to cultivate critical thinking and problem-solving abilities that enable students to address sustainability issues. However, this necessitates a change in teacher development programmes to incorporate ESD concepts and practices. Furthermore, the integration of ESD in mathematics teacher education requires a re-evaluation of assessment techniques. Traditional assessment methods in teacher preparation programmes often prioritise memorisation and procedural skills, which may inadequately represent students' understanding of sustainability concepts or their ability to apply mathematical reasoning in real-world situations (Olawale, 2022; Olawale & Hendricks, 2023). Thus, this chapter explores the concepts of ESD, its key principles, how it could be integrated into mathematics teacher education programmes, as well as its challenges and opportunities.

2. The Concept of Education for Sustainable Development

ESD has become a crucial concept in discussions about sustainable practices and policies, especially regarding global environmental issues. ESD is characterised as a comprehensive educational approach that incorporates sustainability principles into teaching and learning, with the objective of enabling individuals to make informed choices and engage in responsible actions for environmental integrity, economic viability, and social equity (UNESCO, 2014; Agbedahin, 2019). As such, literature on ESD encompasses various facets, including theoretical frameworks such as the model of competence and systems theory (De Haan, 2010; Kopnina & Meijers, 2014; Handtke, Richter-Beuschel, & Bögeholz, 2022), instructional practices such as experiential learning and inquiry-based learning (Kostoulas-Makrakis, 2010; Sinakou, Donche, & Van Petegem, 2022; Sinakou, Donche, & Van Petegem, 2024), and empirical studies that highlight its significance and implications for educational systems worldwide (Mochizuki & Fadeeva, 2010; Ssossé, Wagner, & Hopper, 2021; Mutongoza, Olawale, & Mncube, 2023). Similarly, the transformative learning theory, postulated by Mezirow (1995; 2018), is a foundational paradigm underpinning Education for Sustainable Development (ESD). This concept posits that learning necessitates critical reflection, enabling individuals to scrutinise their assumptions and ideas. In the context of ESD, transformative learning encourages learners to tackle complex sustainability issues, improving critical thinking and fostering a sense of agency (Rieckmann, 2018).

Extensive research has also demonstrated the effectiveness of transformative learning in ESD environments, suggesting that it enhances learners' capacity to address sustainability challenges (Sterling, 2011; Leal Filho et al., 2018; Alam, 2022). Thus, given that pedagogical approaches to ESD are diverse and often customised to specific settings, strategies such as inquiry-based learning, experiential learning, problem-based learning, and project-based learning are frequently acknowledged as effective methodologies that promote active engagement and problem-solving skills in students (Murphy, Smith, Mallon, & Redman, 2020; Daramola et al., 2024; Funa, Roleda, & Prudente, 2024; Cebrián, Palau, & Mogas, 2020; Siphukhanyo & Olawale, 2024). These techniques align with the principles of constructivism, which emphasise the importance of learners constructing their understanding through interaction with their environment. Therefore, Wals and Corcoran (2012) and Daramola et al. (2024) emphasise the significance of experiential learning in enhancing comprehension of sustainability challenges by enabling learners to link theoretical knowledge with practical implementations. Furthermore, the role of educators in advancing Education for Sustainable Development (ESD) is a vital area of investigation within the ESD conceptualisation, as educators are tasked with imparting knowledge and motivating students to adopt sustainable behaviours. For instance, Nikolic, Milutinovic, Nedanovski, and Mrnjaus (2017) assert that professional development programmes for educators can enhance their understanding of ESD and improve their teaching methodologies.

Integrating ESD into teacher training programmes is essential for equipping future educators with the necessary skills and knowledge to effectively impart sustainability principles (Nikolic, Milutinovic, Nedanovski, & Mrnjaus, 2017; Leicht, Combes, Byun, & Agbedahin, 2018). Empirical studies have shown a positive impact of ESD on student achievement, enhancing students' environmental awareness, critical thinking skills, and sense of responsibility toward sustainability (Nousheen, Zai, Waseem, & Khan, 2020; Ernst & Monroe, 2004; O'Flaherty& Liddy, 2018). Furthermore, ESD has been linked to increased motivation and engagement among students, as it often involves practical, real-world projects that resonate with their interests and values (Andersson, Jagers, Lindskog, & Martinsson, 2013; Stössel, Baumann, & Wegner, 2021; Muñoz-García & Villena-Martínez, 2021). However, despite the growing recognition of ESD, challenges remain in its implementation across many educational contexts.

3. Key Principles of Education for Sustainable Development

Education for Sustainable Development (ESD) has become a crucial foundation for addressing the intricate relationships among the environmental, social, and economic aspects of sustainability. The fundamental concepts of ESD focus on cultivating critical thinking, encouraging active engagement, and developing a comprehensive understanding of sustainability challenges (UNESCO, 2014). Table 1 below presents the fundamental concepts of ESD, integrating insights from diverse academic contributions and policy documents.

Table 1: Key Principles of Education for Sustainable Development

Key Principles	Aims
Interdisciplinary Approach	ESD promotes an interdisciplinary perspective that integrates knowledge from various fields, including science, social studies, economics, and ethics (Howlett, Ferreira, & Blomfield, 2016; Annan-Diab & Molinari, 2017; Risopoulos-Pichler, Daghofer, & Steiner, 2020). This approach helps learners grasp the complexities of sustainability issues and the multifaceted solutions needed to address them.
Critical Thinking and Problem-Solving	ESD encourages learners to think critically about the challenges facing society and to develop problem-solving skills (Tilbury, 2011; Leicht et al., 2018). It fosters the ability to analyse situations, evaluate options, and make informed decisions that consider the long-term implications of actions.
Active Participation	ESD - 91 -aption- 91 -in the importance of active participation in learning processes. Students are encouraged to engage in discussions, collaborate with peers, and take part in community initiatives (Wals & Corcoran, 2012; Huckle & Wals, 2015; Olsson, Gericke, & Chang Rundgren, 2016). This participatory approach not only enhances learning outcomes but also empowers individuals to become agents of change within their communities.
Values and Ethics	ESD promotes the development of values and ethics that support sustainable living (Cebrián & Junyent, 2015; Olsson, Gericke, & Chang Rundgren, 2016). It encourages learners to reflect on their beliefs and attitudes towards the environment, society, and future generations, thereby fostering a sense of stewardship and responsibility.
Sustainable consumption	ESD promotes responsible and mindful consumption patterns that contribute to environmental sustainability, social equity, and economic viability (Jackson, 2005; Thøgersen & Zhou, 2012; Rieckmann, 2018). It fosters awareness and knowledge, encourages behavioural change, and - 91 -aption- 91 -in systems thinking, cultural sensitivity, and future-oriented thinking.
Local and Global Perspectives/Holistic Understanding	ESD - 91 -aption- 91 -in the significance of both local and global contexts in understanding sustainability (UNESCO, 2014; Rieckmann, 2018). It encourages learners to appreciate the diversity of cultures, traditions, and practices, while also - 91 -aption- 91 -ing the interconnected nature of global challenges.

The interdisciplinary approach is essential to ESD, as it acknowledges that sustainability concerns are complex and cannot be sufficiently addressed through a single disciplinary perspective. For instance, studies such as Wiek, Xiong, Brundiers, and Van der Leeuw (2014); Howlett, Ferreira, and Blomfield (2016); Annan-Diab and Molinari (2017); and Risopoulos-Pichler, Daghofer, and Steiner (2020) assert that multidisciplinary education enables learners to synthesise knowledge from diverse disciplines, enhancing the critical thinking and problem-solving abilities vital for addressing sustainability issues. Thus, Sterling (2011) emphasises that ESD must surpass conventional topic boundaries to foster a holistic understanding of ecological, social, and economic interdependencies. Furthermore, an interdisciplinary approach fosters collaboration among various stakeholders, including educators, students, and community members. Leal Filho et al. (2018) highlight that collaborative learning environments promote the exchange of ideas and experiences, thereby augmenting the collective potential to tackle sustainability challenges. By amalgamating disciplines such as environmental science, economics, and social studies, ESD can foster a more sophisticated comprehension of sustainability essential for effective intervention.

Another fundamental principle of ESD is the enhancement of critical thinking and problem-solving abilities. Tilbury (2011) asserts that ESD equips learners to critically analyse and assess information, facilitating informed decision-making regarding sustainability issues. The ability for critical reflection is vital in a swiftly evolving environment where misinformation can readily spread. Leicht et al. (2018) assert that promoting critical thinking improves individual decision-making and aids in collective efforts toward achieving sustainable development goals (SDGs).

Active engagement is another fundamental principle of ESD. According to Wals and Corcoran (2012), ESD promotes the active engagement of learners in their communities and environments. This interactive method is essential for fostering a sense of agency in learners, allowing them to engage in sustainable practices and policies. Similarly, values and ethics are crucial in influencing the attitudes and behaviours required for sustainable existence. Thus, integrating ethical considerations into ESD is vital for cultivating a sense of responsibility toward the environment and society. According to Huckle and Wals (2015), ESD must involve learners in ethical contemplation over the consequences of their decisions and actions, fostering a values-oriented perspective on sustainability. UNESCO (2014) emphasises the necessity of incorporating values such as respect for nature, social justice, and intergenerational parity into educational systems. These principles not only influence individual conduct but also establish collective cultural standards that support sustainable behaviours. Moreover, incorporating ethical viewpoints in ESD may enable learners to critically evaluate the consequences of their consumption behaviours and promote more sustainable options (Cebrián & Junyent, 2015; Olsson, Gericke, & Chang Rundgren, 2016).

Sustainable consumption is also an essential aspect of ESD, as it pertains to the influence of individual and group decisions on environmental and social systems. The notion encompasses various practices, such as resource conservation and ethical shopping. For instance, Jackson (2005) asserts that fostering sustainable consumption necessitates an understanding of the fundamental social and economic frameworks that drive unsustainable practices. Hence, educational initiatives centred on sustainable consumption seek to provide learners with the knowledge and skills needed to make informed choices about their consumption behaviours

(Jackson, 2005; Rieckmann, 2018). Thøgersen and Zhou (2012) demonstrate that educational initiatives highlighting the environmental and social ramifications of consumption can markedly affect consumer behaviour. Thus, by cultivating a culture of sustainability, ESD may motivate individuals to embrace actions that reduce their ecological footprint and support overarching sustainability objectives. Lastly, a holistic understanding is also critical in ESD, as it recognises the interconnectedness of environmental, social, and economic systems. According to UNESCO's (2014) Global Action Programme on ESD, a holistic approach to education equips learners with the ability to understand the complex relationships between human activities and ecological health. This perspective aligns with the findings of Rieckmann (2018), who argues that interdisciplinary education is essential for addressing the multifaceted nature of sustainability challenges. Thus, by integrating diverse disciplines, educators can provide learners with a comprehensive understanding of sustainability issues, fostering a more profound commitment to sustainable practices. Therefore, ESD should not simply be an educational programme but a transformative strategy that enables individuals and communities to significantly contribute to sustainable development.

4. Integration of ESD into the Preparation of Pre-service Teachers

The integration of ESD into the preparation of pre-service mathematics teachers is a vital endeavour that has the potential to transform mathematics education. Equipping future educators with the means to address sustainability concerns through mathematical inquiry will cultivate a generation of learners who are both proficient in mathematics and committed to fostering a more sustainable world (Li & Tsai, 2022). However, the successful implementation of ESD in pre-service mathematics teacher education programmes requires a shift in educational practices and policies. This entails integrating sustainability principles into curricula, teaching methods, and school governance. Table 2 below presents several strategies that can facilitate the effective incorporation of ESD into mathematics teacher education programmes.

Table 2: approaches to incorporating ESD in Mathematics Teacher Education Programmes

Strategies	Suggestion for implementation/Integration
Curriculum Development	Educational institutions should revise curricula to include ESD-related content across subjects (Davis & Krajcik, 2005; Bögeholz, Böhm, Eggert, & Barkmann, 2014; Edwards, 2019). This can be achieved by integrating themes such as climate change, social justice, and sustainable economics into existing courses or by developing new interdisciplinary programmes focused on sustainability.
Teacher Training	Educators play a crucial role in facilitating ESD. Professional development programmes should equip teachers with the knowledge and skills needed to teach sustainability concepts effectively (Visser, 2010; Tomas, Girgenti, & Jackson, 2017). Training should emphasise experiential learning, critical pedagogy, and the use of innovative teaching methods.
Community Engagement	Schools should actively engage with local communities to foster partnerships that enhance ESD (Gutstein & Peterson, 2005; Furman, 2012; Valenzuela, 2016; Bennett, 2021). Collaborating with community organisations, businesses, and local governments can provide students with real-world experiences and opportunities for service learning.

Assessment and Evaluation	Measuring the impact of ESD initiatives is essential for continuous improvement (Gulikers, Bastiaens, Kirschner, & Kester, 2008; Gulikers, Biemans, Wesselink, & van der Wel, 2013). Educational institutions should develop assessment frameworks that evaluate not only academic knowledge but also the development of skills, attitudes, and behaviours related to sustainability (Gulikers et al., 2008; 2013).
Policy Support	Governments and educational authorities must create supportive policies that promote ESD at all levels of education (Sterling, 2011; Leal Filho et al., 2018; Kanandjebo, 2024). This includes allocating resources for ESD initiatives, providing incentives for schools to implement sustainable practices, and fostering collaboration among stakeholders.

The urgency of integrating Education for Sustainable Development (ESD) into mathematics teacher training programmes is emphasised by the United Nations' Sustainable Development Goals (SDGs), especially Goal 4, which stresses the importance of providing inclusive and equitable quality education and fostering lifelong learning opportunities for all (United Nations, 2015; United Nations Educational, Scientific and Cultural Organization (UNESCO), 2017). The SDGs offer a comprehensive framework for tackling the complex issues of sustainability, with education acknowledged as an essential facilitator of this initiative. Thus, mathematics teacher education programmes possess a distinct potential to advance the attainment of the SDGs by incorporating sustainability approaches into their training. These strategies collectively facilitate the effective integration of ESD into mathematics teacher education programmes, thereby fostering a generation of educators equipped to address sustainability issues in their teaching.

Curriculum development is a fundamental approach for integrating Education for Sustainable Development (ESD) into mathematics teacher training. For instance, Tilbury (2011) asserts that a well-organised curriculum integrating sustainability principles can augment the applicability of mathematical concepts to real-world issues. In addition, UNESCO (2017) reinforces this viewpoint, highlighting the necessity for a curriculum that encompasses mathematical theories while situating them within sustainable frameworks. Therefore, effective curriculum development must employ multidisciplinary approaches (Bögeholz, 2006; Bögeholz, Böhm, Eggert, & Barkmann, 2014; Edwards, 2019), connecting mathematics with environmental science, social equality, and economic viability to promote a comprehensive understanding of sustainability. Furthermore, the curriculum must be flexible and attuned to local circumstances, as emphasised by Davis and Krajcik (2005) and Barraza et al. (2017), who contend that localised curriculum content can improve student engagement and relevance. Thus, by incorporating project-based learning and problem-solving activities centred on environmental challenges, mathematics teacher education programmes can enhance critical thinking and analytical skills in future educators through real-life scenarios (Barraza et al., 2017).

It is also vital to reiterate that the effectiveness of ESD integration is significantly dependent on the calibre of teacher training programmes. Studies demonstrate that professional development programmes must provide mathematics educators with essential pedagogical competencies to teach sustainability principles effectively (Cohen & Hill, 2000; Malm, 2009; Cebrián & Junyent, 2015). This encompasses instruction in integrative pedagogical techniques, cooperative learning, and the application of technology to enhance Education for Sustainable Development (ESD) (Daramola et al., 2024). As such, studies such as Fraser, Kennedy, Reid, and McKinney (2007) and McKeown and Hopkins (2007) assert that continuous professional development is essential for educators to stay abreast of sustainability matters and pedagogical advancements. Moreover, teacher training programmes ought to prioritise reflective practice, allowing educators to critically evaluate their pedagogical approaches and the effects of their instruction on student learning in relation to sustainability (Visser, 2010; Tomas, Girgenti, & Jackson, 2017). This reflective methodology prompts educators to modify their practices to incorporate ESD concepts into their mathematics teaching more effectively, promoting a culture of ongoing enhancement and adaptability to new sustainability issues (Darling-Hammond, Wei, Andree, Richardson, & Orphanos, 2009).

Furthermore, community engagement is an essential technique for improving Education for Sustainable Development (ESD) in mathematics teacher education programmes. Interacting with local communities enables teacher candidates to comprehend the socio-cultural aspects of sustainability and the role of mathematics in tackling community-specific issues (Furman, 2012). For instance, Vare and Scott (2007) assert that collaborations between educational institutions and community organisations can enhance experiential learning opportunities, enabling future educators to apply mathematical principles to real-world sustainability challenges. Furthermore, community engagement cultivates a sense of social responsibility among teacher candidates, motivating them to perceive themselves as active contributors to the promotion of sustainable practices within their communities (Valenzuela, 2016; Bennett, 2021). This participation may manifest in several ways, including service-learning initiatives, community-based research, and partnerships with local stakeholders. Therefore, integrating community viewpoints into mathematics education enables teacher candidates to cultivate a more sophisticated comprehension of the challenges associated with sustainability (Gutstein & Peterson, 2005; Olawale, Mncube, & Harber, 2021).

Effective assessment and evaluation systems are also crucial for gauging the impact of ESD integration in mathematics teacher education. According to Volante and Fazio (2007) and Fischer et al. (2022), conventional assessment techniques may insufficiently reflect the comprehensive understanding and application of sustainability principles among teacher candidates. Consequently, alternative assessment methods, like performance-based assessments, portfolios, and reflective diaries, ought to be utilised to examine the incorporation of ESD principles (Gulikers, Bastiaens, Kirschner, & Kester, 2008; Gulikers, Biemans, Wesselink, & van der Wel, 2013; Mncube, Ndondo, & Olawale, 2022). Moreover, formative assessments offer continuous feedback to teacher candidates, allowing them to recognise areas needing enhancement and modify their teaching methodologies accordingly (Mncube, Ndondo, & Olawale, 2022). It is therefore essential to design assessment standards that directly reflect sustainability competencies. This entails assessing candidates' competencies in integrating sustainability concerns into their lesson design, pedagogical approaches, and classroom management (Voss, Kunter, & Baumert, 2011; Mncube, Ndondo, & Olawale, 2022).

Lastly, policy support is essential for the incorporation of ESD into mathematics teacher education programmes. For instance, The Global Action Programme on ESD advocates for legislative frameworks that foster the development of curricula, teacher training, and evaluation methods that advance sustainability (UNESCO, 2014). Similarly, studies such as Vare and Scott (2007), Sterling (2011), and Leal Filho et al. (2018) assert that collaboration between educational institutions and policymakers is crucial for establishing conducive settings for the integration of ESD. Research demonstrates that policies promoting innovative teaching methods and allocating funding for ESD projects can substantially improve the quality of mathematics teacher education (Rico, Agirre-Basurko, Ruiz-González, Palacios-Agundez, & Zuazagoitia, 2021; Kanandjebo, 2024). Furthermore, cultivating a sustainability culture at educational institutions can be accomplished through policy initiatives that encourage sustainable activities, including resource conservation and waste minimisation (Adams, Martin, & Boom, 2018).

5. Challenges and Opportunities of ESD in Maths Teacher Education Programmes

Despite the key principles and approaches that support the integration of ESD into mathematics teacher education, several challenges impede this process. According to Laurett and do Paço (2019), a major obstacle is the existing curricular framework, which frequently emphasises content understanding rather than teaching methods that integrate sustainability. As such, mathematics teacher educators may feel restricted by standardised curricula that lack the flexibility necessary to properly integrate ESD concepts. Tilbury (2011) also adds that barriers such as financial limitations, insufficient teacher training, and rigid curricula hinder the effective integration of ESD principles. Similarly, another difficulty is the absence of professional development opportunities for mathematics educators to acquire the requisite skills and expertise for teaching ESD principles. According to Darling-Hammond (1989), Stevenson (2007), and Bakker, Cai, & Zenger (2021), numerous teacher preparation programmes insufficiently equip prospective educators to tackle sustainability concerns via mathematics, resulting in a deficiency in their pedagogical topic understanding. The gap is further intensified by the restricted availability of tools and materials that directly link mathematics with sustainability issues (Kanandjebo, 2024). In addition, there is sometimes a mismatch between academic knowledge and practical implementation (Kopnina & Meijers, 2014). Thus, many mathematics educators may grasp the benefits of ESD but struggle to implement it in their classrooms owing to a lack of confidence or expertise. This mismatch underlines the need for more robust support mechanisms within teacher preparation programmes that enable experimentation and reflection on ESD practices.

Despite the challenges, ample opportunity exists for the incorporation of ESD into mathematics teacher education programmes. A viable strategy involves the integration of interdisciplinary projects that link mathematics with environmental and social concerns. Studies indicate that project-based learning can improve student engagement and comprehension of mathematical concepts and sustainability (Fini, Awadallah, Parast, & Abu-Lebdeh, 2018; Nguyen, Nguyen, Thai, Truong, & Nguyen, 2024). Mathematics educators can develop collaborative projects that tackle real-world issues, fostering significant learning experiences that connect with students' lives. Furthermore, the incorporation of technology in mathematics instruction offers a chance to improve ESD practices (Oladele et al., 2023).The use of digital tools can enhance data analysis and modelling, allowing students to statistically investigate intricate sustainability challenges. For instance, Barwell (2013) argued that utilising statistical tools to examine environmental data enables students to comprehend the mathematical foundations of sustainability metrics, thus enhancing their awareness of mathematics' role in tackling global concerns. In addition, professional development programmes centred on ESD can enable mathematics educators to integrate sustainability into their instructional methodologies (Darling-Hammond, 1989;Stevenson, 2007; Bakker, Cai, & Zenger, 2021). Thus, through workshops and training sessions that focus on practical techniques for incorporating ESD concepts into mathematics curricula, teachers' confidence and competence can be improved (Boeve-de Pauw, Olsson, Berglund, & Gericke, 2022). Lastly, establishing communities of practice among mathematics educators can enhance knowledge exchange and collaboration, fostering a culture of sustainability in mathematics education.

6. Conclusion and Recommendation

This chapter explores the concepts of ESD, its key principles, the integration of ESD in mathematics teacher education programmes, and its challenges and opportunities. It elucidates the essential principles of ESD, highlighting its multidisciplinary nature, the development of critical thinking and problem-solving abilities, and the encouragement of active engagement, values, and ethics. By integrating these themes, it is clear that ESD enhances the mathematics curriculum while aligning it with urgent global environmental concerns. Consequently, the prospective advantages of integrating ESD into mathematics teacher education are numerous. Educators knowledgeable in ESD principles can cultivate a cohort of pre-service teachers who are both skilled in mathematical concepts and proficient in applying these concepts to real-world challenges. This comprehensive understanding is essential for equipping future educators to motivate their pupils towards sustainable habits.

Nevertheless, the chapter underscores considerable challenges, such as resource constraints, inadequate training, and institutional resistance to change, which hinder the effective integration of ESD. To navigate these challenges, collaborative efforts among educators, policymakers, and educational institutions are imperative. By fostering a shared commitment to sustainability in education, stakeholders can create an environment conducive to the successful implementation of ESD principles.

This chapter contributes to the existing body of knowledge by providing a comprehensive overview of ESD integration strategies, serving as a foundational resource for future research and practice. It therefore recommends that mathematics teacher education programmes formulate instructional practices that cultivate critical thinking and problem-solving abilities, empowering future educators to assist students in quantitatively examining and tackling sustainability issues. Similarly, engagement with interdisciplinary fields can augment the relevance of mathematical concepts in sustainability situations, thus enhancing the learning experience. The chapter also recommends integrating practical learning approaches to offer hands-on experiences in which mathematics is utilised for sustainability initiatives, fostering engagement and enhancing comprehension. Furthermore, ongoing professional development for mathematics teacher educators must incorporate ESD training to ensure they are adequately prepared to teach these interconnected ideas effectively. Moreover, empirical research is needed to evaluate the effectiveness of existing ESD-integrated mathematics curricula. Lastly, longitudinal studies that track the impact of ESD-focused teacher training on student outcomes in mathematics and sustainability literacy should be conducted to provide valuable insights into best practices and areas for improvement.

7. Declarations

Funding: This research received no funding.

Conflicts of Interest: The authors declare no conflicts of interest.

References

Adams, R., Martin, S., & Boom, K. (2018). University culture and sustainability: Designing and implementing an enabling framework. *Journal of Cleaner Production, 171*(1), 434–445. https://doi.org/10.1016/j.jclepro.2017.10.032

Agbedahin, A. V. (2019). Sustainable development, education for sustainable development, and the 2030 agenda for sustainable development: Emergence, efficacy, eminence, and future. *Sustainable Development, 27*(4), 669–680. https://doi.org/10.1002/sd.1931

Alam, A. (2022). Mapping a sustainable future through conceptualization of transformative learning framework, education for sustainable development, critical reflection, and responsible citizenship: An exploration of pedagogies for twenty-first century learning. *ECS Transactions, 107*(1), 1–15. https://doi.org/10.1149/10701.9827ecst

Andersson, K., Jagers, S. C., Lindskog, A., & Martinsson, J. (2013). Learning for the future? Effects of education for sustainable development (ESD) on teacher education students. *Sustainability, 5*(12), 5135–5152. https://doi.org/10.3390/su5125135

Annan-Diab, F., & Molinari, C. (2017). Interdisciplinarity: Practical approach to advancing education for sustainability and for the Sustainable Development Goals. *The International Journal of Management Education, 15*(2), 73–83. https://doi.org/10.1016/j.ijme.2017.03.006

Assefa, E. A. (2024). From classrooms to global impact: Leveraging quality education to shape a sustainable, interconnected world. *The Journal of Quality in Education, 14*(24), 1–24. https://doi.org/10.37870/joqie.v14i24.441
Bakker, A., Cai, J., & Zenger, L. (2021). Future themes of mathematics education research: An international survey before and during the pandemic. *Educational Studies in Mathematics, 107*(1), 1–24. https://doi.org/10.1007/s10649-021-10049-w
Barwell, R. (2013). The mathematical formatting of climate change: Critical mathematics education and post-normal science. *Research in Mathematics Education, 15*(1), 1–16. https://doi.org/10.1080/14794802.2012.756633
Bennett, J. (2021). Collective criticality: Towards sustainable socially just teacher education programmes. *Whiteness and Education, 6*(1), 75–91. https://doi.org/10.1080/23793406.2020.1803122
Boeve-de Pauw, J., Olsson, D., Berglund, T., & Gericke, N. (2022). Teachers' ESD self-efficacy and practices: A longitudinal study on the impact of teacher professional development. *Environmental Education Research, 28*(6), 867–885. https://doi.org/10.1080/13504622.2022.2042206
Bögeholz, S. (2006). Nature experience and its importance for environmental knowledge, values and action: Recent German empirical contributions. *Environmental Education Research, 12*(1), 65–84. https://doi.org/10.1080/13504620500526529
Bögeholz, S., Böhm, M., Eggert, S., & Barkmann, J. (2014). Education for sustainable development in German science education: Past–present–future. *Eurasia Journal of Mathematics, Science and Technology Education, 10*(4), 231–248. https://doi.org/10.12973/eurasia.2014.1079a
Cebrián, G., & Junyent, M. (2015). Competencies in education for sustainable development: Exploring the student teachers' views. *Sustainability, 7*(3), 2768–2786. https://doi.org/10.3390/su7032768
Cebrián, G., Palau, R., & Mogas, J. (2020). The smart classroom as a means to the development of ESD methodologies. *Sustainability, 12*(7), 1–18. https://doi.org/10.3390/su12073010
Cohen, D. K., & Hill, H. C. (2000). Instructional policy and classroom performance: The mathematics reform in California. *Teachers College Record, 102*(2), 294–343. https://doi.org/10.1111/0161-4681.00057
Daramola, D. S., Oladele, J. I., & Ndlovu, M. (2024). Impact of technology-enhanced experiential learning on postgraduate students' training in qualitative research. *African Perspectives of Research in Teaching and Learning, 8*(2), 22-38. https://doi.org/10.70875/v8i2article3
Darling-Hammond, L. (1989). *Redesigning teacher education: Opening the door for new recruits to science and mathematics teaching*. The RAND Corporation.
Darling-Hammond, L., Wei, R. C., Andree, A., Richardson, N., & Orphanos, S. (2009). *Professional learning in the learning profession*. National Staff Development Council.
Davis, E. A., & Krajcik, J. S. (2005). Designing educative curriculum materials to promote teacher learning. *Educational Researcher, 34*(3), 3-14. https://doi.org/10.3102/0013189X034003003
De Haan, G. (2010). The development of ESD-related competencies in supportive institutional frameworks. *International Review of Education, 56*(1), 315-328. https://doi.org/10.1007/s11159-010-9157-9
Edwards, G. I. (2019). Multidisciplinary approach to environmental problems and sustainability. In *Encyclopedia of sustainability in higher education* (pp. 1184-1189). https://doi.org/10.1007/978-3-030-11352-0_241

Ernst, J., & Monroe, M. (2004). The effects of environment-based education on students' critical thinking skills and disposition toward critical thinking. *Environmental Education Research, 10*(4), 507-522. https://doi.org/10.1080/1350462042000291038

Fini, E. H., Awadallah, F., Parast, M. M., & Abu-Lebdeh, T. (2018). The impact of project-based learning on improving student learning outcomes of sustainability concepts in transportation engineering courses. *European Journal of Engineering Education, 43*(3), 473-488. https://doi.org/10.1080/03043797.2017.1393045

Fischer, D., King, J., Rieckmann, M., Barth, M., Büssing, A., Hemmer, I., & Lindau-Bank, D. (2022). Teacher education for sustainable development: A review of an emerging research field. *Journal of Teacher Education, 73*(5), 509-524. https://doi.org/10.1177/00224871221105784

Fraser, C., Kennedy, A., Reid, L., & McKinney, S. (2007). Teachers' continuing professional development: Contested concepts, understandings and models. *Journal of In-Service Education, 33*(2), 153-169. https://doi.org/10.1080/13674580701292913

Funa, A. A., Roleda, L. S., & Prudente, M. S. (2024). *Integrated science, technology, engineering, and mathematics-problem-based learning-education for sustainable development (I-STEM-PBL-ESD) framework*. In *A diversity of pathways through science education* (pp. 151-172). Springer Nature. https://doi.org/10.1007/978-981-97-2607-3_9

Furman, G. (2012). Social justice leadership as praxis: Developing capacities through preparation programs. *Educational Administration Quarterly, 48*(2), 191-229. https://doi.org/10.1177/0013161X11427394

Gulikers, J. T., Bastiaens, T. J., Kirschner, P. A., & Kester, L. (2008). Authenticity is in the eye of the beholder: Student and teacher perceptions of assessment authenticity. *Journal of Vocational Education and Training, 60*(4), 401-412. https://doi.org/10.1080/13636820802591830

Gulikers, J. T., Biemans, H. J., Wesselink, R., & van der Wel, M. (2013). Aligning formative and summative assessments: A collaborative action research challenging teacher conceptions. *Studies in Educational Evaluation, 39*(2), 116-124. https://doi.org/10.1016/j.stueduc.2013.03.001

Gutstein, E., & Peterson, B. (2005). *Rethinking mathematics: Teaching social justice by the numbers.* Winsonson: Rethinking Schools.

Handtke, K., Richter-Beuschel, L., & Bögeholz, S. (2022). Self-efficacy beliefs of teaching ESD: A theory-driven instrument and the effectiveness of ESD in German teacher education. *Sustainability, 14*(11), 1-14. https://doi.org/10.3390/su14116477

Howlett, C., Ferreira, J. A., & Blomfield, J. (2016). Teaching sustainable development in higher education: Building critical, reflective thinkers through an interdisciplinary approach. *International Journal of Sustainability in Higher Education, 17*(3), 305-321. https://doi.org/10.1108/IJSHE-07-2014-0102

Huckle, J., & Wals, A. E. (2015). The UN Decade of Education for Sustainable Development: Business as usual in the end. *Environmental Education Research, 21*(3), 491-505. https://doi.org/10.1080/13504622.2015.1011084

Jackson, T. (2005). Live better by consuming less?: Is there a "double dividend" in sustainable consumption? *Journal of Industrial Ecology, 9*(1-2), 19-36. https://doi.org/10.1162/1088198054084734

Kanandjebo, L. N. (2024). Mathematics as a driver for sustainable development: Pre-service teachers' perspectives on awareness, challenges, and implications for teacher training. *Interdisciplinary Journal of Education Research, 6*(1), 1-17. https://doi.org/10.38140/ijer-2024.vol6.26

Kopnina, H., & Meijers, F. (2014). Education for sustainable development (ESD): Exploring theoretical and practical challenges. *International Journal of Sustainability in Higher Education, 15*(2), 188-207. https://doi.org/10.1108/IJSHE-07-2012-0059

Kostoulas-Makrakis, N. (2010). Developing and applying a critical and transformative model to address ESD in teacher education. *Journal of Teacher Education for Sustainability, 12*(2), 17-26. https://doi.org/10.2478/v10099-009-0051-0

Laurett, R., & do Paço, A. (2019). Sustainability barriers. *Encyclopedia of Sustainability in Higher Education*, 1608-1614. https://doi.org/10.1007/978-3-030-11352-0_188

Leal Filho, W., Raath, S., Lazzarini, B., Vargas, V. R., de Souza, L., Anholon, R., & Orlovic, V. L. (2018). The role of transformation in learning and education for sustainability. *Journal of Cleaner Production, 199*, 286-295. https://doi.org/10.1016/j.jclepro.2018.07.017

Leicht, A., Combes, B., Byun, W. J., & Agbedahin, A. V. (2018). From Agenda 21 to Target 4.7: The development of education for sustainable development. *Issues and Trends in Education for Sustainable Development*, 1-15.

Li, H. C., & Tsai, T. L. (2022). Education for sustainable development in mathematics education: What could it look like? *International Journal of Mathematical Education in Science and Technology, 53*(9), 2532-2542. https://doi.org/10.1080/0020739X.2021.1941361

Malm, B. (2009). Towards a new professionalism: Enhancing personal and professional development in teacher education. *Journal of Education for Teaching, 35*(1), 77-91. https://doi.org/10.1080/02607470802587160

McKeown, R., & Hopkins, C. (2007). International network of teacher education institutions: Past, present and future. *Journal of Education for Teaching, 33*(2), 149–155. https://doi.org/10.1080/02607470701259408

Mezirow, J. (1995). *Transformative theory of adult* learning. In M. Welton (Ed.), *In defense of the lifeworld (pp. 1–12). Albany: State University of New York Press.*

Mezirow, J. (2018). Transformative learning theory. In Contemporary theories of learning. Oxfordshire, United Kingdom: Routledge. *https://doi.org/10.4324/9781315147277-8*

Mncube, V., Ndondo, S., & Olawale, B. E. (2022). Chronicling lecturers' and students' experiences in using digital technologies for continuous assessment practice in some South African universities. *In Assessment through digital platforms within higher education studies (pp. 286–304).* Durban, UKZN: CSSALL Publishers.

Mochizuki, Y., & Fadeeva, Z. (2010). Competences for sustainable development and sustainability: Significance and challenges for ESD. *International Journal of Sustainability in Higher Education, 11*(4), 391–403. https://doi.org/10.1108/14676371011077603

Muñoz-García, A., & Villena-Martínez, M. D. (2021). Influences of learning approaches, student engagement, and satisfaction with learning on measures of sustainable behaviour in a social sciences student sample. *Sustainability, 13*(2), 1–14. https://doi.org/10.3390/su13020541

Murphy, C., Smith, G., Mallon, B., & Redman, E. (2020). Teaching about sustainability through inquiry-based science in Irish primary classrooms: The impact of a professional development programme on teacher self-efficacy, competence, and pedagogy. *Environmental Education Research, 26*(8), 1112–1136. https://doi.org/10.1080/13504622.2020.1776843

Mutongoza, B. H., Olawale, B. E., & Mncube, V. (2023). Chronicling lecturers' and stakeholders' experiences of the role of a rural university's sustainable community development project. *Journal of Educational Studies, 22*(3), 130–145. https://doi.org/10.59915/jes.2023.22.3.8
Nguyen, H. T., Nguyen, G. T., Thai, L. T., Truong, D. T., & Nguyen, B. N. (2024). Teaching mathematics through project-based learning in K-12 schools: A systematic review of current practices, barriers, and future developments. *TEM Journal, 13*(3), 1–16. https://doi.org/10.18421/TEM133-33
Nikolic, V., Milutinovic, S., Nedanovski, P., & Mrnjaus, K. (2017). ESD professional development of university educators in Serbia, Croatia, and Macedonia: A comparative analysis. *International Journal of Sustainability in Higher Education, 18*(6), 923–938. https://doi.org/10.1108/IJSHE-02-2016-0036
Nousheen, A., Zai, S. A., Waseem, M., & Khan, S. A. (2020). Education for sustainable development (ESD): Effects of sustainability education on pre-service teachers' attitude towards sustainable development (SD). *Journal of Cleaner Production, 250*, 1–12. https://doi.org/10.1016/j.jclepro.2019.119537
O'Flaherty, J., & Liddy, M. (2018). The impact of development education and education for sustainable development interventions: A synthesis of the research. *Environmental Education Research, 24*(7), 1031–1049. https://doi.org/10.1080/13504622.2017.1392484
Oladele, J. I., Ayanwale, M. A., & Ndlovu, M. (2023). Technology adoption for STEM education in higher education: Students' experience from selected sub-Saharan African countries. *Pertanika Journal of Science & Technology, 31*(1). https://doi.org/10.47836/pjst.31.1.15
Olawale, B. E. (2022). Democratic citizenship in mathematics teacher preparation in South African universities: Contradictions in theory and practice. *International Journal of Pedagogy and Teacher Education, 6*(2), 126-139. https://doi.org/10.20961/ijpte.v6i2.61574
Olawale, B. E., & Hendricks, W. (2023). The role of rural higher education institutions in leading societal transformation through sustainable initiatives. In *Global Education Network (GEN) Conference* (pp. 49-60). Liverpool, UK: Global Education Network (GEN).
Olawale, B. E., Mncube, V. S., & Harber, C. (2021). Critical social pedagogy in mathematics teacher education. *International Journal of Higher Education, 10*(6), 93-104. https://doi.org/10.5430/ijhe.v10n6p93
Olsson, D., Gericke, N., & Chang Rundgren, S. N. (2016). The effect of implementation of education for sustainable development in Swedish compulsory schools—assessing pupils' sustainability consciousness. *Environmental Education Research, 22(*2), 176-202. https://doi.org/10.1080/13504622.2015.1005057
Renert, M. (2011). Mathematics for life: Sustainable mathematics education. *For the Learning of Mathematics, 31*(1), 20-26.
Rico, A., Agirre-Basurko, E., Ruiz-González, A., Palacios-Agundez, I., & Zuazagoitia, D. (2021). Integrating mathematics and science teaching in the context of education for sustainable development: Design and pilot implementation of a teaching-learning sequence about air quality with pre-service primary teachers. *Sustainability, 13*(8), 1-17. https://doi.org/10.3390/su13084500
Rieckmann, M. (2018). Learning to transform the world: Key competencies in education for sustainable development. *Issues and Trends in Education for Sustainable Development, 39*(1), 39-59.

Risopoulos-Pichler, F., Daghofer, F., & Steiner, G. (2020). Competences for solving complex problems: A cross-sectional survey on higher education for sustainability learning and transdisciplinarity. *Sustainability, 12*(15), 1-16. https://doi.org/10.3390/su12156016

Sharma, S. (2023). Towards sustainable education: Integrating environmental and social responsibility into the curriculum. *International Scientific Journal for Research, 5*(5), 1-10.

Sinakou, E., Donche, V., & Van Petegem, P. (2022). Action-orientation in education for sustainable development: Teachers' interests and instructional practices. *Journal of Cleaner Production, 370*, 1-16. https://doi.org/10.1016/j.jclepro.2022.133469

Sinakou, E., Donche, V., & Van Petegem, P. (2024). Teachers' profiles in education for sustainable development: Interests, instructional beliefs, and instructional practices. *Environmental Education Research, 30*(3), 397-418. https://doi.org/10.1080/13504622.2023.2168622

Siphukhanyo, L., & Olawale, B. E. (2024). Chronicling the experiences of life sciences teachers and learners on the usage of enquiry-based learning in enhancing learners' academic performance. *Journal of Culture and Values in Education, 7*(1), 19-36. https://doi.org/10.46303/jcve.2024.2

Ssossé, Q., Wagner, J., & Hopper, C. (2021). Assessing the impact of ESD: Methods, challenges, results. *Sustainability, 13*(5), 1-26. https://doi.org/10.3390/su13052854

Sterling, S. (2011). Transformative learning and sustainability: Sketching the conceptual ground. *Learning and Teaching in Higher Education, 5*(11), 17-33.

Stevenson, R. B. (2007). Schooling and environmental/sustainability education: From discourses of policy and practice to discourses of professional learning. *Environmental Education Research, 13*(2), 265-285. https://doi.org/10.1080/13504620701295650

Stössel, J., Baumann, R., & Wegner, E. (2021). Predictors of student teachers' ESD implementation intention and their implications for improving teacher education. *Sustainability, 13*(16), 1-24. https://doi.org/10.3390/su13169027

Taylor, J. (2014). Shaping the GAP: Ideas for the UNESCO Post-2014 ESD agenda. *Journal of Education for Sustainable Development,* 8(2), 133-141. https://doi.org/10.1177/0973408214548369

Thøgersen, J., & Zhou, Y. (2012). Chinese consumers' adoption of a 'green' innovation: The case of organic food. *Journal of Marketing Management, 28*(3-4), 313-333. https://doi.org/10.1080/0267257X.2012.658834

Tilbury, D. (2011). Are we learning to change? Mapping global progress in education for sustainable development in the lead-up to 'Rio Plus 20'. *Global Environmental Research, 14*(2), 101-107.

Tomas, L., Girgenti, S., & Jackson, C. (2017). Pre-service teachers' attitudes toward education for sustainability and its relevance to their learning: Implications for pedagogical practice. *Environmental Education Research, 23*(3), 324-347. https://doi.org/10.1080/13504622.2015.1109065

UNESCO. (2014). *Education for sustainable development goals: Learning objectives.* Paris: UNESCO.

United Nations Educational, Scientific and Cultural Organization [UNESCO]. (2017). Education for sustainable development goals: Learning objectives. *United Nations Educational, Scientific and Cultural Organization.* https://unesdoc.unesco.org/ark:/48223/pf0000247444

United Nations Educational, Scientific and Cultural Organization. (2018). *Issues and trends in education for sustainable development.* UNESCO.

United Nations. (2015*).* *Transforming our world: The 2030 Agenda for sustainable development A/RES/70/1*. United Nations.

Valenzuela, A. (2016). *Growing critically conscious teachers: A social justice curriculum for educators of Latino/a youth*. Teachers College Press.

Vare, P., & Scott, W. (2007). Learning for a change: Exploring the relationship between education and sustainable development. *Journal of Education for Sustainable Development, 1*(2), 191-198. https://doi.org/10.1177/097340820700100209

Visser, W. (2010). Schön: Design as a reflective practice. *Collection, 2*(1), 21-25.

Volante, L., & Fazio, X. (2007). Exploring teacher candidates' assessment literacy: Implications for teacher education reform and professional development. *Canadian Journal of Education, 30*(3), 749-770. https://doi.org/10.2307/20466661

Voss, T., Kunter, M., & Baumert, J. (2011). Assessing teacher candidates' general pedagogical/psychological knowledge: Test construction and validation. *Journal of Educational Psychology, 103*(4), 1-14. https://doi.org/10.1037/a0025125

Wals, A. E., & Corcoran, P. B. (2012). Re-orienting, re-connecting and re-imagining: Learning-based responses to the challenge of (un)sustainability. In *Learning for sustainability in times of accelerating* change (pp. 19-32). Wageningen Academic. https://doi.org/10.3920/9789086867578_0004

Wiek, A., Xiong, A., Brundiers, K., & Van der Leeuw, S. (2014). Integrating problem- and project-based learning into sustainability programs: A case study on the School of Sustainability at Arizona State University. *International Journal of Sustainability in Higher Education, 15(*4), 431–449. https://doi.org/10.1108/IJSHE-02-2013-0013

CHAPTER SEVEN

The integration of ICT pedagogy: A panacea to mathematics teacher training in South African universities

Buhle S. Nhlumayo[1*]
Kereng G. Pule[2]

AFFILIATIONS

[1&2]College of Education, University of South Africa, Pretoria, South Africa.

CORRESPONDENCE

Email: Nhlumbs@unisa.ac.za*

Reference Formats (APA 7th Edition)
Nhlumayo, B. S., & Pule, K. G. (2025). The integration of ICT pedagogy: A panacea to mathematics teacher training in South African universities. In B. E. Olawale (Ed.), *Building the Foundations: Effective Approaches in Mathematics Teacher Preparation* (pp. 105-117). ERRCD Forum. https://doi.org/10.38140/obp3-2025-07

Abstract: Information and Communication Technology (ICT) pedagogy involves exposing pre-service teachers (PSTs) to the integration of ICT across the curriculum as a teaching and learning strategy. Research in university teacher training indicates that there is little to no instruction on ICT integration in the teacher training programmes of South African universities. While today's PSTs appear to be skilled ICT users, it is a miscon- - 105 -aption to assume that they have developed adequate skills outside their teacher training programmes. Therefore, universities need to capitalise on ICT pedagogy for mathematics PSTs by equipping them with essential technology tools for teaching. ICT integration into mathematics teaching and learning promotes collaboration, communication, and knowledge-sharing among learners. Although there is access to ICT tools at universities today, it is important to note that access is not synonymous with competency; hence, mathematics PSTs require skills to integrate these ICT tools into their teaching. Adopting a critical literature review, this chapter aims to analyse the approaches that can be used to teach pre-service mathematics teachers to integrate ICT into their instruction. This will be achieved by exploring the importance of exposing PSTs to ICT pedagogy, investigating the extent and ways in which technology is used in teacher-training institutions, understanding how these institutions prepare mathematics PSTs for the integration of ICT in their future classrooms, and exploring the approaches teacher educators can use to teach mathematics PSTs to integrate ICT in their teaching.

Keywords: ICT integration, ICT pedagogy, innovative technologies, mathematics teaching, pre-service teachers.

1. Introduction

ICT is an integral part of the teacher education system as it helps pre-service teachers meet the challenges of a digitalised educational environment. Over 80% of communication now occurs digitally, emphasising the need for ICT-based training (Morrison et al., 2023). Moreover, an emphasis on basic numeracy and critical thinking skills is essential for fostering problem-solving skill development and preparing learners for science and technology careers. Universities should realign with current educational needs to address the gaps in mathematics teacher education training, where equitable practical and technology-based methods are the centrepiece of teacher education. South Africa will adequately strengthen its pre-service mathematics teachers to effectively ignite and educate generations to come by cultivating an intersection of content and pedagogical skills.

Effective teacher training is fundamental to achieving excellence in mathematics education. Mainali (2021) postulates that teacher educators need to understand representations of their students' learning to teach mathematics most effectively. This is because proficiency in mathematics alone does not translate into effective teaching, as both require distinct skill sets. Teacher training for mathematics pre-service teachers aims to produce educators who enhance learner outcomes in the subject, a challenge compounded by historical and socio-political factors in South Africa (Khoza & Biyela, 2020; Morrison et al., 2023). This chapter critically examines approaches to ICT pedagogy in South African universities and pedagogical frameworks in mathematics teacher training, integrating historical and contemporary perspectives to propose forward-looking strategies.

The legacy of apartheid bequeathed deep inequalities in education, which framed teacher training policies and practices. Some initiatives and policy reforms, such as the Employment of Educators Act of 1998, aimed to homogenise teacher education across major disciplines toward greater equity and integration (Morrison et al., 2023). Furthermore, a dynamic training approach is required in South Africa, particularly due to the variety of educational backgrounds within the country. Alarming differences persist between theoretical training and classroom practices even after reforms. It has been noted that over 60% of secondary school graduates are taught only elementary mathematics, highlighting the critical need for strong teacher preparation (Morrison et al., 2023). In addition, rural and under-resourced schools must tackle the growing challenge of implementing innovative strategies that incorporate ICT in teaching.

1.1 Chapter outcomes

This chapter aims to expose teacher-educators to the importance of teaching mathematics to pre-service teachers (PSTs) how to use ICTs in their teaching. After reading the chapter, teacher-educators are expected to be able to:

- Recognise the significance of ICT pedagogy in teaching preservice teachers. Prepare PSTs for the integration of ICT in teaching mathematics.
- Be aware of the challenges and opportunities in integrating ICTs into mathematics instruction.
- Understand the pedagogical frameworks relevant to mathematics teacher training.
- Identify the approaches teacher-educators can employ to teach mathematics PSTs how to integrate ICTs into their instruction.

1.2 Clarification of concepts

- ICT pedagogy: A teaching approach that integrates ICT into the curriculum. It involves embracing technological advancements in teaching and learning to ensure growth, advancement, and sustainability in higher education (Adtani et al., 2023).

- Pre-service teacher: An individual who is undergoing training and education to become a teacher but has not yet begun teaching professionally. Abedi et al. (2024) indicate that PSTs are teachers in training.
- ICT integration: Mnisi et al. (2024) define ICT integration as the use of technology in teaching and learning, including computers, the internet, and communication networks, to enhance the quality of instruction, making it more accessible and cost-effective.
- Innovative educational technologies: New or improved products, processes, or services that utilise technology to solve problems or enhance existing practices, thereby facilitating teaching and learning (Chugh et al., 2023).

2. The Importance of Exposing PSTs to ICT Pedagogy

The role of ICT in the classroom is becoming increasingly important, as learners need to develop skills that will empower them in contemporary society, and because of the potential value of such technologies as tools for learning. The Minimum Requirements for Teacher Education Qualifications (MRTEQ) policy document stipulates that a teacher's ability to use ICT for innovation in teaching and learning is essential (DHET, 2015). One of the many reasons to expose pre-service teachers (PSTs) to ICT pedagogy is the need to equip them with the skills to contribute to and thrive in an information society, as well as to produce a technologically skilled and flexible workforce. Furthermore, as technology becomes more deeply embedded in teaching and learning, future educators must understand how to effectively integrate ICT tools and strategies into their practice. Jita and Dhliwayo (2024) indicate that ICT-integrated pedagogy enhances teaching and learning, increases engagement and motivation for PSTs, and promotes active and personalised learning. Schools are increasingly adopting digital platforms for communication, lesson delivery, and administrative tasks; therefore, PSTs must be comfortable navigating these systems to work effectively within these environments. This perspective is echoed by Ma et al. (2024), who indicate that as technology continues to evolve, teachers must be adaptable and flexible in their approach. Learning ICT pedagogy prepares PSTs to stay up-to-date with emerging technologies and apply them in creative ways. When PSTs are well-prepared to incorporate ICT into their teaching, they not only enhance their teaching effectiveness but are also equipped to empower their learners to become critical, engaged, and competent users of technology in a rapidly changing world. Integrating ICT pedagogy not only encourages PSTs to learn how to use technology for their professional development, but also teaches them how to apply these skills in their future classrooms (Jita & Sintema, 2022).

Preparing PSTs to use ICT in their future classrooms to facilitate a variety of learning styles has been a challenge for most teacher training institutions. Mncube and Olawale (2020) indicate that integrating ICT pedagogy at institutions of higher learning is fundamental to modernising higher education and equipping students with the tools and skills necessary for success in an increasingly digital world. By embracing technology in teaching, learning, assessment, and administration, universities can foster more engaging, inclusive, and personalised educational experiences. Moreover, Wessels (2020) asserts that ICT pedagogy enables universities to remain adaptable in a rapidly changing educational landscape. Resonating with this view are Akour and Anelezi (2022), who assert that ICT pedagogy ensures that students graduate with the critical digital and collaborative skills they will need in their future classrooms. While some universities face the challenge of insufficient ICT infrastructure and equipment for their students, others encounter the problem of inadequate training and experience of teacher-educators in teaching with ICT, all of which hinder the effective incorporation of ICT into teachers' education (Kennedy, 2023).

Mathematics PSTs can benefit from ICT pedagogy by providing differentiated instruction to their learners through representation, engagement, and expression. Through various types of digital assessments, and in their future classrooms, PSTs can quickly gather formative data, allowing them to adjust their instruction based on learners' needs. Marbán and Sintema (2021) suggest that using software enables teachers to customise lessons for diverse learners, such as employing voice narration or visual aids for students with varying learning requirements. When mathematics PSTs are exposed to ICT pedagogy, they can utilise it to track learners' progress, identify areas of difficulty, and provide immediate support. Using technology like graphing tools, interactive visualisations, or problem-solving apps, PSTs can explore mathematical concepts and build their knowledge (Makamure & Jojo, 2021). This approach encourages inquiry-based learning, where they can hypothesise, experiment, and reflect, deepening their understanding of mathematical concepts. Sithole and Mbukanma (2024) further state that incorporating ICT pedagogy into the training of mathematics PSTs equips them with essential skills and strategies to enhance student engagement, foster deeper learning, and prepare their students for future academic and professional success. Additionally, ICT pedagogy supports PSTs' continuous professional growth in a rapidly evolving educational landscape. With the aid of relevant technologies, PSTs will be able to solve complex mathematical problems, think creatively and critically, communicate, and collaborate with others from diverse backgrounds. However, a key challenge for teacher educators is delivering learning experiences at university that allow PSTs to develop technological, pedagogical, and content knowledge skills and capacities.

3. The Current State of Mathematics Teacher Training in South Africa

Education is the cornerstone of social development and economic growth; as a result, improving mathematics teacher training is necessary. Researchers often cite that learners' mathematics achievement is directly correlated to mathematics teaching competency (Khoza & Biyela, 2020; Morrison et al., 2023). This context provides an opportunity for us, as teacher educators, to ensure that teacher education programmes adequately prepare prospective mathematics teachers to respond meaningfully to the various forms of learning they will encounter in South African classrooms.

Mathematics teacher training in South Africa is conducted across various universities and universities of technology, aligning with the General Education and Training (GET) and Further Education and Training (FET) curricula (Morrison et al., 2023). The GET programme culminates in the final degree, the Bachelor of Education (B.Ed.), which equips beginning teachers with foundational mathematics pedagogical skills for teaching in primary schools. However, many primary school teachers, even after four years of training, have insufficient exposure to ICT. This gap frequently results in fears about the use of technology in the learning of mathematics, which is often aggravated by historical teaching methods that discourage the use of ICT (Shava, 2022). This suggests that they may be passing on poor attitudes toward mathematics and technology to learners, which is an important consideration in existing training models.

The challenges faced by teacher training institutions are compounded by the diverse contexts of South African schools. Mojapelo and Durodolu (2022) argue that some schools, particularly in rural areas, are under-resourced and already at a disadvantage due to insufficient infrastructure necessary for ICT integration. This further marginalises school users in such contexts. Unless specific measures are taken to address the lack of access to quality mathematics education, the inequalities will persist (Azhari & Farji, 2022), undermining the overall purposes of the South African education system. In a different context, Xu and Ouyang (2022) indicate that integrating ICT into education may signal a transformative leap, particularly in mathematics education, creating reflective teaching and interactive environments for learners. ICT tools not only encourage active learner participation but also align with constructivist approaches, positioning learners as active participants in constructing their knowledge. Some methodologies, such as Problem-Based, Inquiry-Based, and Project-Based Learning, adopt technology as a means to achieve meaningful communication and high-level problem-solving skills.

Moreover, integrating ICT pedagogy can strengthen the link between theoretical knowledge and its applications in teacher education. When ICT is embedded in lesson planning, teachers can use this technology effectively by showing learners real-life examples that help them learn important mathematical concepts. Such an approach is not just engaging; it equips learners with the skills needed in a technology-led economy (Morrison et al., 2023; Xu & Ouyang, 2022). For the integration of ICT in mathematics teaching to be effective, teachers need to be knowledgeable in both pedagogy and technology. ICT-based teaching strategies are most relevant to the needs of knowledge adaptability and 21st-century skills, with a drive to provide learners with as much information as they seek (Mofokeng, 2022). Furthermore, these strategies foster the ability to identify necessary information by adopting critical thinking. These are all essential skills in preparing learners to thrive in an increasingly complex and interconnected world.

4. Challenges, Opportunities and Advantages of ICT Integration in Mathematics Teacher Training

4.1 Challenges

Marban and Sintema (2021) indicate that integrating ICT pedagogy into mathematics teacher training presents a transformative opportunity to enhance educational outcomes. ICT also provides pre-service teachers (PSTs) with access to a diversity of resources, innovative environments, and collaborative ventures that support personalised learning and digital literacy (Morrison et al., 2023; Xu & Ouyang, 2022). However, challenges still exist, particularly in contexts like South Africa, where infrastructure constraints and systemic barriers to implementation complicate the situation. This article discusses the key issues, approaches, and potential benefits of integrating ICT into mathematics teacher education programmes. For instance, in under-resourced schools and universities, unreliable internet connectivity, hardware, and software represent significant barriers (Aktaş & Özmen, 2020). The ICT experience of many PSTs in South Africa upon entering training programmes is minimal, and only once early experience has been established can more advanced pedagogical applications be introduced. Moreover, in-service teachers tend to overlook ICT integration, as they did not receive sufficient exposure during their training. This discrepancy is further compounded by inflexible curricula that address the immediate crises facing education rather than the technological innovations on the horizon. Additionally, limited professional development opportunities and the absence of a cohesive institutional support system have left many teachers underprepared to implement innovative practices.

4.2 Opportunities

Although ICT integration presents several challenges, it can provide numerous benefits and opportunities that positively impact mathematics teacher training. For instance, it encourages personalised learning and helps PSTs conveniently access tailored resources that meet their needs and address their learning preferences. Not only does this solidify their understanding of mathematical concepts, but it also enables them to deliver differentiated instruction to learners in their classrooms (Aktaş & Özmen, 2020; Morrison et al., 2023). ICT integration promotes collaboration among teachers, which is a critical element in developing communities of practice that support the exchange of ideas, resources, and best practices. Such networks offer a wealth of information that is particularly useful in mathematics education, where novel approaches to teaching and problem-solving are essential for learners' academic success. Finally, ICT integration into teaching and learning helps teachers engage students more effectively, promoting critical thinking, creativity, and problem-solving skills. Innovations using technologies like interactive simulations, virtual manipulatives, and data analysis software have been shown to make mathematics more accessible to learners.

4.3 Advantages

ICT integration offers significant advantages that can transform mathematics teacher training. First, it facilitates personalised learning, allowing preservice teachers to access tailored resources that address their specific needs and learning styles. This approach not only improves their understanding of mathematical concepts but also enhances their ability to deliver differentiated instruction in the classroom (Aktaş & Özmen, 2020). ICT fosters collaboration among educators, creating communities of practice that encourage the sharing of ideas, resources, and best practices. These networks are particularly valuable in mathematics education, where innovative teaching strategies and problem-solving techniques are essential for student success. ICT equips teachers with the tools to engage students actively, promoting critical thinking, creativity, and problem-solving skills. By incorporating technologies such as interactive simulations, virtual manipulatives, and data analysis software, educators can make mathematics more accessible and engaging for learners. Hence, context-specific strategies are needed for successful ICT integration, as teachers and learners have different requirements (Morrison et al., 2023). For example, a South African university adapted its first-year mathematics pedagogy to illustrate data-led activities, such as spreadsheet-based data analysis and algorithm visualisation, which can supplement traditional arithmetic instruction. This adaptation enhanced learners' ability to compute while also developing logical reasoning and communication skills. An additional example is a project called Schools in Partnerships, which showcased the power of ICT to improve the quality of teacher professional growth and learner learning. This initiative empowered teachers to effectively integrate technology into their teaching practices by focusing on planning, execution, and ongoing support.

5. Pedagogical Frameworks in Mathematics Teacher Training

Reports based on recent research, conferences, and postgraduate studies offer crucial insights into the pedagogical frameworks informing mathematics teacher education at South African universities. These frameworks must align with the educational goals outlined in the post-1994 reforms while addressing the immediate developmental needs of South African schools, particularly those in underserved areas (Bakar et al., 2020; Morrison et al., 2023). Given that education is one of the most powerful functions for transforming society, reimagining teacher training in South Africa presents an opportunity with vast potential to enhance learning outcomes and address systemic challenges on their own terms.

Mathematics teacher training frameworks are based on three key pillars: sound theory, practical applications, and responsiveness to society's needs (Jaeger et al., 2023). The existing models need to transition from solely sociological or cognitive approaches to transformative perspectives that focus on developmental practices (Morrison et al., 2023). This transformation is especially important for schools in impoverished communities, where teachers face challenges such as poverty and the continuing fallout of the HIV/AIDS epidemic. Furthermore, frameworks need to embrace new pedagogy and a variety of learning environments. These frameworks can provide effective tools for teachers to navigate the complexities of modern classrooms by focusing on challenges specific to their communities.

Constructivist principles are foundational to modern mathematics teacher training. These methods encourage active learning, knowledge construction, and practical application, which, in turn, foster critical thinking and collaborative problem-solving (Morrison et al., 2023). Taken alone, constructivist frameworks demand a complete rethinking of how teachers interact with one another and with learners, promoting more interactive and learner-centred methodologies. Such methods move away from the traditional approach of rote memorisation and instead cultivate lifelong learning habits and the capacity to adapt to change.

While these frameworks have great potential, systemic obstacles impede their implementation. Resistance to change within educational institutions, insufficient pedagogical training for teachers, and budget constraints significantly affect the dissemination of innovative methodologies. Moreover, these limitations are compounded by a lack of partnerships with local organisations and the failure to provide adequate access to relevant contextual information, particularly in rural schools.

Some promising models, such as institutional-workplace collaboration (IWC) (Arinaitwe, 2021), have emerged, showcasing how collaboration can improve teacher training. The IWC refers to how teachers reflect on their co-development of questions concerning effective teaching and learning, such as through reflective practice workshops designed to empower teachers. To this end, regional stakeholders, including university representatives, Department of Basic Education (DBE) officials, and teacher unions, have come together to share best practices and build consensus on reform initiatives. Such practices demonstrate the value of collaboration and the need for customised initiatives that address specific institutional issues. They serve as a case study of how teacher training programmes can be rethought to meet local, unique concerns while also aligning with international education standards. Each of these collaborative efforts reinforces the idea that trust-based relationships are essential for advancing institutional reform. This process helps ensure that teacher training programmes remain both relevant and impactful by involving diverse stakeholders in the process.

6. Approaches Teacher-Educators Can Use to Teach Mathematics Preservice Teachers to Integrate ICTs in Their Teaching

6.1 Modelling

Teaching mathematics PSTs to integrate ICTs into their teaching requires a blend of theoretical knowledge, hands-on experience, and reflective practice. Teacher-educators can adopt several approaches to ensure that PSTs are well prepared to integrate ICTs effectively into their teaching practices. One approach is for teacher-educators to model ICT integration in their own teaching (Adnan et al., 2024). Dewa and Ndlovu (2022) argue that while teacher-educators possess knowledge of how to teach with digital technologies in their lecture rooms, they often do not model specialised mathematics teaching skills for their students. Therefore, through active demonstration, teacher-educators can showcase the use of ICT tools during their lessons. By demonstrating how to integrate technology seamlessly into lessons, PSTs can observe the practical application of ICT tools, such as graphing software like GeoGebra, dynamic geometry tools, or online platforms such as Desmos and Wolfram Alpha. Teaching PSTs to integrate ICTs in their mathematics teaching involves equipping them with both technical skills and pedagogical strategies. Supporting this view, Stockless et al. (2022) argue that university curricula should take responsibility for equipping PSTs with both technical and pedagogical knowledge.

6.2 ICT-infused lesson planning

To support mathematics PSTs in integrating ICT into their teaching, Lim et al. (2011) indicate that each programme they are involved in, which includes curriculum, assessment, and practicum, should be ICT-infused. This involves teaching PSTs to design lesson plans that effectively integrate ICT. Key elements of lesson planning include aligning the use of technology with learning objectives, designing tasks that leverage technology to visualise or explore mathematical concepts, and incorporating interactive or formative assessment tools. PSTs can be introduced to real-world data sets that can be analysed using ICT tools. For example, they can use software like Excel or Google Sheets to create statistical models or solve real-world mathematical problems, demonstrating the relevance of mathematics in everyday life. By thoughtfully incorporating ICT tools, PSTs can enhance teaching and learning, creating engaging, interactive, and differentiated lessons that cater to the diverse needs of learners. Janssen et al. (2019) indicate that ICT can be aligned with lesson objectives and incorporated to support content delivery. Mathematics PSTs should be taught how to plan for differentiation of instruction in mathematics classrooms and leverage ICT tools to adjust teaching and learning to individual learner needs (van den Kieboom & Groleau, 2022).

6.3 Workshops for PSTs

The advent of the Fourth Industrial Revolution (4IR) necessitated a shift in the alignment of pedagogical activities worldwide. As van Wyk and Waghid (2023) indicate, South African pre-service teachers (PSTs) generally deem themselves unqualified and incompetent to teach in South African schools using ICT, particularly in mathematics. South African universities can provide workshops that introduce PSTs to a range of ICT tools suitable for the mathematics classroom. These workshops should focus on practical applications, such as creating interactive lessons, assessments, and visual aids using these tools. Although Pule and Raxangana (2024) indicate that, as a developing country, South African teachers face challenges in using mathematical software to enhance teaching and learning in schools, Nhlumayo (2024) asserts that most teachers were never guided on using ICT for teaching in their training programmes. Khoza and Biyela (2020) postulate that PSTs arrive at universities owning both technological hardware (smartphones and/or laptops) and software (Facebook, X, WhatsApp, Instagram, etc.). However, since ownership is not synonymous with ability, it is the responsibility of universities to introduce PSTs to mathematical ICT tools and simulations. These may include innovative technologies such as GeoGebra, Desmos, Microsoft Excel, Google Classroom, Moodle, PhET Interactive Simulations, and Wolfram Alpha, which can model complex mathematical concepts and visualisations that might otherwise be difficult to explain. As part of ICT pedagogy, South African universities must include the basics of digital literacy and computational thinking in their teacher preparation programmes, thus helping PSTs to become proficient not only in using mathematical software but also in evaluating online resources.

7. Conclusion

To prepare mathematics PSTs for ICT-integrated pedagogy, South African universities must offer a curriculum that integrates ICT training with content and pedagogy, provides hands-on experience with technology, and emphasises the real-world application of ICT in mathematics classrooms. Integrating ICT pedagogy into mathematics teacher training is essential for modernising education and ensuring that PSTs are well-equipped to meet the evolving needs of their learners. This chapter argues that the integration of ICT pedagogy improves mathematical conceptual understanding, particularly for concepts that may be difficult to grasp, such as calculus, algebraic structures, or geometry. Moreover, it contends that the integration of ICT pedagogy prepares future mathematics teachers for the digital age. Furthermore, this chapter posits that ICT pedagogy also facilitates different forms of assessment for mathematics, such as online quizzes and digital portfolios. This can assist PSTs in easily tracking their learners' progress, identifying areas where learners are struggling, and adjusting their teaching accordingly. By fostering technical proficiency, pedagogical expertise, and awareness of the socio-economic context, universities can ensure that PSTs are well-prepared to teach mathematics using ICT effectively, enhancing student engagement and learning outcomes across South Africa's diverse educational settings.

Integrating ICT pedagogy into mathematics teacher training is vital for developing a more effective, engaging, and inclusive educational experience. Equipping PSTs with the skills to use technology effectively enhances their academic outcomes, promotes digital literacy, and prepares them for future challenges. Furthermore, it supports diverse learning styles, fosters collaboration, and offers tools that make mathematical concepts more accessible and engaging. Ultimately, the integration of ICT pedagogy empowers teachers and students, preparing them for the demands of the modern digital world.

8. Declarations

Funding: This research did not receive any external funding.

Conflicts of Interest: The authors declare no conflict of interest.

References

Abedi, E. A., Prestridge, S., Geelan, D., & Hodge, S. (2024). Preparing pre-service teachers to teach with information technology: mapping knowledge patterns in what is included and omitted in Ghana. *Cambridge Journal of Education*, *54*(3), 337-356. https://doi.org/10.1080/0305764X.2024.2355215

Adnan, M., Tondeur, J., Scherer, R., & Siddiq, F. (2024). Profiling teacher educators: ready to prepare the next generation for educational technology use? *Technology, Pedagogy and Education*, 1-18. https://doi.org/10.1080/1475939X.2024.2322481

Adtani, R., Neelam, N., Raut, R., Deshpande, A., & Mittal, A. (2023). Embracing ICT in academia: adopting and adapting to the new normal pedagogy. *Global Knowledge, Memory and Communication*. https://doi.org/10.1108/GKMC-03-2023-0089

Akour, M., & Alenezi, M. (2022). Higher education future in the era of digital transformation. *Education Sciences*, *12*(11), 784. https://doi.org/10.3390/educsci12110784

Aktaş, I., & Özmen, H. (2020). Investigating the impact of TPACK development course on pre-service science teachers' performances. *Asia Pacific Education Review, 21(4),* 667-682. https://doi.org/10.1007/s12564-020-09653-x

Arinaitwe, D. (2021). Practices and strategies for enhancing learning through collaboration between vocational teacher training institutions and workplaces. *Empirical Res Voc Ed Train, 13,* 13. https ://doi.org/10.1186/s40461-021-00117-z

Azhari, B., & Fajri, I. (2022). Distance learning during the COVID-19 pandemic: School closure in Indonesia. *International Journal of Mathematical Education in Science and Technology*, *53*(7), 1934-1954. https://doi.org/10.1080/0020739X.2021.1875072

Bakar, N.S.A., Maat, S.M., & Rosli, R. (2020). Mathematics Teacher's Self-Efficacy of Technology Integration and Technological Pedagogical Content Knowledge. *Journal on Mathematics Education*, *11*(2), 259276. http://doi.org/10.22342/jme.11.2.10818.259-276

Chugh, R., Turnbull, D., Cowling, M. A., Vanderburg, R., & Vanderburg, M. A. (2023). Implementing educational technology in Higher Education Institutions: A review of technologies, stakeholder perceptions, frameworks and metrics. *Education and Information Technologies*, *28*(12), 16403-16429. https://doi.org/10.1007/s10639-023-11846-x

Department of Higher Education and Training. (2015). *Minimum Requirements for Teacher Education Qualifications.* Government Printers.

Dewa, A., & Ndlovu, N. S. (2022). Use of information and communication technologies in mathematics education lecturers: Implications for preservice teachers. *Journal for Transdisciplinary Research in Southern Africa, 18*(1), 1-8. http://dx.doi.org/10.4102/td.v18i1.1165

Jaeger, J., Masselot, C., Greshake Tzovaras, B., Senabre Hidalgo, E., Haklay, M., & Santolini, M. (2023). An epistemology for democratic citizen science. *Royal Society Open Science*, *10*(11), 231100. https://doi.org/10.1098/rsos.231100

Janssen, N., Knoef, M., & Lazonder, A. W. (2019). Technological and pedagogical support for pre-service teachers' lesson planning. *Technology, Pedagogy and Education, 28*(1), 115-128. https://doi.org/10.1080/1475939X.2019.1569554

Jita, T., & Dhliwayo, A. (2024). Narratives of Teacher Educators on the Integration of Information and Communication Technologies to Prepare Pre-service Teachers in Specific Subjects. In A. Chigona. H. Crompton & N. Tunjera (Eds.), *Global Perspectives on Teaching with Technology* (pp. 60-74). Routledge. https://doi.org/10.4324/9781003406631

Jita, T., & Sintema, E. J. (2022). Pre-service teachers' self-concept and views toward using ICT for teaching Science. *EURASIA Journal of Mathematics, Science and Technology Education*, *18*(9). https://doi.org/10.29333/ejmste/12396

Kennedy, G. M. (2023). Challenges of ICT integration in teachers' education: A case study of the College of Education, University of Liberia. *International Journal of Social Science and Education Research Studies*, *3*(5), 860-870. https://doi.org/10.55677/ijssers/V03I5Y2023-15

Khoza, S.B., & Biyela, A.T. (2020). Decolonising technological pedagogical content knowledge of first-year mathematics students. *Education and Information Technologies*, *25*, 2665–2679. https://doi.org/10.1007/s10639-019-10084-4

Lim, C. P., Chai, C. S., & Churchill, D. (2011). A framework for developing pre-service teachers' competencies in using technologies to enhance teaching and learning. *Educational Media International*, *48*(2), 69-83. https://doi.org/10.1080/09523987.2011.576512

Ma, K., Bello, A., Dong, J., & Chutiyami, M. (2024). Pre-service teachers' adaptability to blended teaching and its predictors. *Distance Education*, 1-20. https://doi.org/10.1080/01587919.2024.2338710

Mainali, B. (2021). Representation in teaching and learning mathematics. *International Journal of Education in Mathematics, Science, and Technology, 9*(1), 1-21. https://doi.org/10.46328/ijemst.1111

Makamure, C., & Jojo, Z. M. (2021). Visual-spatial skills and mathematics content conceptualisation for pre-service teachers. *Indonesian Journal of Science and Mathematics Education*, *4*(3), 223-241. http://dx.doi.org/10.24042/ijsme.v4i3.9842

Marbán, J. M., & Sintema, E. J. (2021). Pre-service teachers' TPACK and attitudes toward integration of ICT in mathematics teaching. *International Journal for Technology in Mathematics Education*, *28*(1), 37-46.

Mnisi, B. R., Mtshali, T. I., & Moses, M. (2024). Moving beyond the challenges of learning through technologies: The current status of ICT integration in South African schools. *Journal of Education and E-Learning Research, 11*(1), 128–134. https://doi.org/10.20448/jeelr.v11i1.5396

Mncube, V., & Olawale, E. (2020). Exploring Readiness of South African Universities for the Fourth Industrial Revolution. In *EDULEARN20 Proceedings* (pp. 8989-8995). IATED. https://doi.org/10.21125/edulearn.2020.1794

Mofokeng, P. M. (2022). *A multiple comparative institutional case study exploring the extent to which final year B. Ed. Students are prepared for ICT usage in the classroom* [Doctoral dissertation, University of the Witwatersrand]. https://hdl.handle.net/10539/34772

Mojapelo, S. M., & Durodolu, O. O. (2022). Information and communications technologies in library facilities in disadvantaged rural schools in South Africa: Lessons from Limpopo province. *Education for Information*, *38*(2), 113-131. https://doi.org/10.3233/EFI-211541

Morrison, S., Graven, M., Venkat, H., & Vale, P. (2023). A 20-year review of South African Early Grade Mathematics Research Articles. *African Journal of Research in Mathematics, Science and Technology Education*, *27*(3), 304-323. https://doi.org/10.1080/18117295.2023.2226547

Nhlumayo, B. S. (2024). Rural primary school principals' leadership strategies for ICT integration. *Research in Social Sciences and Technology, 9*(1), 171-184. https://doi.org/10.46303/ressat.2024.10

Pule, K. G., & Raxangana, L. (2024). The Challenges Entry-Level Mathematics Teachers Face in Conducting Blended Teaching. *Research in Social Sciences and Technology, 9*(1), 124-140. https://doi.org/10.46303/ressat.2024.7

Shava, E. (2022). Reinforcing the role of ICT in enhancing teaching and learning post-COVID-19 in tertiary institutions in South Africa. *Journal of Culture and Values in Education*, *5*(1), 78-91. https://doi.org/10.46303/jcve.2022.7

Sithole, V. L., & Mbukanma, I. (2024). Prospects and Challenges to ICT Adoption in Teaching and Learning at Rural South African Universities: A Systematic Review. *Research in Social Sciences and Technology*, *9*(3), 178-193. https://doi.org/10.46303/ressat.2024.54

Stockless, A., Villeneuve, S., Bisaillon, J., Fournier, F., & Venant, F. (2022). Pre-Service Teachers' Competence and Pedagogical Use of ICT: Are They Ready to Develop Collaborative Activities with Students? *Computers in the Schools*, *39*(3), 203-229. https://doi.org/10.1080/07380569.2022.2071223

van den Kieboom, L. A., & Groleau, S. V. (2022). Pre-service teacher planning for differentiation of instruction in mathematics classrooms. *Educational Studies in Mathematics, 111*(2), 225-252. https://doi.org/10.1007/s10649-022-10149-1

van Wyk, M.D., & Waghid, Z. (2023). South African pre-service teachers' preparedness for Fourth Industrial Revolution teaching and learning. *Education and Information Technologies*, *28*, 2887-2907. https://doi.org/10.1007/s10639-022-11287-y

Wessels, L. (2020). *How South African universities can contribute to preparing the future workforce for the fourth industrial revolution* (Doctoral dissertation, Stellenbosch: Stellenbosch University). http://hdl.handle.net/10019.1/108143

Xu, W., & Ouyang, F. (2022). The application of AI technologies in STEM education: a systematic review from 2011 to 2021. *International Journal of STEM Education*, *9*(1), 59. https://doi.org/10.1186/s40594-022-00377-5

CHAPTER EIGHT

Aftermath of Overlooking Foundation Pre-Service Mathematics Teachers' Beliefs: A Self-Study in Post-Apartheid South Africa

Linda van Laren[1*]

AFFILIATIONS

[1]School of Education, University of KwaZulu-Natal, Edgewood Campus, South Africa.

CORRESPONDENCE

Email: vanlarenl@ukzn.ac.za*

Reference Formats (APA 7th Edition)

van Laren, L. (2025). Aftermath of overlooking foundation pre-service mathematics teachers' beliefs: A self-study in post-apartheid South Africa. In B. E. Olawale (Ed.), *Building the Foundations: Effective Approaches in Mathematics Teacher Preparation* (pp. 118-137). ERRCD Forum. https://doi.org/10.38140/obp3-2025-08

Abstract: Mathematics is often described as a "high status" discipline and in primary schools, mathematics is compulsory. However, in educational research, myriad performance issues in primary mathematics teaching and learning have been emphasised. The foundations of future mathematics learning are laid for young learners by their teachers who are often the learners' main resource, and foundation mathematics problems snowball as learners progress through schooling levels. Teachers are certified by higher education institutions so it is thus worthwhile for primary mathematics teacher educators to explore constructive, reflexive, optimistic, starting-with-ourselves approaches to reflect on what is offered in mathematics education. This looking inward approach contrasts with research in which recommendations are proposed to encourage improvements in professional practice. The research question for this study was: "What primary mathematics education professional learning can I (re)construct by reflecting on teacher education modules offered at my higher education institution, using a self-reflexive approach?" My primary study source was the foundation phase mathematics education modules detailed in University of KwaZulu-Natal's Handbook for 2024. This qualitative study explored the competences offered by these modules in the university's foundation phase mathematics teacher education programme, and included a set of objects used as a metaphor for self-reflexivity and to represent the importance of attending to the often-overlooked pre-service teachers' beliefs about mathematics education in post-apartheid South Africa.

Keywords: Beliefs, post-apartheid South Africa, pre-service mathematics education, self-reflexive metaphor, starting with ourselves.

1. Introduction

Mathematics is often described as a "high status" discipline and as Valero (2013, p. 2) noted, in a 21st-century school curriculum, it is "unthinkable" to omit mathematics. In South African primary schools, mathematics is compulsory with foundation phase (FP) learners required to spend approximately 32% of school time doing mathematics and intermediate phase (IP) learners, 22% of school time (Department of Basic Education, 2011). Reasons usually given for why mathematics is an essential subject are that mathematics knowledge and skills are valuable in many aspects of everyday life and positively influence life opportunities.

Myriad performance issues in South African primary mathematics teaching and learning are constantly emphasised in education research. Some of these were summarised by economist, Servaas van der Berg, who asserted that poverty and financial inequality are directly related to the unsatisfactorily poor quality of South African education systems. Van der Berg further contended: "If you want to change the economy, you have to start with the education system" (as quoted in Thompson, 2024, para. 7).

There is no doubt that even in post-apartheid South Africa, school learner education is generally unequal and differentiated according to family income (Beckmann, 2021). Three decades after the birth of South African democracy, most schools in low-income areas, where learners are mainly of African origin, still experience apartheid after-effects. Hlatshwayo (2023) acknowledged that although there are limited improvements in learners' numeracy levels, many have not benefitted significantly from being schooled in the post-apartheid context.

The foundations of future mathematics learning are laid for young learners by their teachers who are often learners' main resource in South Africa. Thus learners depend on their teachers' mathematics subject content knowledge (SCK), pedagogical content knowledge (PCK), and beliefs about mathematics teaching and learning for their development of "appropriate knowledge, skills and values" (Department of Education, 2006, p. 7). In the early grades, foundation mathematics difficulties commence and snowball as learners progress through schooling levels (Lénárt et al., 2022). Given that pre-service teachers are certified by higher education institutions, their education programmes impact on primary, secondary, and tertiary levels of learners' mathematics learning.

Some of the most important role players in school education are the teachers, who "are central to the success of education outcomes" Hlatshwayo (2023, p. 389). However, if the teachers believe that mathematics is only for the gifted few, then that attitude towards mathematics achievement could be internalised by their learners. Teachers' attitudes and the general psycho-social environment in mathematics classrooms play a significant role in mathematics learning; learners' achievement is influenced by learners' mathematics attitudes and beliefs (Mazana et al., 2019). Those scholars further noted that suitable pedagogical strategies and appropriate social psychological environments promote enjoyment and positive attitudes towards mathematics, which in turn, benefit learners' performance.

On reviewing South African educational research on teaching and learning for a World Bank report, van der Berg and Hofmeyr (2018, p. 16) concluded that teacher education programmes are "not succeeding in adequately preparing students for the teaching profession." They bluntly pointed out that mathematics teachers are underprepared in teacher education programmes, certified teachers have insufficient mathematics SCK and PCK, pre-service teachers enrolled for teacher education are not top-performing candidates, and teacher development in higher education institutions is unsuccessful in building pedagogical skills for teaching young learners mathematics in African languages. Van der Berg and Hofmeyr (2018, p. 23) further declared that teacher development programmes have failed and hence require "independent evaluation as to whether they are working and why," and that "rigorous evaluation of the effectiveness of such pedagogical training must be done regularly."

After reading findings, conclusions, and recommendations in mathematics educational research, I realised that many disheartening, uncomfortable, and disconcerting issues have been raised. Furthermore, in mathematics education literature there are worthy (but sometimes complex) recommendations offered for improving mathematics teaching and learning. However, there is no point in dwelling on these saddening, problematic issues without doing something to make a difference in my professional practice as a South African primary mathematics teacher educator.

The *National Qualifications Framework Act* (Department of Higher Education and Training, 2015), which specifies expertise required by South African teachers in fundamental disciplines and subject areas, states that standards to define competence for specific subjects or specialisation are not defined in that government policy. Rather, specific subjects, such as primary mathematics education, need to develop their own standards within relevant teacher education communities of practice. In other words, it is important that mathematics teacher education community members take up the opportunity to communicate and make public improvement possibilities. Thus, instead of thinking, "there is nothing I can do about transformation in primary mathematics teacher education," it was preferable to explore constructive, reflexive, optimistic "starting with ourselves" (van Manen, 1990, p. 43) approaches to reflect on what is offered in mathematics teacher education at my own higher education institution. The knowledge gained through looking inward could then be used to improve my professional practice and hopefully inspire others in my educator community to also commit to using improvement-orientated self-study approaches to address the criticisms levelled at mathematics teacher education.

An approach where self-improvement considerations are key, contrasts with research where recommendations are presented to encourage changes in professional practice. The research question for this starting-with-ourselves study was: "What primary mathematics education professional learning can I (re)construct by reflecting on teacher education modules offered at my higher education institution, using a self-reflexive approach?"

2. Theoretical Framework for Mathematics Teacher Education Programmes

Many mathematics teacher educator researchers (for example, Shulman, 1987; Tatto, 2012; 2018) have suggested competence requirements for primary mathematics education curricula. Manouchehri (1995) addressed various areas of mathematical competence including SCK, PCK, pedagogical reasoning, and beliefs. Manouchehri (1995, p. 1) further defined SCK as "key facts, concepts, principles and explanatory framework in the discipline," PCK as the "knowledge of students and learning, knowledge of curriculum and school context, and knowledge of teaching," pedagogical reasoning as the "process of transforming content knowledge into forms that are pedagogically powerful and adaptive to particular groups of students," and beliefs as the "attitudes and assumptions of entering preservice teachers, beginning teachers, and experienced teachers."

2.1 Affect in mathematics and mathematics teacher education

Tatto (2018, p. 210), in her international (13 countries, including Botswana) study of primary mathematics teacher education programmes, pointed out that teachers' beliefs are an "essential component of what makes a teacher effective and thus have been an important feature of teacher education." And Manouchehri (1995) indicated that often, pre-service teachers' beliefs about pedagogy were developed prior to their enrolment in teacher education programmes, thereby having a profound influence on their learning in those programmes. Additionally, Tatto (2018) highlighted the significance of pre-service teachers' beliefs about learning mathematics—how they perceived the nature/vision of mathematics, how they experienced learning mathematics in schools, and what their approach to teaching mathematics might be.

In the South African post-apartheid context, the past experiences of pre-service teachers' mathematics school learning may not have been ideal because of the distressing and haunting influences of 37 years of apartheid government that ended in 1994. The government enforced racism, using separate and unequal social, political, and educational opportunities to further its apartheid ideals. It employed pervasive ideology systems that imposed perverse "symbols, images, beliefs, feelings, thoughts, and attitudes" (Wills, 2011, p. 14). The watered-down government school curriculum for most South African learners (generally, of African origin) was a political strategy, and the dumbing-down of mathematics could regretfully result in teachers and learners developing a vision of mathematics as being only for gifted learners, too difficult to achieve, and reserved for other scholars.

From Manouchehri's (1995, p. 1) definition of beliefs, it appears that the author considered "beliefs" to be synonymous with "attitudes and assumptions." Research in mathematics education related to affect (see for example, Di Martino & Zan, 2011; Zhang & Morselli, 2016) has noted that in the affect field, there is no agreement on the definitions of constructs such as beliefs, attitudes, emotions, and values because of the changing nature of teaching and learning environments. However, this does not detract from the need to make pre-service teachers aware of beliefs that influence practice. Zhang and Morselli (2016) pointed out that teacher education programmes should stress exploring belief awareness rather than only belief changing.

Di Martino and Zan (2011) emphasised the relationship between beliefs and emotions because emotions bring physiological reactions into play, which interfere with cognitive processes and may stifle thinking. Furthermore, emotions play a significant role in "coping, adapting and decision making" (Di Martino & Zan, 2011, p. 472). After analysing the narrative and autobiographic descriptions of well over a thousand research participants' relationships with mathematics, these authors proposed a model that explained the relationships between affect constructs and concluded that attitude acts as a "bridge between beliefs and emotions" (Di Martino & Zan, 2011, p. 484).

Di Martino (2024) then developed a theoretical framework to study the attitudes of pre-service primary teachers required to teach mathematics in their Italian schooling context. His study "confirmed how negative attitudes towards mathematics and towards the idea [of] having to teach mathematics are very common" (Di Martino, 2024, p. 434) among prospective primary school teachers.

2.2 Mathematics education literature addressing pre-service teachers' beliefs

Di Martino and Zan (2011) questioned assessment approaches that focus mainly on accuracy. They promoted the notion that the mathematics vision in solving mathematics problems should be focused on the processes rather than just the accuracy of the final solution. In other words, possible mathematics assessment shifts are required to promote the "idea of success from the production of correct answers to the enactment of meaningful thinking processes" (Di Martino & Zan, 2011, p. 487). Accuracy emphasis, along with other classroom experiences, influence learners' attitude to mathematics because some learners consequently perceive themselves as innately unable to succeed in mathematics. The authors acknowledged the difficulties of teachers and teacher educators in accepting the emphasis on thinking process required for this vision, but emphasised the importance of overcoming the "distorted vision of mathematics and possibly low perceived competence" of learners in order to address their affect in mathematics education (Di Martino & Zan, 2011, p. 487).

Tatto's (2018, p. 244) study indicated that lower levels of performance of newly qualified teachers were associated with those teachers believing that mathematics problem solving required "memorizing a collection of rules and procedures," prescribing methods, and applying "definitions, formulas, mathematical facts and procedures." This means that these teachers subscribed to the belief that mathematics teaching mainly involved encouraging instrumental understanding with a "rule-based . . . focus on how to do something" as opposed to relational understanding (Herheim, 2023, p. 390), which relates to "understanding structures, searching for patterns, and relating new concepts to previous understanding" (Herheim, 2023, p. 391). According to Herheim (2023, p. 391), "both instrumental and relational understanding have their advantages, but the instrumental one is more short-term, rigid, and context-dependent."

Furthermore, Manouchehri (1995, p. 12) encouraged teacher education programmes to "address the prospective teachers' pre-existing beliefs," assist pre-service teachers in making their "implicit beliefs about teaching, learning, subject matter and learning to teach explicit, . . . challenge the adequacy of those beliefs," and facilitate the integration of new ideas and information into their existing beliefs. Tatto et al. (2012, p. 172) noted that belief "change is unlikely to occur unless teacher-preparation programs explicitly address beliefs about mathematics and mathematics learning." And Manouchehri (1995) recommended the use of reflection for challenging pre-service teachers' beliefs. In answering his question: "Can reflective practice be taught?" Russell (2005, p. 202) concluded that reflective practice can and should be taught. He noted that explicit, direct, thoughtful, and patient self-reflection-in-action interpretations improved reflective practice teaching.

In addition, Tatto et al. (2012) found that future teachers' belief patterns matched their mathematics teacher educators' beliefs, which implies that altering the teacher education curriculum is unlikely to change beliefs of pre-service teachers. Tatto et al. (2012, p. 172) contended that a change in the beliefs of pre-service teachers "if it is to occur, would probably require a significant investment in professional development for practicing teachers as well as for teacher educators."

Various mathematics education researchers consider pre-service teacher belief reflections to be purposeful in mathematics education. This means that in mathematics teacher education, attending to affect should be an integral part of teacher education programmes because mathematics is the subject that "triggers the strongest negative emotions" (Di Martino & Zan, 2011, p. 471). Moreover, there appears to be a need to inspire mathematics teacher educators to make use of reflexive self-study strategies for professional learning in order to seek opportunities to improve their professional practice in sustainable, doable, and context appropriate ways (Manouchehri, 1995; Tatto, 2018). The aims of my study were to contribute to the development of my own usable self-learning related to social justice issues, to improve my own professional practice as a primary mathematics teacher educator, and to make public my self-study for scrutiny by the mathematics education community.

3. Self-Study Research

3.1. Situating myself as a mathematics teacher educator

I have been a mathematics teacher educator for almost four decades, teaching at South African higher education institutions in both the apartheid and post-apartheid contexts. I spent most of my mathematics teacher educator career at a higher education institution in Durban, South Africa. I am a White South African woman and prior to 1994, was a primary mathematics teacher educator in an institution where only White pre-service teachers were enrolled. After 1994, primary pre-service teachers were predominantly of African origin, and I realised that the context in which they had been schooled could influence the way they experienced mathematics education studies. In other words, the contexts in which most pre-service teachers were educated at high schools could be substantially different from what White privileged pre-service teachers experienced. The interplay between their cognitive and emotional aspects in mathematics education (affect) could be somewhat different from what White pre-service teachers and their teacher educators experienced. Thus, I saw the need to (re)construct my professional learning as a post-apartheid South African mathematics teacher educator. I subscribe to Griffiths' (2003, p. 167) slogan, "social justice is a verb," and align my views with her approach because social justice is "always action oriented, [and] always unfinished," and for social justice and self-improvement reasons, chose to examine FP teacher mathematics education at my higher education institution using a self-reflective approach.

3.2. Characteristics of self-study

Although Tatto (2017, p. 622) advocated self-study by mathematics teacher educators, she cautioned that "reflective researchers are especially vulnerable to criticisms by policymakers" and if a bottom-up approach is pursued, then "to be taken seriously, their research needs to demonstrate rigour." In self-study, researchers consider the term "vigour" to be more appropriate than rigour to gauge research quality (Faulkner, 2019). This means that self-study research should rather be assessed in terms of, for example, worthiness, competence, enthusiasm, and robustness.

A criticism levelled at self-study is that it is naval-grazing, meaning that the researcher is self-absorbed and has a narrow outlook with a "limited desire to move [and] change" or relate to worldwide perspectives (Darian-Smith, 2023, p. 226). However, a defining characteristic of self-study methodology is that it is improvement-aimed (LaBoskey, 2004), which requires the researcher to actively pursue professional practice changes. Other self-study characteristics include being self-initiated and self-focused, being interactive, using mainly qualitative methods, and using authentically supported teaching practices (LaBoskey, 2004). Furthermore, self-study for improvement is usually facilitated through relearning, rethinking, and reimagining, using self-reflexivity within "discourses of the social construction of knowledge, reflective practice and action for social change" (Gu∂jónsdóttiri & Dalmau, 2006, p. 108).

3.3. Data sources and methods

University of KwaZulu-Natal's (UKZN, n.d.) *College of Humanities: Handbook for 2024* detailed information about the five mathematics education modules specified for pre-service teachers enrolled in the FP specialisation for the Bachelor of Education (BEd) curriculum. I copied information dispersed within the handbook, and tabulated the aims and contents of these modules to facilitate easy, aggregated data access. Thereafter, I used an interpretive method of close data reading to gain information about what is offered across these modules in relation to SCK, PCK, and beliefs. That was the first qualitative method utilised.

My second qualitative method was an arts-inspired strategy to provoke self-reflexivity. I selected a set of objects as a means of making sense of my learnt knowledge about the competences addressed in the five modules. These objects facilitated reflexivity to convey a "new perspective on a situation" by viewing a "concept from a target domain [primary concept] in terms of another, apparently dissimilar concept from a source domain [secondary concept]" (Akula et al., 2023, p. 23202). The target domain was the competences indicated in UKZN's primary teacher education curriculum and the source domain was my selected object set. Furthermore, an object's visual image serving as a metaphor is powerful in "conveying the metaphorical message" (Akula et al., 2023, p. 23202). Thus, I used the metaphor to convey my understanding of the importance of teaching and learning beliefs in mathematics education.

I adapted Samaras' (2011, p. 105) writing prompts to explain how my self-selected objects and associated metaphor expressed my learning in order to improve my professional practice as a primary mathematics teacher educator in our post-apartheid context. Thus, to elaborate on my metaphor reflection, I used the following four self-asked prompts to capture my professional learning after interpreting the offerings in UKZN's FP primary mathematics education modules.

- Prompt 1: Describe what metaphor you chose to capture the main idea of your research interest linked to the FP mathematics education curriculum at UKZN. Explain why you chose this metaphor. Give it a title that signifies the core meaning of your metaphor.
- Prompt 2: Indicate what in the metaphor represents you, the teacher educator, and what represents your professional learning.
- Prompt 3: Does culture play a role your metaphor?
- Prompt 4: Are there others involved in your metaphor memories? What roles do they play? What is their influence on your thinking? Do they see things the way you do? (van Laren & Mudaly, 2024).

3.4. FP mathematics education modules in UKZN's *Handbook for 2024*

UKZN's 4-year bachelor's degrees in education offer three specialised curricula: FP (Grades R–3), IP (Grades 4–6), and a combined senior phase (Grades 7–9) and further education and training (Grades 10–12). For each BEd specialisation, the required credit minimum is 512. An academic performance score (APS) of 31 points is the minimum for acceptance into Bed specialisations. For pre-service teachers specialising in the FP, UKZN offers two language options: English Home Language and isiZulu First Additional Language, or isiZulu Home Language and English First Additional Language.

Because of the importance of mathematics foundations, my interest lay in the primary mathematics education modules. I drew up a summary (see Figure 1) of the FP specialisation, indicating the minimum mathematics APS required for registration; number of credits; module study year, title, aim, content; and page number of the information in the UKZN (n.d.) handbook (see Figure 1).

Figure 1: *UKZN FP primary mathematics education*

FOUNDATION PHASE
For admission to specialization: Minimum Academic Performance Score on NSC for Mathematical Literacy Level 4 (50% – 59%) or Mathematics Level 3 (40% - 49%)
First year - Numeracy in the Early Years (16 Credits): **Aim:** •To introduce students to fundamental concepts of early mathematics to strengthen their basic mathematical knowledge. •To provide students with knowledge of number sense and how to deal with numbers in number relationships and in operations in the early years of schooling. To explore early geometric thinking, geometric concepts and to develop students' reasoning about space and shape in the early years of schooling. •To provide students with an understanding of the indigenous concepts of measurement; the measuring process and the use of mathematics vocabulary in the early years of schooling. • To equip students with knowledge of early data handling including collection, organisation, representing and interpreting data. **Content:** • Fundamental concepts of mathematics to strengthen students' mathematical knowledge. • Number sense and how to deal with numbers in number relationships and in operations. • Geometric thinking, geometric concepts and reasoning about space and shape. • The concepts of measurement; the measuring process and the use of mathematics vocabulary. • Data handling including collection, organisation, representing and interpreting of data. (UKZN, 2024, p. 427)
Second year - Mathematics Education for Foundation and Intermediate Phase 1 (16 Credits): **Aim:** The aim of this module is to provide a sound basis of the educational theories that underpin mathematics teaching in the Foundation Phase and Intermediate Phases, and in depth understanding of the mathematics content areas taught in the Foundation Phase and Intermediate Phases. **Content:** Introduction to theoretical basis of mathematics education: • Views on the nature of mathematics • Views on how children learn mathematics • Mathematical proficiency • Development of algebraic thinking Study of important and foundational mathematics education ideas applicable to Foundation and Intermediate Phase teaching of • Number • Operations • Patterns • Early algebra • Proportional reasoning (UKZN, 2024, pp. 408, 409)
Second year - Mathematics Education for Foundation and Intermediate Phase 2 (16 Credits): **Aim:** The aim of this module is to provide a sound basis of the educational theories that underpin mathematics teaching in the Foundation and Intermediate phases, and in depth understanding of the mathematics content areas taught in the Foundation and Intermediate phases. **Content:** Introduction to theoretical basis of mathematics education: • Development of geometric thinking, and theories of learning geometry • Views on how children learn measurement concepts • Development of statistical literacy • Proportional reasoning • Study of important and foundational mathematics education ideas applicable to Foundation and Intermediate Phase teaching of • Shape and Space • Measurement. (UKZN, 2024, p. 409)
Third year - Mathematics for Foundation Phase Method 1 (16 Credits): **Aim:** • To provide students with theoretical understanding of how Foundation Phase Learners learn Mathematics • To empower students to plan and teach the Mathematics curriculum in the Foundation Phase (Grades R-3) • To introduce students to various approaches for teaching Foundation Phase learners: - Number sense and number relationships - Patterns and Algebra - Geometry (Space and Shape) - Data handling. • To equip students with knowledge and strategies of assessing Mathematics in the Foundation Phase. • To provide students with knowledge and skills to develop Foundation Phase Mathematics resources. **Content:** •Theoretical understanding of how Foundation Phase Learners learn Mathematics • Planning and teaching Mathematics curriculum in the Foundation Phase (Grades R-3) • A variety of approaches for teaching: - Number sense and relationships - Patterns and Algebra - Geometry (Space and Shape) - Data Handling. • Knowledge and strategies of assessing Mathematics in the Foundation Phase. • Knowledge and skills of developing Foundation Phase Mathematics resources. (UKZN, 2024, p. 430)
Third year - Mathematics for Foundation Phase Method 2 (16 Credits): **Aim:** • To develop students' comprehensive theoretical understanding of how Foundation Phase learners learn Mathematics. • To empower students to plan and teach the Mathematics curriculum in the Foundation Phase (Grades R-3). • To equip students with knowledge of critiquing observed mathematics through theoretical lenses • To provide students with a variety of strategies for teaching: - Number relationships and place values - Geometry (Space and Shape) - Measurement. • To equip students with knowledge to identify and use relevant assessment strategies for Mathematics in the Foundation Phase. • To develop students' in-depth understanding of preparing and designing resources for Mathematics in the Foundation Phase. • To empower students with knowledge of identifying barriers and applying intervention strategies to learning Mathematics in the Foundation Phase. **Content:** • Theoretical concepts about how Foundation Phase learners learn Mathematics. • Mathematics CAPS curriculum in the Foundation Phase (Grades R-3). • A variety of approaches for teaching: - Number relationships and place values - Geometry (Space and Shape) - Measurement. • Knowledge of identifying and using relevant assessment strategies for Mathematics in the Foundation Phase. • An in-depth understanding of preparing and designing resources for Mathematics in the Foundation Phase. • Knowledge of identifying barriers and applying intervention strategies to learning Mathematics in the Foundation Phase. (UKZN, 2024, p. 431)

3.5. Analysis of FP primary mathematics education

This qualitative analysis focuses on competences offered across the modules in the FP. Thereafter I explain the self-selected set of objects' metaphor I chose to express my professional learning gained by exploring FP UKZN primary mathematics education. Then I used this metaphor to assist in answering my research question.

The South African National Senior Certificate achievement levels determine the FP BEd admission. To register for this phase, pre-service teachers require a minimum of 40–49% in mathematics or 50–59% in mathematical literacy. The total number of credits allocated to mathematics education modules is 80.

The first-year Numeracy in the Early Years module concentrates on FP SCK to "strengthen their basic mathematical knowledge" (UKZN, n.d., p. 427). The study topics to reinforce pre-service teachers' mathematics knowledge include number sense development, number relationships, number operations, geometric thinking, mathematics vocabulary, indigenous measurement concepts, measurement, and data handling. The emphasis is on developing pre-service teachers' SCK essential for teaching young mathematics learners.

The two second-year modules Mathematics Education for Foundation and Intermediate Phase 1, and Mathematics Education for Foundation and Intermediate Phase 2, attend to PCK using educational theories that pertain to "mathematics content areas" (UKZN, n.d., pp. 408–409) taught in the FP and IP. In the former module, FP and IP PCK for the teaching of early algebraic thinking, number operations, patterns, and proportional reasoning are included. In the latter module, FP and IP PCK for development of geometric thinking, measurement concepts, statistical literacy, and proportional reasoning are covered.

The two third-year modules, Mathematics for Foundation Phase Method 1 and Mathematics for Foundation Phase Method 2, focus on FP PCK utilising theoretical understanding of how FP "learners learn mathematics" for planning to teach the curriculum using a variety of approaches (UKZN, n.d., pp. 430–431). In the former module, FP PCK for teaching number sense and relationships, patterns and algebra, geometry, and data handling are offered. Other topics in this module include strategies for assessment and resource development. In the latter module, FP PCK focuses on teaching number relationships and place, value, geometry, and measurement. Additional topics explored are assessment methods, resource development, and identification of barriers to learning mathematics together with intervention strategies.

My analysis showed that 20% of this total number of credits is allocated to FP SCK, 40% to combined FP and IP PCK, and 40% to FP PCK. The approximate credit percentage of mathematics education modules in the FP specialisation is 16% of the total number of credits (512) required for the BEd degree.

4. My Professional Learning Gained: A Self-Reflexive Metaphor

The professional learning I gleaned from my analysis of the competencies offered in primary mathematics education modules led me to recognise possibilities for improving my teacher education practice in the post-apartheid context. I selected objects related to "lighting a candle" (see Figure 2) to illustrate my learning, to convey improvement possibilities for UKZN, and to answer my research question. I used self-asked writing prompts to interrogate my metaphor.

Figure 2: *Objects to light a candle*

4.1 Prompt 1

I chose candleholders, wax candles, and a box of matches as objects for my metaphor because these are familiar items found in most South African homes. The main function of this object set is to shed light in the darkness. The two candleholders are identical and made of sturdy, substantial, opaque metal. For stability, the base of the holder is wider than its apex where the candle is secured. The holder keeps the candle vertical to prevent it from tipping, thus ensuring that it can safely and effectively provide light without being a fire hazard. The candle on the left-hand side of the image is broken into two pieces. The wick—a usually unseen yet a vital component of a candle— is embedded within it, running through the entire centre of the cylindrical wax column connecting the two broken pieces. This candle will not be effective when the wick at the candle tip is set alight. The intact candle will be effective when the candlewick is set alight. The box of matches is an essential requirement for lighting the wick to make a flame to illuminate darkness.

Although a candle's primary function is to shed light in darkness, there are many other symbolisms associated with a candle and candlelight. Candle flames are a symbol of enlightenment, clarity, knowledge, wisdom, optimism, hope, truth searching, anticipating new beginnings, and joy (Axiom Home, 2023). For example, the birthday tradition of making a wish after blowing out all birthday cake candles facilitates positive, joyful, personal reflection. Furthermore, the proverb "It is better to light a candle than to curse the darkness" emphasises the need for constructive action when faced with bleak circumstances. An appropriate title for my object set was thus "Igniting Positive Inner Illumination."

4.2 Prompt 2

I consider the candles to be the competences of a FP pre-service mathematics education teacher. In the damaged candle, the bottom piece of candle is the mathematics SCK offered in mathematics education modules and the top piece is the mathematics PCK covered in these modules. The wick, which is an essential yet almost invisible, overlooked element of the candle, is the pre-service teacher's mathematics teaching and learning beliefs. These beliefs form an integral part of their competence because it connects, and is central to, their SCK and PCK. Although beliefs and affect, in general, are abstract and difficult to pinpoint or physically observe, it is possible to interrogate and reflect on beliefs.

The ignited match is me, the mathematics teacher educator, and the box contains my current professional beliefs.

The opaque candleholder is the SCK the pre-service acquired as a school learner. The opaque, dense colour is fitting because the mathematics teaching and learning in many post-apartheid school classrooms may still feel the influences of unequal, unjust apartheid circumstances that may not always be observable and transparent. The shape of the candleholder shows the importance of having a stable, firm foundation to successfully support the candle—pre-service teachers' competences required for effective mathematics teaching.

The invisible oxygen surrounding the objects is the pre-service teacher's personal experiences of teaching and learning in post-apartheid primary and secondary school mathematics classrooms.

The flame is the pre-service teacher who enlightens by effective teaching.

To make the intact candle's flame effective, there are three essential components: fuel, oxygen, and heat. The fuel is the pre-service teacher's mathematics SCK, mathematics PCK, and belief systems influenced by the modules presented in the BEd programme that makes up the candle wax. The oxygen is the pre-service teacher's own relationships with mathematics influenced by 12 years of schooling in a post-apartheid context, which is not visible. The heat to ignite the flame is the mathematics teacher educator who presents and develops appropriate knowledge, skills, and beliefs in university lecture halls and during practice teaching. In other words, the candle's flame illuminates because of action taken by me, the mathematics teacher educator lecturing mathematics education content to merge mathematics PCK, SCK, and mathematics beliefs. Therefore, it is important that I attend to the pre-service teacher's hidden, yet significant, beliefs to benefit their prospective mathematics teaching.

4.3 Prompt 3

My culture is my way of life. As a South African who has lived and taught as a mathematics teacher and mathematics teacher educator in both apartheid and post-apartheid contexts it is important that I embrace the proverb: "It is better to light a candle than to curse the darkness."

In terms of lighting a candle, I realise that it is possible to take constructive action through positive, hopeful, optimistic, self-improvement endeavours to confront social justice issues using a starting-with-ourselves approach.

However, it is impossible and imprudent to turn a blind eye to social injustices underlying teacher development in mathematics education when taking action to support and encourage appropriate mathematics education belief reflections. Balfour (quoted in Naidu, 2024, para. 26) noted that the consequences of apartheid still impact and influence one's "experience of how you learn, how you teach, and what your chances are of succeeding." In other words, lingering social, political, and educational inequalities related to, for example, "access to resources, historical legacies of advantage, [and] historical legacies of disadvantage" (Balfour, quoted in Naidu, 2024, para. 26) may still haunt many South African learners, pre-service teachers, teachers, teacher educators, and others.

Looking inwards, by analysing my own higher education institution's modules, gave me opportunity to explore, reflect, and look forward to envisaging my professional practice improvements as a primary mathematics teacher educator in a post-apartheid context, but has also made me mindful of looking backwards. I need to take cognisance of pre-service teachers' past schooling experiences, where they may have developed discouraging beliefs about mathematics learning and teaching. Past beliefs are of particular importance for prospective FP schoolteachers because these generalists may not have anticipated having to teach mathematics for approximately one third of their practice.

Furthermore, looking inwards enabled looking outwards. This outward looking facilitated consciousness of possible alternative pre-service teacher experiences vastly different from my privileged opportunities. In addition to opening possibilities of modelling reflective practice (Sweeney et al., 2023) in teacher education, thinking outward after self-reflection made me realise how my mathematics teacher educator practice can make a difference in the teaching and learning of pre-service teacher education. Being a life-long learner and a reflective teacher is an educator requirement but as a mathematics teacher educator, these approaches support my professional practice by understanding my own PCK and beliefs, which are grounded in my post-apartheid way of life.

4.4 Prompt 4

The many primary pre-service teachers enrolled in mathematics education modules are in my metaphor memories. FP pre-service teachers are destined to become generalist teachers with vital roles to play in laying mathematics foundations in primary schools. Some generalist teachers may not be passionate about, or have personal confidence in, mathematics.

Looking back, I remember a tutorial activity that we, a team of UKZN primary mathematics educators, once used. We invited primary pre-service mathematics teachers, enrolled in their first mathematics education module, to share their mathematics school learning and teaching memories. I recall being perturbed and shocked by the openness of some who declared that they "hated" mathematics. As a mathematics education team, we did not anticipate this disadvantageous response and did not address the issue adequately because we were unsure how to support those pre-service teachers in overcoming their strong emotions. However, if prospective teachers harbour such intense unfavourable feelings about mathematics, then it is possible that they could adversely influence their young impressionable learners' relationship with mathematics whilst laying vital, influential mathematics foundations.

4.5 Found poem

To summarise and condense my prompt reflections, and to move towards answering my research question, I created a found poem from pertinent reflection words and phrases (Butler-Kisber, 2010) in a shape that visually resembled a flaming candle (see Figure 3). I chose this shape because of the many everyday symbolisms associated with a flaming candle and because, in my metaphor, the candle flame is the pre-service teacher.

Figure 3: *Summary of pertinent prompt reflections*

fuel, oxygen, heat
Igniting positive inner illumination
better to light a candle than to curse the darkness
heat to ignite flame is mathematics teacher educator
fuel is pre-service teacher's SCK, PCK and belief systems
flame is pre-service teacher that enlightens by effective teaching
oxygen is pre-service teacher's relationships with mathematics
influenced by 12 years schooling in post-apartheid context
may harbor intense unfavorable feelings
beliefs difficult to physically observe
confronting social justice issues
invisible overlooked
constructive action
looking backwards
looking outwards
looking inwards
look forward

5. Answering Research Question, Gazing Forward and Outward

By analysing UKZN's FP enrolment requirements, language focus, and specialisation subject packages I have learnt that UKZN is addressing unfavourable comments (van der Berg & Hofmeyr, 2018) aimed at South African teacher education institutions. The UKZN School of Education enrols pre-service teachers with an APS higher than the minimum UKZN requirement of 28 points. Moreover, in the FP specialisation the importance of mathematics is recognised because a pass in NSC mathematics or mathematical literacy are prerequisites for enrolment. This indicates that school mathematics SCK achievement is an important consideration for improving FP teacher education. However, school SCK achievement does not necessarily provide information about learners' mathematics beliefs developed during their 12 years of schooling.

In addition, I realised that substantial progress is being made in building and strengthening language and pedagogical skills in isiZulu. UKZN School of Education's efforts in providing teacher education in English and isiZulu at university level is, according to Hlongwa (quoted in Greenleaf Walker, 2021, p. 1) a "linguistic revolution," particularly in the post-apartheid teacher education context.

My module analysis showed that at least three quarters of the UKZN module content is devoted to educational theory-based PCK in the FP specialisation. In the mathematics education modules, the SCK and PCK development appear to be taught separately, which according to Manouchehri (1995), influences the pre-service teachers' beliefs because they may consider these knowledges to be unrelated and disconnected. It is thus necessary to model what is advocated in PCK modules in mathematics SCK modules. For example, emphasise in mathematics SCK assessment that awarding process marks for meaningful use of problem-solving processes deserves acknowledgement.

Furthermore, in FP primary mathematics education modules, no mention or exploration of the central influence of affect is explicitly stated. This omission suggests that the significance of the vital links between cognition, emotions, and beliefs in mathematics may have been overlooked. In other words, disregard for prior beliefs of primary mathematics pre-service teachers is an aspect that is not given sufficient consideration (Olawale, 2024). Perhaps affect is explored in other UKZN teacher education disciplines but beliefs and emotions are noteworthy in relation to mathematics education. Affect is relevant in mathematics teaching and learning because of possible strong, adverse emotional reactions associated with mathematics that have far reaching cognitive influences.

However, there are possibilities within the existing mathematics education modules to allow for inclusion of mathematics teaching and learning beliefs. For example, exploration of pre-service teachers' beliefs could be comfortably situated in the third- year FP module where "knowledge of identifying barriers" (UKZN, n.d., p. 431) is covered. Here the pre-service teachers could explore their beliefs as possible barriers. However, this means that pre-service teachers' beliefs would only be addressed in their last year of mathematics education. It would be beneficial to include these beliefs in an earlier mathematics education module so that they can benefit from their learning over an extended period to develop refreshed, new, additional, or alternate beliefs. Other possibilities for inclusion of personal mathematics teaching and learning beliefs could be effectively addressed during pre-service teachers' annual practice sessions when they teach mathematics in schools.

Whilst exploring personal mathematics teaching and learning beliefs, pre-service teachers would benefit from using metaphor drawings to assist in their belief reflection and development in primary mathematics education. This arts-inspired method of metaphor drawing is well documented in mathematics education teaching and learning (see, for example, Hobden, 1999; van Laren, 2007). By making use of personal hand-drawn metaphors to explore their own beliefs, where there are no correct or incorrect responses, primary pre-service teachers can be offered opportunities to incorporate additional, novel, creative self-reflexive self-study methods. Reflection possibilities for rethinking, reinventing, or revising personal beliefs about teaching and learning foundational mathematics provides agency and insight into this key yet overlooked component of mathematics education.

Mathematics teacher educators may also enjoy and benefit from self-reflexive methods, and model what is required of pre-service teachers who need to explore their mathematics teaching and learning beliefs. Discussions and dialogue amongst mathematics teacher educators and pre-service teachers could benefit this central influence on the learning of mathematics SCK and PCK. As Olawale (2024, para. 6) noted, "limited effectiveness of training [pre-service teachers] might be attributed" to the beliefs about mathematics teaching and learning that are inculcated during their 12 years of foundational school mathematics, post-apartheid experiences.

5.1 What is the Relevance of my Self-Study?

As a mathematics teacher educator, I acknowledge my prior privileged learning influences but, as a self-study researcher, I am committed to positively lighting the candle. My use of a metaphor to reflect on the central importance of pre-service teachers' personal mathematics teaching and learning beliefs (candlewick) facilitated my professional learning as a teacher educator (igniting match) by highlighting my role in preparing pre-service teachers (flame) for effective mathematics teaching (illuminating flame). This study made me more aware of the daunting responsibility of being a primary mathematics teacher educator in the post-apartheid context, how influential my professional beliefs (matchbox) are, and how these beliefs could inadvertently be conveyed to pre-service mathematics teachers. The challenging task of remedying the overlooking of mathematics beliefs provided an opportunity to do something about the social injustices many pre-service teachers may have endured during their mathematics schooling.

I used a self-study approach to learn more about what FP pre-service teachers are offered in mathematics education at my higher education institution because these teachers will lay the foundations for mathematics learning in their prospective generalist teaching. My inward-looking professional learning facilitated the envisioning of positive, constructive, workable improvement possibilities for attending to the influential, under-explored, mathematics teaching and learning beliefs of pre-service primary mathematics teachers in our post-apartheid context. I offer and make public my findings for scrutiny by the mathematics teacher education community, and hope to pave the way for other starting-with-ourselves reflective studies that envisage sustainable self-improvement possibilities in primary mathematics teacher education.

6. Conclusion

My study explored what is offered to generalist FP pre-service teachers in mathematics education modules at the higher education institution where I am a mathematics teacher educator. Module content presented in UKZ-N's *Handbook for 2024*, suggested that pre-service teachers' belief awareness in mathematics teaching and learning was not covered. Given that they are prospective teachers in post-apartheid South Africa, overlooking pre-service teachers' beliefs ignores how they might perceive the nature/vision of mathematics, how they experienced learning mathematics in schools, and what their approach to teaching mathematics may be. Furthermore, this means that the important links between cognition and beliefs in mathematics and mathematics education were underestimated.

Using a qualitative, reflexive, constructive, optimistic starting-with-ourselves approach, I sought possibilities to improve what is offered to FP pre-service teachers because they will influence young learners' beliefs, which are significant for their subsequent mathematics learning. The study thus offers self-improvement possibilities for the integration and incorporation of belief awareness in existing modules offered to FP mathematics pre-service teachers at UKZN. It concludes that self-reflexive re-examination and re-evaluation of what is offered in one's own mathematics education programmes allows for constructive rethinking and self-improvement possibilities to make a difference in mathematics pre-service teacher education.

7. Declarations

Funding: This research did not receive any external funding.

Conflicts of Interest: The author declares no conflict of interest.

References

Akula, A. R., Driscoll, B., Narayana, P., Changpinyo, S., Jia, Z., Damle, S., Pruthi, G., Basu, S., Guibas, L., Freeman, W. T., Li, Y., & Jampani, V. (2023, June 17–24). *MetacLUE: Towards comprehensive visual metaphors research*. IEEE/CVF conference on computer vision and pattern recognition (pp. 23201–23211), Vancouver, Canada. https://doi.org/10.1109/CVPR52729.2023

Axiom Home. (2023). *Candle symbolism unveiled: Discover the mystical meanings associated with candles.* https://tinyurl.com/5dnt6j64

Beckmann, J. (2021). International involvement and education in South Africa: From hope to disenchantment. In N. Popov, C. Wolhuter, L. de Beer, G. Hilton, J. Ogunleye, E. Achinewhu-Nworgu, & E. Niemczyk (Eds.), *New challenges to education: Lessons from around the world* (pp. 45–52). Bulgarian Comparative Education Society.

Butler-Kisber, L. (2010). *Qualitative inquiry: Thematic, narrative and arts-informed perspectives*. SAGE.

Darian-Smith, E. (2023). Academic navel-gazing. In R. Benedikter, M. B. Steger, H. Pechlaner, & I. Kofler (Eds.), *Globalization: Past, present, future* (pp. 222–241). University of California Press.

Department of Basic Education. (2011). *National curriculum statement Grades R–12 (January 2012).*

Department of Education. (2006). *The national policy framework for teacher education and development in South Africa*. https://tinyurl.com/58srt9sf

Department of Higher Education and Training. (2015). *National Qualifications Framework Act: Revised policy on the minimum requirements for teacher education qualifications* (Government Gazette Vol. 596, No. 38487). https://tinyurl.com/bdcjr5ey

Di Martino, P. (2024). Attitudes in mathematics education. In J. Wang (Ed.), *Proceedings of the 14th International Congress on Mathematical Education: Volume II: Invited lectures* (pp. 427–436). East China Normal University Press & World Scientific Publishing. https://doi.org/10.1142/13700-vol2

Di Martino, P., & Zan, R. (2011). Attitude towards mathematics: A bridge between beliefs and emotions. *ZDM-Mathematics Education*, *43*, 471–482. http://dx.doi.org/10.1007/s11858-011-0309-6

Faulkner, S. L. (2019). *Poetic inquiry: Craft, method and practice*. Routledge.

Greenleaf Walker, J. (2021, October, 4). The use of isiZulu at university level is a linguistic revolution. *Universities South Africa*. https://usaf.ac.za/the-use-of-isizulu-at-a-university-level-is-a-linguistic-revolution/

Griffiths, M. (2003). *Action for social justice in education: Fairly different*. Open University Press.

Guðjónsdóttiri, H., & Dalmau, M. C. (2006). Parallel engagements: Self-study of active group practice with educators. In L. Fitzgerald, M. Heston, & D. Tidwell (Eds.), *Collaboration and community: Pushing boundaries through self-study. Proceedings of the 6th International Conference on Self-Study of Teacher Education Practices, Herstmonceux Castle, England* (pp. 108-112). University of Northern Iowa.

Herheim, R. (2023). On the origin, characteristics, and usefulness of instrumental and relational understanding. *Educational Studies in Mathematics, 113,* 389–404. https://doi.org/10.1007/s10649-023-10225-0

Hlatshwayo, P. (2023). The indelible Verwoerd's edicts: A critical review of their indirect impact on South African basic mathematics education reforms. *African Perspectives of Research in Teaching & Learning, 7*(2), 387–398.

Hobden, S. D. (1999). *The beliefs of preservice teachers about mathematics teaching and learning* [Unpublished master's dissertation, University of Natal] South Africa.

LaBoskey, V. K. (2004). The methodology of self-study and its theoretical underpinnings. In J. J. Loughran, M. L. Hamilton, V. K. LaBoskey, & T. Russell (Eds.), *International handbook of self-study of teaching and teacher education practices* (Part 2, pp. 817–869). Kluwer.

Lénárt, I., Barmby, P., Brodie, K., Graven, M., Hahne, Lerman, S., Mosimege, M., Olivier, W., Roberts, N., Rybak, A., Webb, P., & N. Roberts (2022). *The pedagogy of mathematics in South Africa: Is there a unifying logic?* Mapungubwe Institute for Strategic Reflection. https://mistra.org.za/mistra-research-projects/the-pedagogy-of-mathematics-in-south-africa/

Manouchehri, A. (1995, June). *Theory and practice: Implications for mathematics teacher education programs* [Paper presentation]. Sixth National Forum Association of Independent Liberal Arts Colleges for Teacher Education, St. Louis, Missouri. https://files.eric.ed.gov/fulltext/ED460933.pdf

Mazana, M. Y., Montero, C. S., & Casmir, R. O. (2019). Investigating students' attitude towards learning mathematics. *International Electronic Journal of Mathematics Education, 14*(1), 207–231. https://doi.org/10.29333/iejme/3997

Naidu, E. (2024, September 26). *Incoming vice-chancellor reflects on transformation's lessons*. University World News. https://www.universityworldnews.com/post.php?story=20240924131547479

Olawale, B. E. (2024). Impact of pre-service teacher education programme on mathematics student teachers' teaching practices during school experiences. *Education Sciences*, *14*(7), 762. https://doi.org/10.3390/educsci14070762

Russell, T. (2005). Can reflective practice be taught?. *Reflective Practice*, *6*(2), 199–204. https://tinyurl.com/4mrhtapc

Samaras, A. P. (2011). *Self-study teacher research: Improving your practice through collaborative inquiry*. Sage Publications.

Shulman, L. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review*, *57*(1), 1–23.

Sweeney, T., Bowles, R., & Coulter, M. (2023). Looking back to move forward identifying value in collaborative self-study. In A. Cameron-Standerford, B. Bergh, & C. U. Edge (Eds.), *Pausing at the threshold: Self-study in teacher education* (pp. 609–618). Equity Press.

Tatto, M. T. (2017). The role of comparative and international research in developing capacity to study and improve teacher education. In M. A. Peters, B. Cowie, & I. Menter (Eds.), *A companion to research in teacher education* (pp. 621–636). Springer. http://dx.doi.org/10.1007/978-981-10-4075-7_42

Tatto, M. T. (2018). The mathematical education of primary teachers. In M. T. Tatto, M. Rodriguez, W. Smith, M. Reckase, & K. Bankov (Eds.), *Exploring the mathematics education of teachers using TEDS-M data* (pp. 205–256). Springer. https://link.springer.com/book/10.1007/978-3-319-92144-0

Tatto, M. T., Schwille, J., Senk, S. L., Ingvarson, L., Rowley, G., Peck, R., Bankov, K, Rodriguez, M. C., & Reckase, M. (2012). *Policy, practice, and readiness to teach primary and secondary mathematics in 17 countries: Findings from the IEA Teacher Education and Development Study in Mathematics*. International Association for the Evaluation of Educational Achievement.

Thompson, D. (2024, July 30). "I want to change what happens," says legendary researcher. *University World News.*

University of KwaZulu-Natal. (n.d.). College of Humanities: Handbook for 2024. https://saa.ukzn.ac.za/wp-content/uploads/2024/02/Humanities-Handbook-Final-Draft-signed-Copy-Mbali-002-For-Printing2.pdf

Valero, P. (2013). Mathematics for all and the promise of a bright future. In B. Ubuz, Ç. Haser, & M. A. Mariotti (Eds.), *Proceedings of the Eighth Congress of the European Society for Research in Mathematics Education* (pp. 1–10). European Society for Research in Mathematics Education.

Van der Berg, S., & Hofmeyr, H. (2018). *Education in South Africa.* World Bank. https://documents1.worldbank.org/curated/en/339291529320964248/pdf/127304-Education-in-South-Africa.pdf

van Laren, L. (2007). Using metaphors for integrating HIV and AIDS education in mathematics curriculum in pre-service teacher education: An exploratory classroom study. *International Journal of Inclusive Education*, *11*(4), 461–479. https://doi.org/10.1080/13603110701391451 van Laren, L., & Mudaly, R. (2024). Collaboratively stepping back as a method in an arts-inspired self-study to move forward in professional learning for social change. *International Journal of Qualitative Methods, 23*, 1–13. https://doi.org/10.1177/16094069241236218

van Manen, M. (1990). *Researching lived experience: Human science for an action sensitive pedagogy*. SUNY Press.

Wills, I. R. (2011). *The history of Bantu Education: 1948–1994* [Unpublished doctoral dissertation, Australian Catholic University] Australia. https://tinyurl.com/5n6kjr4f

Zhang, Q., & Morselli, F. (2016). Summary and looking ahead. In G. Kaiser (Ed.), *Attitudes, beliefs, motivation and identity in mathematics education: An overview of the field and future directions* (pp. 23–26). Springer.

CHAPTER NINE

Equipping Pre-service Mathematics Teachers for Diverse Classrooms: Best Practices and Innovations

Felix Egara[1*]
Moeketsi Mosia[2]

AFFILIATIONS
[1&2]Faculty of Education, University of the Free State, Bloemfontein, South Africa.

CORRESPONDENCE
Email: felix.egara@unn.edu.ng*

Reference Formats (APA 7th Edition)
Egara, F., & Mosia, M. (2025). Equipping pre-service mathematics teachers for diverse classrooms: Best practices and innovations. In B. E. Olawale (Ed.), *Building the Foundations: Effective Approaches in Mathematics Teacher Preparation* (pp. 138-152). ERRCD Forum. https://doi.org/10.38140/obp3-2025-09

Abstract: This chapter explores innovative approaches and best practices in preparing mathematics teachers for diverse classrooms by employing four key theoretical frameworks: Culturally Relevant Pedagogy, Culturally Responsive Teaching, Differentiated Instruction, and Universal Design for Learning. These frameworks serve as guiding principles for equipping pre-service educators to effectively address the diverse cultural, academic, and learning needs in today's classrooms. The study employed a comprehensive review of literature, focusing on empirical studies and practical implementations of these inclusive pedagogies in mathematics education. The analysis examined how teacher preparation programmes integrate these frameworks into their curricula to foster equity, inclusivity, and academic excellence. The chapter also discusses challenges encountered in the implementation of inclusive pedagogies, such as resistance to pedagogical shifts and limitations in teacher training programmes. To address these challenges, recommendations are provided for enhancing teacher preparation, including the incorporation of cultural competence training, differentiated instructional strategies, and universally designed learning environments. The findings suggest that these pedagogical frameworks, when implemented effectively, lead to improved student engagement, achievement, and a greater sense of belonging among learners from diverse backgrounds. By aligning teacher preparation programmes with CRP, CRT, DI, and UDL, this chapter emphasises the need for mathematics educators to adopt flexible, responsive teaching practices that cater to the needs of all students, ultimately fostering a more equitable learning environment in diverse classrooms.

Keywords: Cultural competence, diversity in education, inclusive pedagogy, mathematics teacher preparation, teacher training innovations.

1. Introduction

Classroom diversity has become a central issue in today's rapidly evolving educational landscape, particularly in mathematics education. Increasingly, classrooms comprise students from various cultural, linguistic, and socioeconomic backgrounds. This diversity poses challenges and opportunities for mathematics teachers, who must be equipped with the skills to meet the needs of all learners (Mahlambi, 2023). In such environments, the success of mathematics education mainly rests on how well programmes for preparing teachers address concerns of inclusion, equity, and diversity (Faragher et al., 2016). However, despite ongoing reforms in education, significant gaps remain in how preservice teachers are trained to handle the demands of diverse classrooms (Weiss et al., 2024).

Teacher preparation programmes provide prospective mathematics educators with content knowledge and the pedagogical skills to deliver that knowledge in ways that resonate with diverse learners. However, many programmes still focus on traditional approaches, with insufficient emphasis on cultural competence and inclusive pedagogies (Eden et al., 2024; Olawale, 2022). Cultural competence is crucial for ensuring that teachers are aware of their students' varied cultural contexts and are prepared to adjust their instructional strategies accordingly (Karatas, 2020). Inclusive pedagogy, which seeks to accommodate all learners, especially those from marginalised communities, is vital for effective teaching in diverse classrooms (Sanger, 2020).

An increasing amount of scholarly literature indicates that creative and comprehensive methods of preparing mathematics teachers are required (Cochran-Smith & Reagan, 2021). Some promising developments include integrating culturally responsive teaching practices into the curriculum and providing preservice teachers with opportunities to engage in reflective practice and real-world classroom experiences (Anyichie, 2024). Additionally, emerging technologies and collaborative models are beginning to reshape teacher training, creating more dynamic and interactive learning environments (Haleem et al., 2022; Olawale, 2024; Olawale & Hendricks, 2024). Despite these advances, many challenges persist. For instance, research indicates that teacher preparation programmes often fail to offer adequate mentorship and field experiences that reflect the diversity of modern classrooms (Olawale, 2024; Mbhiza et al., 2024). Moreover, implicit biases and systemic barriers can limit the effectiveness of even the most well-intentioned training initiatives (FitzGerald et al., 2019).

This chapter addresses these gaps by exploring best practices and innovative approaches to equipping preservice mathematics teachers for diverse classrooms. It will examine how teacher preparation programmes can be restructured to better integrate cultural competence, inclusive pedagogies, and differentiated instruction, thus fostering teacher effectiveness and improving student outcomes in diverse educational settings. Through theoretical insights and practical examples, this chapter aims to provide educators, researchers, and policymakers with valuable strategies for enhancing mathematics teacher education in an increasingly multicultural world.

2. Theoretical Insights

This section draws on four vital theoretical frameworks to explore best practices and innovations in preparing mathematics teachers for diverse classrooms: Culturally Relevant Pedagogy (Ladson-Billings, 1995), Culturally Responsive Teaching (Gay, 2000), Differentiated Instruction (Tomlinson, 2001), and Universal Design for Learning (CAST, 2008). Each framework contributes to understanding how teacher preparation programmes can equip educators with the tools to address the cultural, academic, and learning diversity present in today's classrooms.

2.1 Culturally relevant pedagogy and culturally responsive teaching

Culturally Relevant Pedagogy (CRP), introduced by Gloria Ladson-Billings in 1995, and Culturally Responsive Teaching (CRT), developed by Geneva Gay in 2000, provide foundational frameworks for addressing the diverse needs of students in mathematics education. CRP emphasises leveraging students' cultural backgrounds to enhance academic success, while CRT builds on this by focusing on practical teaching strategies that integrate students' cultural identities into classroom practices and learning environments. Both CRP and CRT share three core principles: (1) Promoting academic success while valuing students' cultural assets – recognising students' cultural identities as a resource to support their academic growth; (2) Developing cultural competence – encouraging teachers and students to learn about and appreciate each other's cultural experiences; (3) Fostering critical consciousness – empowering students to understand and challenge social inequalities within and beyond the classroom. In the context of mathematics teacher preparation, CRP and CRT highlight the importance of equipping educators with the skills to acknowledge and incorporate students' cultural experiences into the teaching process. Mathematics is often perceived as abstract and culturally neutral; however, culturally responsive teaching counters this notion by using culturally relevant examples, connecting mathematical content to real-world contexts, and promoting collaborative learning strategies reflective of diverse cultural norms (Gay, 2022). Effective teacher preparation programmes must incorporate training on cultural competence, which includes understanding students' cultural backgrounds and applying this knowledge to develop lesson plans, manage classrooms, and design assessments. Such training helps educators create learning environments that foster inclusivity and engagement. Evidence suggests that implementing CRP and CRT improves student achievement and engagement in mathematics, making these frameworks essential components of mathematics teacher preparation programmes.

2.2 Differentiated Instruction

Differentiated Instruction (DI), developed by Carol Ann Tomlinson (2001), is a pedagogical approach designed to address the diverse needs, abilities, and learning preferences of students within a single classroom. It is grounded in the principle that effective teaching requires adjustments to the content (what students learn), process (how students learn), and product (how students demonstrate their learning). These adjustments enable teachers to meet the unique learning profiles of individual students while maintaining high academic expectations for all. DI emphasises the importance of ongoing assessment, which allows teachers to identify and respond to variations in students' readiness, interests, and learning styles. For instance, in mathematics education, teachers can use pre-assessments to gauge students' proficiency levels and then tailor lessons to provide appropriate levels of challenge. Strategies may include offering tiered assignments, flexible grouping, and scaffolding to ensure students receive the support they need to grasp mathematical concepts.

This framework aligns with the principles of Culturally Relevant Pedagogy (CRP) by fostering inclusivity and equity. It enables teachers to integrate students' cultural and linguistic backgrounds into differentiated tasks, making learning experiences both accessible and meaningful. For example, incorporating real-world problems that reflect students' lived experiences can make abstract mathematical concepts more relatable. Teacher preparation programmes play a critical role in equipping educators with the knowledge and skills needed to implement DI effectively. Coursework should emphasise the principles of differentiation, including strategies for modifying instruction and managing diverse classrooms. Field experiences should provide preservice teachers with opportunities to practise these techniques under the guidance of experienced mentors (Olawale, 2024). Additionally, professional development workshops focused on DI can further support teachers in refining their practices. By integrating Differentiated Instruction into teacher preparation programmes, educators are better equipped to navigate the complexities of diverse classrooms. This approach ensures that all students, regardless of their background or starting point, have the opportunity to succeed in mathematics.

2.3 Universal design for learning

Universal Design for Learning (UDL), developed by the Centre for Applied Special Technology (CAST, 2008), is a framework designed to ensure that all students have equitable access to learning opportunities. UDL emphasises the proactive design of curriculum, instruction, and assessments to address the diverse needs of learners. Its foundation rests on three guiding principles: multiple means of engagement, multiple means of representation, and multiple means of action and expression (Meyer et al., 2014). These principles collectively support the creation of inclusive learning environments.

Multiple Means of Engagement: This principle focuses on stimulating students' interest and motivation to learn by offering options for how they interact with the material. In mathematics education, this might include real-world problem-solving scenarios that relate to students' cultural and social contexts or gamified learning experiences that maintain student engagement. Teachers can use tools such as interactive platforms or culturally relevant examples to cater to diverse student interests and sustain attention.

Multiple Means of Representation: Recognising that students process information in different ways, this principle encourages the use of varied methods to present mathematical concepts. Visual aids, hands-on manipulatives, and digital simulations can help demystify abstract ideas for students who struggle with traditional instructional approaches. For instance, using graphs, videos, or multilingual resources ensures that learners with linguistic or cognitive differences can access and comprehend mathematical content.

Multiple Means of Action and Expression: This principle emphasises providing students with diverse ways to demonstrate their understanding. In mathematics, this might involve allowing students to solve problems through oral explanations, written solutions, or visual representations, such as diagrams or charts. Offering flexibility in assessments ensures that all students, including those with disabilities or language barriers, can showcase their learning effectively.

Integrating UDL into mathematics teacher preparation is essential for fostering inclusive practices. Teacher preparation programmes must include courses and field experiences that train educators to design flexible lesson plans, use adaptive technologies, and implement varied instructional strategies. For example, prospective teachers might practice creating lesson plans that incorporate visual aids for geometry lessons or digital tools for data analysis, ensuring they address the needs of diverse learners.

UDL complements frameworks such as Culturally Relevant Pedagogy (CRP) and Differentiated Instruction (DI) by encouraging teachers to anticipate and remove barriers to learning from the outset. This proactive approach is particularly crucial in mathematics education, where abstract concepts and traditional instructional methods often create challenges for diverse learners. By adopting UDL principles, mathematics teachers can create equitable and accessible learning environments that empower all students to succeed. This framework underscores the importance of flexibility and innovation in addressing the diverse needs of today's classrooms, making it an indispensable component of teacher preparation programmes.

These theoretical frameworks—Culturally Relevant Pedagogy, Differentiated Instruction, and Universal Design for Learning—collectively provide a comprehensive foundation for equipping mathematics teachers to thrive in diverse classrooms. Together, they guide the development of inclusive and responsive teaching practices that can be embedded in teacher preparation programmes. By integrating these frameworks, this chapter seeks to demonstrate how innovative approaches to teacher preparation can foster equity and excellence in mathematics education, ensuring that all students, regardless of their backgrounds, can succeed.

3. Methodology: Reviewing Best Practices and Case Studies

In this chapter, we employed a meta-analysis approach to synthesise and evaluate the effectiveness of best practices and innovations in equipping mathematics teachers for diverse classrooms. Meta-analysis is a systematic method used to aggregate and statistically analyse findings from multiple studies to assess the overall effectiveness of interventions and strategies. This approach enables us to draw more robust conclusions by combining results across studies, identifying trends, and exploring variations in outcomes.

3.1 Study selection criteria

A comprehensive search was conducted using electronic databases, including Web of Science, Scopus, and Education Research Complete. Web of Science and Scopus were chosen for their extensive coverage of peer-reviewed literature, including the social sciences, while Education Research Complete focuses exclusively on leading educational research. The keywords used in the search were: "mathematics teacher preparation," "diverse classrooms," "culturally relevant pedagogy," "inclusive pedagogy," "differentiated instruction," and "Universal Design for Learning." The inclusion criteria were: studies focused on mathematics teacher preparation for diverse classrooms; peer-reviewed empirical studies published in English between 2015 and 2024; studies that evaluated the effectiveness of interventions or strategies used to prepare mathematics teachers; and studies that reported quantitative, qualitative, or mixed-methods data on outcomes such as student achievement, teacher effectiveness, or cultural competence.

3.2 Data extraction and coding

For each study that met the inclusion criteria, the following data were extracted: author(s) and year of publication, title of the study, focus or objective of the study, research design and data analysis methods used, tools or instruments employed to collect data, key findings regarding the impact of the intervention on mathematics teachers or students, challenges reported in the implementation of the strategies, and recommendations for future practice or research.

The search terms yielded twenty-eight journal articles from three databases published between 2015 and 2024. Following PRISMA guidelines (Moher et al., 2009), thirty-eight articles were screened by titles and abstracts. Twenty-three articles were excluded for not meeting the criteria, often due to irrelevance or non-secondary or primary school contexts. Fifteen full-text articles were then assessed, with three more excluded for lacking clear details on the implementation of inclusive pedagogy. Ultimately, twelve articles were reviewed. Figure 1 below shows the overall selection process for the review.

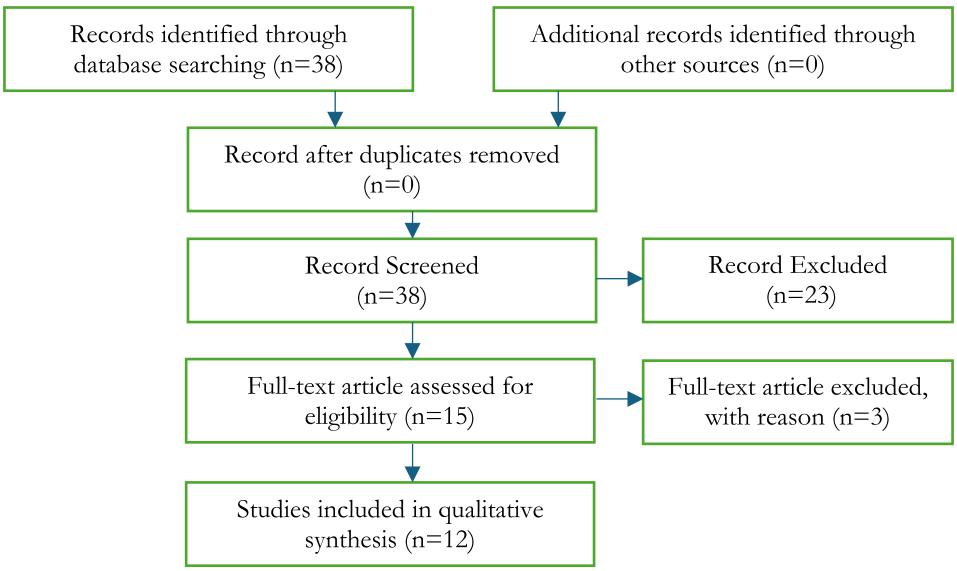


Figure 1: Meta-Analysis Approach Diagram

Table: Summary of Reviewed Studies on Inclusive Pedagogy Implementation

Name of Author(s)	Title	Focus/ Objectives	Design/ Data Analysis	Tools Used	Findings	Challenges and Recommendation(s)
Byrd, C. M. (2016)	Does Culturally Relevant Teaching Work? An Examination From Student Perspectives	To examine how students perceive culturally relevant teaching and its effectiveness in the classroom	Qualitative study using student interviews and surveys	Student interviews, surveys	Students responded positively to culturally relevant teaching, reporting increased engagement and belonging.	Teachers need better training on cultural relevance, and some resist changing traditional methods.
Mburu, J. M. (2022)	"All Children Matter": A Preservice Teacher's Understanding and Practice of Culturally Responsive Teaching in a Third-Grade Mathematics Classroom	To explore a preservice teacher's understanding and application of culturally responsive teaching in math	Case study, qualitative interviews, and classroom observations	Classroom observations, interviews	The preservice teacher successfully incorporated culturally responsive teaching, increasing student participation.	Further professional development is needed to help preservice teachers refine their skills in culturally responsive education.
Shultz, M., et al. (2024)	Enacting Culturally Relevant Pedagogy when "Mathematics Has No Color": Epistemological Contradictions	To explore the contradictions faced by teachers implementing culturally relevant pedagogy in mathematics	Qualitative study, teacher reflections	Teacher interviews, classroom observations	Teachers struggled to reconcile culturally relevant pedagogy with traditional epistemologies in math.	Recommends more consistent integration of culturally relevant pedagogy in mathematics teacher preparation programs
Almeqdad, Q. I., et al. (2023)	The effectiveness of universal design for learning: A systematic review of the literature and meta-analysis	To systematically review the effectiveness of UDL on academic achievement	Systematic review and meta-analysis	Meta-analysis of multiple studies	UDL significantly improved academic outcomes across diverse learner populations	Challenges in teacher implementation due to lack of training and resources; recommends extensive professional development.
Kusumaha, I. P., & Ani, Y. (2021)	Universal design learning approach to overcome	To investigate how UDL can help overcome barriers in mathematics learning	Qualitative study, case studies	Classroom observations, teacher reflections	UDL strategies improved student participation and comprehension in mathematics	Teachers faced challenges adapting UDL in traditional classroom settings; more

Name of Author(s)	Title	Focus/ Objectives	Design/ Data Analysis	Tools Used	Findings	Challenges and Recommendation(s)
	mathematics learning					teacher training and collaboration are recommended.
Root, J. R., et al. (2020)	Applying the Universal Design for Learning Framework to Mathematics Instruction for Learners With Extensive Support Needs	To assess how UDL can be applied to math instruction for students with extensive support needs	Qualitative study, case studies	Classroom observations, interviews	UDL improved math learning for students with disabilities by providing multiple means of engagement	Difficulty in adapting UDL materials for learners with varied needs. The study suggests collaboration between special educators and math teachers
Bal, A. P. (2016)	The Effect of the Differentiated Teaching Approach in the Algebraic Learning Field on Students' Academic Achievements	To investigate the effects of differentiated teaching on students' academic performance in algebra	Quasi-experimental design, pretest-posttest analysis	Algebraic achievement tests	Differentiated teaching led to significant improvements in students' algebraic achievements.	Teachers face difficulties managing diverse student needs; they recommend more teacher training in differentiation.
Bal, A. P. (2023)	Assessing the impact of differentiated Instruction on mathematics achievement and attitudes of secondary school learners	To explore how differentiated Instruction impacts mathematics achievement and student attitudes	Quasi-experimental design, pretest-posttest, attitude scale	Mathematics achievement tests, attitude questionnaires	Differentiated Instruction improved both achievement and positive attitudes toward math	The lack of resources and teacher knowledge remains challenging, so further professional development is recommended.
Kamarulzaman, M. H., et al. (2022)	Impact of Differentiated Instruction on the Mathematical Thinking Processes of Gifted and Talented Students	To evaluate the effect of differentiated Instruction on the mathematical thinking of gifted students	Mixed-methods approach using quantitative and qualitative data	Observations, interviews, mathematical thinking assessments	Differentiated Instruction enhanced mathematical thinking processes among gifted students	Limited teacher experience with differentiated Instruction; suggests providing more targeted training for teachers

Name of Author(s)	Title	Focus/ Objectives	Design/ Data Analysis	Tools Used	Findings	Challenges and Recommendation(s)
Mićanović, V., et al. (2023)	Effects and challenges to implement differentiated mathematics teaching among fourth graders in Montenegro	To assess the effectiveness and challenges of implementing differentiated Instruction in mathematics	Qualitative study, interviews with teachers and students	Classroom observations, interviews	Differentiated Instruction improved student engagement, but lacking instructional materials and time posed challenges.	Recommends additional resources and time allocations for effective implementation
Nurasiah, L., et al. (2020)	The effect of differentiated Instruction on student mathematical communication ability	To determine how differentiated Instruction influences students' mathematical communication skills	Quasi-experimental design, pretest-posttest	Mathematical communication tests, classroom observations	Differentiated Instruction positively impacted students' mathematical communication abilities	Teachers struggled with time management when implementing DI; recommends further research on DI in standard curricula
Prast, E. J., et al. (2018)	Differentiated Instruction in primary mathematics: Effects of teacher professional development on student achievement	To examine how teacher professional development on differentiation impacts student math achievement	Randomised controlled trial, pretest-posttest	Student achievement tests, teacher PD workshops	Students of teachers who received PD on differentiated Instruction performed significantly better in mathematics.	Long-term PD programs remain challenging; continuous support is recommended for teachers implementing DI.

3.3 General description of reviewed studies

The reviewed studies on inclusive pedagogies (CRT, UDL, and DI) span a variety of contexts and educational levels, primarily focusing on mathematics instruction and its impact on diverse learner populations. For instance, CRT studies, such as Byrd (2016) and Mburu (2022), examine the effectiveness of culturally responsive teaching strategies. Byrd (2016) highlights how these strategies foster academic engagement and a sense of representation among students. Similarly, Mburu (2022) illustrates the successful implementation of CRT in a third-grade mathematics classroom, demonstrating increased student participation and engagement. Additionally, UDL research, including Root et al. (2020), investigates how flexibility in instructional design and the use of multiple means of engagement, representation, and action/expression accommodate students with extensive support needs in mathematics classrooms. Differentiated Instruction, examined by Bal (2023) and others, assesses how tailoring instruction to students' readiness levels, interests, and learning profiles improves student achievement and attitudes toward mathematics. Overall, these studies (Bal, 2023; Byrd, 2016; Mburu, 2022; Root et al., 2020) show that inclusive pedagogies address a wide range of student needs, from cultural inclusivity (CRT) to instructional flexibility (UDL) and personalisation of learning paths (DI). Each approach seeks to improve equity in education by providing accessible and relevant learning experiences for all students, particularly those from underrepresented or marginalised backgrounds.

3.4 Inclusive pedagogy implementation of reviewed studies

In terms of implementation, the studies highlight various strategies for integrating inclusive pedagogies in the classroom. Culturally Relevant Teaching (CRT) is often implemented through curriculum redesign, where teachers incorporate culturally relevant materials and examples that reflect the backgrounds of their students (Mburu, 2022). Teachers also foster critical thinking and engage in social justice discussions, linking mathematics learning to real-world issues relevant to students' communities. In Universal Design for Learning (UDL), flexible curriculum design is central to its implementation. The study by Kusuma and Ani (2021) highlights how digital tools and adaptive technologies enable teachers to present mathematical concepts in multiple formats, thereby supporting students with diverse learning needs. Differentiated Instruction (DI) emphasises ongoing assessment and flexible groupings based on students' learning requirements. The research conducted by Prast et al. (2018) underscores the importance of professional development in equipping teachers to adapt their instructional strategies effectively, ensuring they can meet the needs of various learning profiles.

3.5 Effectiveness of the inclusive pedagogy approaches of reviewed studies

The reviewed studies consistently find that inclusive pedagogies improve student outcomes when implemented effectively. For instance, Culturally Relevant Teaching has been shown to enhance student engagement and academic performance by fostering a sense of belonging and validation (Byrd, 2016). Students in culturally responsive classrooms are more likely to perceive the relevance of mathematics to their own lives, which boosts motivation and achievement. Universal Design for Learning has been found to improve access to mathematics content for students with learning disabilities or other special needs (Root et al., 2020). The flexibility of UDL allows students to engage with content in ways that suit their strengths, leading to better comprehension and retention. Differentiated Instruction has demonstrated positive effects on student achievement and attitudes toward mathematics. Bal (2023) reports that students in DI classrooms outperform their peers in traditional settings, especially when instruction is tailored to their readiness levels and interests. This approach fosters a positive learning environment in which students feel supported and appropriately challenged.

3.6 Challenges encountered with the inclusive pedagogy of reviewed studies

Despite the promise of these inclusive pedagogies, several challenges have emerged in their implementation. Systemic barriers, such as a lack of institutional support and adequate professional development, prevent teachers from fully adopting inclusive practices. For example, Byrd (2016) highlights that culturally relevant pedagogy requires teachers to possess a deep knowledge of their students' cultural backgrounds, which many educators lack due to insufficient training. A significant challenge for Universal Design for Learning (UDL) is the resource constraints related to access to technology and specialised teacher training. Almeqdad et al. (2023) note that schools with limited funding face difficulties in implementing UDL effectively due to the high costs of essential digital tools and instructional materials. In the case of Differentiated Instruction, teachers often encounter time constraints, making it difficult to prepare lessons that cater to the diverse needs of students (Bal, 2023). Differentiation demands continuous assessment and adaptation, which can overwhelm teachers who lack sufficient planning time or support.

4. Lessons Learned and Recommendations

The reviewed studies on inclusive pedagogies, namely CRT, UDL, and DI, provide critical perspectives on enhancing educational practices for diverse learners. One significant lesson learned is the importance of professional development. Continuous training equips teachers with the skills and confidence necessary to effectively implement CRT, UDL, and DI. Research has shown that ongoing professional development improves teachers' instructional strategies and positively impacts student engagement and achievement (Almeqdad et al., 2023; Bal, 2023). Another critical recommendation emphasises the need for inclusive leadership and advocacy. School leaders and policymakers play a vital role in promoting inclusive practices within teacher preparation programmes. Their support can create a conducive environment that prioritises inclusivity and provides the necessary resources for successful implementation (Byrd, 2016; Iwuanyanwu, 2023). Additionally, fostering a collaborative classroom culture is essential. Encouraging open communication between teachers and students allows educators to tailor their approaches to meet individual learning needs. Engaging students in discussions about their experiences and preferences can enhance the relevance of their learning (Bal, 2023; Kamarulzaman et al., 2022).

The development of comprehensive assessment tools is also crucial. Schools should focus on creating diverse assessment methods that accurately reflect students’ understanding in inclusive settings. Such assessments should account for various learning styles and offer insights into the social-emotional impacts of teaching practices (Prast et al., 2018; Root et al., 2020). Moreover, leveraging technology can significantly enhance UDL implementation. Digital tools that provide adaptive learning experiences can help educators better meet the needs of diverse learners, facilitating greater differentiation in instruction (Kusumaha & Ani, 2021; Nurasiah et al., 2020). It is equally important to ensure equity in resource distribution across schools. Addressing disparities can facilitate the successful implementation of inclusive pedagogies. Targeted funding and interventions should prioritise schools in marginalised communities to ensure equitable access to high-quality education (Almeqdad et al., 2023; Mićanović et al., 2023).

Cultural competency training must also be a priority in teacher education. By understanding their students' cultural backgrounds, teachers can implement CRT more effectively, creating inclusive classroom environments that respect and value diversity (Mburu, 2022; Shultz et al., 2024). Lastly, incorporating peer mentorship and community engagement can bolster the effectiveness of inclusive pedagogy. Developing mentorship programmes and fostering partnerships with community members provide additional resources and perspectives, enriching the educational experience for students (Byrd, 2016; Mićanović et al., 2023). By embracing these lessons and recommendations, educators and institutions can significantly enhance their approaches to inclusive pedagogy, ultimately providing a more equitable and effective mathematics education for diverse learners.

5. Conclusion

This chapter has demonstrated that inclusive pedagogies such as Culturally Responsive Teaching, Universal Design for Learning, and Differentiated Instruction are vital for preparing mathematics teachers to meet the diverse needs of their students. These approaches enhance engagement, support equity, and improve mathematics achievement by valuing students' cultural identities, accommodating varied learning styles, and providing personalized instruction. However, challenges such as resource limitations and inadequate training must be addressed to fully realise their potential. The broader implications are clear: adopting inclusive teaching practices fosters an equitable education system that empowers diverse learners to succeed. Moving forward, teacher training programmes must emphasise cultural competence, hands-on experience, and collaboration between institutions and communities. By championing inclusivity and equity, pre-service mathematics educators can transform classrooms into spaces where every student has the opportunity to thrive.

6. Declarations

Funding: This research did not receive any external funding.

Conflicts of Interest: The authors declare no conflict of interest.

References

Almeqdad, Q. I., Alodat, A. M., Alquraan, M. F., Mohaidat, M. A., & Al-Makhzoomy, A. K. (2023). The effectiveness of universal design for learning: A systematic review of the literature and meta-analysis. *Cogent Education, 10*(1), 1–24. https://doi.org/10.1080/2331186X.2023.2218191

Anyichie, A. C. (2024). Engaging preservice teachers with culturally relevant microteaching activity: A university professor's experience and perception. *Journal of Pedagogical Sociology and Psychology, 6*(3), 125-137. https://doi.org/10.33902/jpsp.202429217

Bal, A. P. (2016). The effect of the differentiated teaching approach in the algebraic learning field on students' academic achievements. *Eurasian Journal of Educational Research, 63*, 185–204. https://doi.org/10.14689/ejer.2016.63.11

Bal, A. P. (2023). Assessing the impact of differentiated instruction on mathematics achievement and attitudes of secondary school learners. *South African Journal of Education, 43*(1), 1–10. https://doi.org/10.15700/saje.v43n1a2065

Byrd, C. M. (2016). Does culturally relevant teaching work? An examination from student perspectives. *SAGE Open, 6*(3). https://doi.org/10.1177/2158244016660744

Centre for Applied Special Technology. (2008). *Universal design for learning guidelines version 1.0.* Centre for Applied Special Technology.

Cochran-Smith, M., & Reagan, E. M. (2021). *"Best practices" for evaluating teacher preparation programs.* National Academy of Education Committee on Evaluating and Improving Teacher Preparation Programs.

Eden, C. A., Chisom, O. N., & Adeniyi, I. S. (2024). Cultural competence in education: Strategies for fostering inclusivity and diversity awareness. *International Journal of Applied Research in Social Sciences, 6*(3), 383–392. https://doi.org/10.51594/ijarss.v6i3.895

Faragher, R., Hill, J., & Clarke, B. (2016). Inclusive practices in mathematics education. In *Research in mathematics education in Australasia 2012–2015* (pp. 109–131). Springer. https://doi.org/10.1007/978-981-10-1419-2_7

FitzGerald, C., Martin, A., Berner, D., & et al. (2019). Interventions designed to reduce implicit prejudices and implicit stereotypes in real-world contexts: A systematic review. *BMC Psychology, 7*, 29. https://doi.org/10.1186/s40359-019-0299-7

Gay, G. (2022). *Culturally responsive teaching: Theory, research, and practice* (3rd ed.). Teachers College Press.

Haleem, A., Javaid, M., Qadri, M. A., & Suman, R. (2022). Understanding the role of digital technologies in education: A review. *Sustainable Operations and Computers, 3*, 275-285. https://doi.org/10.1016/j.susoc.2022.05.004

Iwuanyanwu, P. N. (2023). Preparing teachers for culturally responsive education. *Indilinga African Journal of Indigenous Knowledge Systems, 22*(1), 13. https://hdl.handle.net/10520/ejc-linga_v22_n1_a2

Kamarulzaman, M. H., Kamarudin, M. F., Sharif, M. S. A. M., Esrati, M. Z., Saali, M. M. S. N., & Yusof, R. (2022). Impact of differentiated instruction on the mathematical thinking processes of gifted and talented students. *Journal of Education and E-Learning Research, 9*(4), 269–277. https://doi.org/10.20448/jeelr.v9i4.4253

Karatas, K. (2020). The competencies of the culturally responsive teacher: What, why, and how? *i.e.: Inquiry in Education, 12*(2), Article 2. https://digitalcommons.nl.edu/ie/vol12/iss2/2

Kusumaha, I. P., & Ani, Y. (2021). Universal design learning approach to overcome mathematics learning. *Global Conferences Series: Social Sciences, Education and Humanities (GCSSSEH), 8*, 33–39. https://doi.org/10.32698/GCS-11513

Ladson-Billings, G. (1995). Toward a theory of culturally relevant pedagogy. *American Educational Research Journal, 32*(3), 465–491. https://doi.org/10.3102/00028312032003465

Mahlambi, S. B. (2023). Mathematics teachers' use of assessment for learning to promote classroom diversity of learners. *Pythagoras, 44*(1). https://doi.org/10.4102/pythagoras.v44i1.708

Mamedova, S., & Orosco, M. J. (2024). Culturally responsive mathematics: Characteristics in classroom settings. In *The Palgrave Encyclopedia of Disability* (pp. 1–9). https://doi.org/10.1007/978-3-031-40858-8

Mbhiza, H., Nkambule, T., & Masinire, A. (2024). Student teachers' mentorship experiences during teaching practice: A comparison of mentor-student dynamics in rural and urban schools. *Interdisciplinary Journal of Education Research, 6*, 1-14. https://doi.org/10.38140/ijer-2024.vol6.05

Mburu, J. M. (2022). "All children matter": A preservice teacher's understanding and practice of culturally responsive teaching in a third-grade mathematics classroom. *International Journal of Multicultural Education, 24*(1), 27–46. https://doi.org/10.18251/ijme.v24i1.2623

Meyer, A., Rose, D. H., & Gordon, D. (2014). *Universal design for learning: Theory and practice*. CAST Professional Publishing.

Mićanović, V., Vučković, D., Maslovarić, B., Šakotić, N., & Novović, T. (2023). Effects and challenges to implement differentiated mathematics teaching among fourth graders in Montenegro. *South African Journal of Education, 43*(3), 1–11. https://doi.org/10.15700/saje.v43n3a2100

Moher, D., Liberati, A., Tetzlaff, J., & Altman, D. G. (2009). Preferred reporting items for systematic reviews and meta-analyses: The PRISMA statement. *Research Methods and Reporting, 1*–8. https://doi.org/10.1136/bmj.b2535

Nurasiah, L., Priatna, B. A., & Priatna, N. (2020). The effect of differentiated instruction on student mathematical communication ability. *Journal of Physics: Conference Series, 1469*, 012160. https://doi.org/10.1088/1742-6596/1469/1/012160

Olawale, B. (2024). Collaborative virtual environments (CVEs) in the preparation of pre-service mathematics teachers. *Journal of Learning and Teaching in Digital Age, 9*(2), 55–63. https://doi.org/10.53850/joltida.1359772

Olawale, B. E. (2022). Democratic citizenship in mathematics teacher preparation in South African universities: Contradictions in theory and practice. *International Journal of Pedagogy and Teacher Education*, *6*(2), 126-139. https://doi.org/10.20961/ijpte.v6i2.61574

Olawale, B. E. (2024). Impact of pre-service teacher education programme on mathematics student teachers' teaching practices during school experiences. *Education Sciences, 14*(7), 762. https://doi.org/10.3390/educsci14070762

Olawale, B. E., & Hendricks, W. (2024, December). Artificial intelligence in teacher education: Unlocking the tools and benefits of a transforming pedagogy. Paper presented at *21st Century Pedagogies for Sustainable Development in Education Conference*, Liverpool Hope University, Liverpool, UK.

Prast, E. J., Van de Weijer-Bergsma, E., Kroesbergen, E. H., & Van Luit, J. E. H. (2018). Differentiated instruction in primary mathematics: Effects of teacher professional development on student achievement. *Learning and Instruction, 54*, 22–34. https://doi.org/10.1016/j.learninstruc.2018.01.009

Root, J. R., Cox, S. K., Saunders, A., & Gilley, D. (2020). Applying the universal design for learning framework to mathematics instruction for learners with extensive support needs. *Remedial and Special Education, 41*(4), 194–206. https://doi.org/10.1177/0741932519887235

Sanger, C. S. (2020). Inclusive pedagogy and universal design approaches for diverse learning environments. In C. Sanger & N. Gleason (Eds.), *Diversity and inclusion in global higher education* (pp. 55-71). Palgrave Macmillan. https://doi.org/10.1007/978-981-15-1628-3_2

Shultz, M., Close, E., Nissen, J., & Van Dusen, B. (2024). Enacting culturally relevant pedagogy when "mathematics has no color": Epistemological contradictions. *International Journal of*

Research in Undergraduate Mathematics Education, 10(2), 486–515. https://doi.org/10.1007/s40753-023-00219-x
Tomlinson, C. A. (2001). *How to differentiate instruction in mixed-ability classrooms* (2nd ed.). ASCD.
Weiss, S., Braun, A., Pacher, M., Schlegel, C. M., & Kiel, E. (2024). Adapting feedback to the demands of teaching in diverse classrooms: Novel feedback concepts for preservice teachers during practicum. *Assessment & Evaluation in Higher Education*, 49(3), 285–304. https://doi.org/10.1080/02619768.2024.2338846

CHAPTER TEN

Conceptual Foundation for Ethnomathematics Instructional Design in Mathematics Teacher Preparation

Joshua Abah Abah[1*]

AFFILIATIONS

[1]Faculty of Education, University of Zululand, KwaDlangezwa, South Africa.

CORRESPONDENCE

Email: AbahJ@unizulu.ac.za*

Reference Formats (APA 7th Edition)

Abah, J. A. (2025). Conceptual foundation for ethnomathematics instructional design in mathematics teacher preparation. In B. E. Olawale (Ed.), *Building the Foundations: Effective Approaches in Mathematics Teacher Preparation* (pp. 153-175). ERRCD Forum. https://doi.org/10.38140/obp3-2025-10

Abstract: Preparing mathematics teachers who can deliver the change we are witnessing and meet the needs of the future requires a concerted effort from all stakeholders in the mathematics education ecosystem. Such efforts must take into account the mediating role of culture-based instructional design in addressing the varying needs of indigenous communities, digital technology users, proposals from early adopters, and the expanding range of opportunities provided by new and future digital technologies. Culture-based educational enhancements depend on how well researchers' and designers' interventions satisfy end-users and the educational and interactional effects the designed tools necessitate. On this basis, this chapter presents a conceptual foundation for ethnomathematics instructional design in mathematics education by first elaborating on the concept of ethnomathematics and the framework for realistic mathematics education. This is followed by a focus on the role of ethnomathematics in mathematics teacher preparation, alongside research-based contextual deployments of culture-based mathematics instructional design. The details of digital content management in instructional environments are then considered, followed by a presentation of empirical case studies of instructional design in teacher preparation. The conceptual presentation in this chapter aims to encourage researchers and practitioners in the field of Mathematics Education to explore the full opportunities and benefits of ethnomathematics instructional design.

Keywords: Digital content management, ethnomathematics, global competence, instructional design, mathematics teacher preparation.

1. Introduction

Education is a conscious effort by society to inculcate its existing body of knowledge, values, norms, science and technology into the younger generation to encourage active participation in society. To achieve this objective, society engages the services of educational institutions where children are guided through well-planned structures. Thus, one of the most urgent aims of education is to facilitate social and economic development while exposing learners to the scientific and analytical thinking skills they need to understand global issues, build a sustainable world, and innovate new technologies (Iji, Abah & Anyor, 2017). Present-day education that is desirable fosters global competence.

The term "global competence" is used in educational research to describe a body of knowledge about world regions, cultures, and global issues, along with the skills and dispositions necessary to engage responsibly and effectively in a global environment (Longview Foundation, 2008). A globally competent student possesses knowledge of and curiosity about the world's history, geography, cultures, environmental and economic systems, and current international issues. Such a student has the language and cross-cultural skills needed to communicate effectively with people from other countries, understand multiple perspectives, and utilise primary sources from around the globe. A globally competent student is committed to ethical citizenship and cultural awareness.

To help students become globally competent, it is essential that teachers possess the knowledge, skills, and dispositions expected of their students. In addition to these capacities, globally competent teachers must understand the international dimensions of their subject matter and a range of global issues; possess pedagogical skills to teach their students to analyse primary sources from around the world, appreciate multiple viewpoints, and recognise stereotyping; and show a commitment to assisting students in becoming responsible citizens of both the world and their communities (Longview Foundation, 2008). Given the era in which we live, digital technology appears to be the fulcrum on which progress in knowledge rests.

Present levels of digital penetration are evolving as technology advances, making access to and use of Information and Communication Technologies (ICTs) an essential element for participation in society, democracy, and the economy. Furthermore, digital equity is the ultimate outcome of ongoing efforts toward digital inclusion, focusing on actions and investments to eliminate historical, systematic, and structural barriers that perpetuate disadvantage among individuals and communities. Globalisation through digital equity acknowledges the moral obligation to harness ICT to address the needs of disadvantaged individuals, communities, neighbourhoods, community-based organisations, and small businesses (Iji & Abah, 2019). This reality must be factored into any programme of teacher preparation.

Thus, training teachers for the global age requires teacher educators, who are preparing future teachers in higher education institutions, to develop the aforementioned capacities, along with the knowledge, skills, and dispositions necessary for teacher candidates to acquire them (Longview Foundation, 2008). For mathematics teacher educators, these capacities entail a kind of meta-knowledge that encompasses at least some of the knowledge that mathematics teachers require, just as teachers of mathematics need to know more than the foundational knowledge required to help students learn mathematics as a school subject (Beswick & Goos, 2018).

The extensive use of digital technology not only raises the need for skills that complement what computers do but also influences the relevance of science, technology, engineering, and mathematics in our society (Gravemeijer et al., 2017; Mncube & Olawale, 2020). Preparing mathematics teachers who can respond to the changes we are witnessing and meet future needs requires a concerted effort from all stakeholders in the mathematics education ecosystem. This effort must take into account the mediating role of culture-based instructional design in addressing the varying needs of indigenous communities, digital technology users, proposals from early adopters, and the expanding range of opportunities offered by new and emerging digital technologies. Culture-based educational enhancements rely on how effectively researchers' and designers' interventions meet the needs of end users and the educational and interactional effects that the designed tools produce (Nouri, Spikol & Cerratto-Pargman, 2016). The lifecycle, therefore, must consider well-defined collaboration among mathematics teacher educators, mathematics teachers, and designers, particularly during the implementation of the artefact (Iversen & Jonsdottir, 2018).

For digital tools aimed at amplifying the recognition that every cultural group generates its own mathematics, ethnomathematics instructional design must crowdsource primary resources, tested classroom practices, culture-based lesson templates, and other localised content from practising mathematics teachers. This entails context-sensitive design elements drawn from numerous micro-communication processes, resulting in a research-based consequences feedback loop, both positive and negative, expected and unexpected.

This chapter takes a holistic view of the necessity for a sound conceptual foundation for ethnomathematics instructional design in mathematics teacher preparation and the necessary adjustments that can be made to mathematics teacher preparation schemes across different levels. The presentation begins with an overview of ethnomathematics, followed by a consideration of a framework for realistic mathematics education and an exploration of the dimensions of ethnomathematics in contemporary mathematics teacher preparation. The work also examines instances of culture-based instructional design and concludes with a discussion on digital content management in instructional environments, accompanied by case studies from empirical research.

2. Ethnomathematics

One of the most relevant reasons for teaching Mathematics is the consideration of Mathematics as an expression of human development, culture, and thought, highlighting its integral role in the cultural heritage of humankind. Although contemporary society places great value on a Western-oriented approach to science and mathematics, ethnomathematics has demonstrated that mathematics comprises many diverse and distinct cultural traditions, not just those emerging from the Mediterranean basin mathematics tradition (Rosa & Orey, 2010). Mathematical thinking has been influenced by the vast diversity of human characteristics, such as languages, religions, morals, and economic, social, and political activities. In view of these multidimensional influences, humans have developed logical processes related to universal needs to quantify, measure, model, and explain, all operating within different socio-historical contexts (Abah & Chinaka, 2024).

Since each cultural group has its own way of doing Mathematics, the connections often come to represent a given cultural system, particularly in the manner in which they quantify and use numbers, geometric forms and relationships, and measure or classify objects in their own environment. Rosa and Orey (2010) state that for all these reasons, each cultural group has developed its unique way of mathematizing its own realities. In this sense, D'Ambrosio (2001a) defines Ethnomathematics as the Mathematics practiced by cultural groups, such as urban and rural communities, groups of workers, professional classes, children in a group, indigenous societies, and numerous other groups identified by the objectives and traditions common to these groups. In simple terms, ethnomathematics is used to express the relationship between culture and mathematics.

The everyday life of groups, families, tribes, communities, associations, professions, and nations occurs in different regions of the planet, in various ways and at different paces, with the individuals within them sharing knowledge such as language, systems of explanation, myths and legends, customs, and culinary habits. The behaviours of members are made compatible with and subordinated to value systems agreed upon by the group, thereby forming a culture. In sharing knowledge and aligning behaviours, the characteristics of a culture are synthesised (D'Ambrosio, 2001a). Essentially, the distinct way of doing (practice) and knowing (theory) that characterises a culture is part of the shared knowledge and behaviours that have become compatible.

Everyday life is imbued with the knowledge and practices of a culture. At all times, individuals are comparing, classifying, quantifying, measuring, explaining, generalising, inferring, and, in some way, evaluating, using material and intellectual instruments that belong to their culture (D'Ambrosio, 2001a). These individuals are actively employing the arts or techniques of explaining, understanding, and coping with their environments that they have learned in their cultural settings. This inherited knowledge of their cultural group constitutes the ethnomathematics of the group (D'Ambrosio, 1994).

3. The Framework for Realistic Mathematics Education (Freudenthal, 1968)

Realistic Mathematics Education (RME) is a domain-specific instructional framework for mathematics. Essentially, RME emphasises the importance of rich, "realistic" situations in the learning process. These situations serve as a source for initiating the development of mathematical concepts, tools, and procedures, and as a context in which learners can later apply their mathematical knowledge, which gradually becomes more formal and general, and less context-specific. Although "realistic" situations, in the sense of "real-world" contexts, are important in RME, the term "realistic" has a broader connotation here.

The foundation of Realistic Mathematics Education (RME) is credited to Hans Freudenthal (1905–1990), a mathematician born in Germany who became a Professor of Pure and Applied Mathematics and the Foundations of Mathematics at Utrecht University in the Netherlands in 1946. Later in his career, Freudenthal (1968, 1973, 1991) became interested in mathematics education and advocated for teaching mathematics that is relevant for learners, conducting thought experiments to explore how learners can be given opportunities for the guided re-invention of mathematics.

One of the foundational concepts of RME is Freudenthal's idea of mathematics as a human activity. For him, mathematics is not a static body of knowledge but the activity of solving problems and seeking new problems. More broadly, it is the activity of organising elements from reality or mathematical concepts, which he referred to as "mathematization" (Freudenthal, 1968). This activity-based interpretation of mathematics has significant implications for how mathematics education is conceptualised. Specifically, it influences both the goals of mathematics education and the teaching methods employed (Van den Heuvel-Panhuizen, 2003). According to Freudenthal (1968, 1971, 1973), mathematics can best be learned by doing, and mathematization is the core goal of mathematics education. Rather than presenting mathematics as a ready-made product, the objective should be to engage learners in mathematics as an activity. Thus, similar to how the mathematical activities of mathematicians have shaped mathematics as it is known today, the activities of learners should lead to the construction of mathematics, effectively allowing them to invent mathematics (Gravemeijer, 2008).

The tenets of RME distinguish between two ways of mathematizing in an educational context, namely "horizontal" and "vertical" mathematizing. In the case of horizontal mathematizing, mathematical tools are employed to organise and solve problems situated in daily life. Vertical mathematizing, in contrast, refers to various reorganisation and operations undertaken by learners within the mathematical system itself. According to Freudenthal (1991), to mathematize horizontally means to transition from the world of life to the world of symbols, while to mathematize vertically involves movement within the world of symbols. The latter implies, for instance, making shortcuts, discovering connections between concepts and strategies, and utilising these findings (Van den Heuvel Panhuizen, 2003). In RME, emphasis is placed on the notion that both forms of mathematizing hold equal value and can occur at all levels of mathematical activity.

Essentially, ethnomathematics as a programme of mathematics education highlights the importance of providing realistic situations within the instructional process. The design of ethnomathematics instruction serves as a means to project the mathematical concepts and knowledge embedded in learners' real lives and cultural practices. Frequent references to the history of mathematics and storytelling within developed instructional tools present problem situations that learners can envision and relate to as experientially real (Abah, Iji & Abakpa, 2018). The content coverage of digital tools designed on this foundation aims to humanise mathematics for users, guiding them in ways to re-invent Mathematics for themselves.

In terms of mathematization, the educational products developed within the scope of ethnomathematics instructional design are tools that can be creatively utilised to organise and solve problems situated in students' daily lives. This makes the platform an indispensable companion for teachers engaging in horizontal mathematization. For instance, the blog articles, lesson plans, adaptable classroom activities, and ethnomathematics forum provided by a web-based ethnomathematics repository have been shown to assist mathematics teachers in seamlessly transitioning from the world of life to the world of mathematical symbols (Abah, Iji, Abakpa & Anyagh, 2021; Abah, 2024). The engaging activities available on the platform demonstrate that Mathematics is best learned by doing mathematics.By building on the reality principle of Realistic Mathematics Education, Abah, Iji, Abakpa, and Anyagh (2021) and Abah (2024) designed the Ethnomathematics Instructional Content Repository to present resources that encourage mathematics teachers to start from problem situations that are meaningful to students and are derived from the learners' rich cultural contexts. This approach allows students to move from context-related situations to constructing practical mathematical strategies, progressing through a spiral of intertwined curricular content to achieve success. In learning and doing mathematics, technology in the form of real-world interfaces, such as the Ethnomathematics Instructional Content Repository, can assist students with problem-solving, support the exploration of mathematical concepts, provide dynamically linked representations of ideas, and encourage general metacognitive abilities such as planning and checking (Barkatsas, 2004). The adaptable teaching templates available from the repository can aid mathematics teachers in providing the necessary guided re-invention and group discussions that will develop students' conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition (Kilpatrick & Findell, 2001). Consequently, culture-based digital tools mediate for practising mathematics teachers the process of constructing knowledge, with an emphasis on pupils' hands-on activities and everyday life.

4. Ethnomathematics in Mathematics Teacher Preparation

Ethnomathematics requires a dynamic interpretation because it describes concepts that are neither rigid nor singular, namely "ethno" and "mathematics" (D'Ambrosio, 1987). The term "ethno" encompasses all the elements that constitute the cultural identity of a group: language, codes, values, jargon, beliefs, food, dress, habits, and physical traits. Mathematics expresses a broad view of itself, which includes ciphering, arithmetic, classifying, ordering, inferring, and modelling (D'Ambrosio, 1987). Many educators may be unfamiliar with the term; however, a basic understanding of it allows teachers to expand their mathematical perceptions and instruct their students more effectively.

Teachers, and the public in general, do not commonly acknowledge the connection between mathematics and culture (D'Ambrosio, 2001b). When teachers do recognise a connection, they often engage their students in multicultural activities merely as a curiosity. Such activities typically refer to a culture's past and to cultures that are very remote from those of the children in the class. This situation arises because teachers may not understand how culture relates to children and their learning. An important component of mathematics teacher preparation today should be to reaffirm, and in some instances restore, the cultural dignity of children (D'Ambrosio, 2001b). Although multicultural mathematics activities are important, they should not be our ultimate goal. As students engage in multicultural mathematical activities that reflect the knowledge and behaviours of people from diverse cultural environments, they may not only learn to value mathematics but, just as importantly, develop a greater respect for those who are different from themselves.

To acquire requisite skills while maintaining cultural dignity and be prepared for full participation in society requires more than what is offered in a traditional curriculum. Much of today's curriculum is so disconnected from the child's reality that it is impossible for the child to be a full participant in it (D'Ambrosio, 2001b). The mathematics taught in many classrooms has practically nothing to do with the world that the children are experiencing. Just as literacy has come to mean much more than reading and writing, mathematics must also be considered as more than, and indeed different from, counting, calculating, sorting, or comparing. Considering that today's children live in a civilisation dominated by mathematically based technology and unprecedented means of communication, it is safe to assert that much of the content of current mathematics programmes does little to help students learn the information and skills necessary to function successfully in this new world.

The goal of Mathematics Education should be to foster students' ability to successfully use modern technology to solve problems and communicate their thinking and answers while gaining an awareness of the capabilities and limitations of technological instruments. The school system can help learners realise their full mathematical potential by acknowledging the importance of culture to the identity of the child and how culture affects the way children think and learn. Children must be taught to value diversity in the mathematics classroom and to understand both the influence that culture has on Mathematics and how this influence results in different ways in which mathematics is used and communicated (D'Ambrosio, 2001b). Such understanding is gained through the study of ethnomathematics.

As a programme, ethnomathematics studies the cultural aspects of Mathematics. It acknowledges that there are different ways of doing Mathematics by considering the appropriation of academic mathematical knowledge developed by various sectors of society, as well as the different modes in which various cultures negotiate their mathematical practices. Ethnomathematics researchers investigate how different cultural groups comprehend, articulate, and apply ideas, procedures, and techniques identified as mathematical practices (Rosa & Shirley, 2016). These mathematical practices refer to forms of Mathematics that vary as they are embedded in cultural activities. Ethnomathematics presents the mathematical concepts of the school curriculum in a way that relates these concepts to the cultural backgrounds of students, thereby enhancing their ability to make meaningful connections and deepening their understanding of Mathematics.

Development within the field of ethnomathematics represents a methodology for ongoing research and analysis of the processes that transmit, diffuse, and institutionalise mathematical knowledge, ideas, processes, and practices that originate from diverse cultural contexts throughout history. This context has enabled the development of six important dimensions of ethnomathematics, which are interrelated and aim to analyse the socio-cultural roots of mathematical knowledge. Rosa and Shirley (2016) summarise the six dimensions as follows:

- *Cognitive:* This dimension concerns the acquisition and dissemination of mathematical knowledge across generations. Mathematical ideas such as comparison, classification, quantification, measurement, explanation, generalisation, modelling, and evaluation are understood as social, cultural, and anthropological phenomena that trigger the development of knowledge systems elaborated by members of distinct cultural groups. In this regard, it is not possible to evaluate the development of cognitive abilities apart from social, cultural, economic, environmental, and political contexts.
- *Conceptual:* The challenges of everyday life provide members of distinct cultural groups with the opportunity to answer existential questions by creating procedures, practices, methods, and theories based on their representations of reality. These actions constitute a fundamental basis for the development of essential knowledge and decision-making processes. Survival depends on immediate behaviour in response to routines inherent to the development of group members. Thus, mathematical knowledge emerges as an immediate response to the needs for survival and transcendence.
- *Educational:* This dimension does not reject knowledge and behaviour acquired academically, but incorporates human values such as respect, tolerance, acceptance, caring, dignity, integrity, and peace into the teaching and learning of Mathematics to humanise it and bring it to life. In this context, ethnomathematics promotes the strengthening of academic knowledge when students understand the mathematical ideas, procedures, and practices present in their daily lives. These are the main ideas of "nonkilling mathematics" as proposed by D'Ambroiso in his search for peace and transcendence.
- *Epistemological:* This dimension deals with knowledge systems, which are sets of empirical observations developed to understand, comprehend, explain, and cope with reality. Thus, three questions arise regarding the evolution of mathematical knowledge in relation to diverse forms of generation, organisation, and dissemination: (a) how to move from ad hoc observations and practices to experimentation and methods; (b) how to move from experimentation and method to reflection and abstraction; and (c) how to proceed towards inventions and theories. These questions guide reflections regarding this evolution by considering the unique interplay between people and their own reality.
- *Historical:* It is necessary to study the links between the history of mathematics and the reality of the learners. This dimension leads students to examine the nature of Mathematics in terms of understanding how mathematical knowledge is allocated in their individual and collective experiences. Thus, knowledge is constructed from the interpretations of how humanity has analysed and explained mathematical phenomena throughout history. This is why it is necessary to teach mathematics within a historical context, so students can understand the evolution of and the contributions made by other peoples to the ongoing development of mathematical knowledge
- *Political:* This dimension aims to recognise and respect the history, tradition, and mathematical thinking developed by members of distinct cultural groups. It stresses the

importance of recognising and respecting the socio-cultural roots of others while reinforcing these roots through dialogue in cultural dynamism. It also aims to develop political actions that guide students in transitioning from subordination to autonomy, helping them to gain a broader understanding of their rights as citizens.

These dimensions show that the ethnomathematics programme has an agenda that offers a broader view of mathematics that embraces ideas, processes, methods, and practices that are related to different cultural environments. This aspect leads to increased evidence of cognitive processes, learning capabilities, and attitudes that may direct the learning process occurring in our classrooms. Ethnomathematics offers an important perspective for a dynamic and globalised modern society that recognises that all cultures and all people develop unique methods and explanations that allow them to understand, act, and transform their own reality. The need to imbibe this perspective in present-day mathematics instructional design cannot be overemphasised.

5. Culture-based Mathematics Instructional Design

Culture is a learned behaviour consisting of thoughts, feelings and actions and is transferred in social interaction (Vanio, Walsh & Varsaluoma, 2014). Within every culture, there exists indigenous knowledge which encompasses the complex, intergenerational and cumulative experiences and teachings of the indigenous peoples (Jacob, Sabzalian, Jansen, Tobin, Vincent & LaChance, 2018). However, many contemporary educational researchers agree that there is a discontinuity between the home or community culture of students and the education they receive in mainstream schools (Ezeife, 2011). The lack of relevance of school mathematics to the learners' everyday life and culture suggests that there is a need to incorporate into the mathematics curriculum such cultural practices, ideas and beliefs that would connect the school to the community in which it exists and functions. Educators, academics and policymakers have called for more research that addresses gaps in understanding of culture-based Mathematics Education (Kanaiaupuni, 2007).

Culture-based Mathematics Education is the teaching and learning of mathematics that takes into consideration the context of the learners, blending academic and vocational competencies. Contextualisation is based on the proposition that people learn more effectively when they are learning about something that they are interested in, that they already know something about, and that affords them the opportunity to use what they already know to figure out new things (Epper & Baker, 2009 and CUNY, 2003). The use of locally relevant contexts—situations and phenomena that have local and personal meaning to students and teachers for whom a curricular product is designed—provides access to educational and social participation and opportunity at multiple levels of practice (Ebby et al., 2011). In other words, Culture-based Mathematics Education, in addition to attending to academic goals, must take seriously the ways students' experiences are structured by policies, institutions and societal practices and work with students to confront them.

Evidently, new tools and media can be extremely helpful to many mathematics teachers who would otherwise struggle to provide culture-based mathematics instruction. If schools are to provide such forms of instruction effectively and at scale, they will require a new technology infrastructure such as e-learning and other digitally designed tools (Dede, 2014). E-learning can be defined as the use of computer and Internet technologies to deliver a broad array of solutions to enable learning and improve performance (Ghirardini, 2011). However, e-learning is a cultural artefact and, as such, it is infused with characteristics that reflect those of the designing culture. In other words, any e-learning application will possess characteristics that reflect the culture of its originators and users, from the types of pedagogies they prefer to their cultural expectations and values (Masoumi & Lindstrom, 2009). Accepting this view that culture is an integral part of every instructional design makes it important to consider social and cultural differences in designing and providing mathematics education and instruction.

Bringing culture to the nexus of discussions and enactments (that is, what people do and how they do it) in designing e-learning and seeking to align teaching and instruction to the cultural contexts of ethnically diverse learners challenges mainstream notions of teaching and learning (Masoumi & Lindstrom, 2009). Cross-cultural design is designing technology for different cultures, languages, and economic standings, ensuring usability and user experience across cultural boundaries (Vanio, Walsh & Varsaluoma, 2014). Such a user-centred design approach supports the cross-cultural product development process with user-centred activities identifying the need for internationalisation and localisation.

In the field of instructional technology, “development” has a somewhat unique connotation. A typical definition views “development” as the process of translating the design specifications into physical form (Richey, Klein & Nelson, 2004). In other words, it refers to the process of producing instructional materials, interventions, and even web-based products. On this premise, design-based research (DBR) methods focus on designing and exploring the whole range of designed innovations: artefacts as well as less concrete aspects such as activity structures, institutions, scaffolds, and curricula (The Design-Based Research Collective, 2003). Interventions such as web-based educational products embody specific theoretical claims about teaching and learning and reflect a commitment to understanding the relationships among theory, designed artefacts, and practice.

Basically, design-based research (DBR) is a process that integrates design and scientific methods to allow researchers to generate useful products and effective theory for solving individual and collective problems in education (Easterday, Lewis & Gerber, 2014). Ethnomathematics instructional design, like other DBR processes, consists of six (6) iterative phases in which designers focus on the problem, understand the problem, define goals, conceive the outline of a solution, build the solution, and test the solution. Following this blueprint, mathematics education researchers like Mosimege (2004) report outcomes of a South African programme that calls upon curriculum planners and implementers to incorporate indigenous knowledge systems within Mathematics. The extent to which mathematical knowledge is exhibited in cultural villages, both by the workers and in the artefacts made, was discussed in relation to how these can be used in mathematics classrooms. Mosimege (2004) lists mathematical concepts identified in the making of a grass container and beadwork to include estimation, lines, shapes, patterns, and angles.

Further design-based studies have shown that culture-based mathematics education can have significant positive effects for students, including improved retention, graduation rates, college attendance rates, and standardized test scores (Best & Dunlap, 2013). Fenyvesi, Koskimaa, and Lavicza (2014) show that creating visual illusions, paradox structures, and "impossible" figures through playful and artistic procedures holds an exciting pedagogical opportunity for raising students' interest in mathematics. To anchor this, innovatively designed games were deployed to clarify mathematical concepts related to visual illusions, such as symmetry, perspective, and isometric projection (Fenyvesi, Koskimaa & Lavicza, 2014).

Relatedly, Neel (2010) carried out a study in the culture-based mathematics instructional design paradigm, in which members of the Haida Role Model Programme on the islands of Haida Gwaii were interviewed to determine how they "Do the Math" in their daily lives. The programme "consists of elders, professionals and community members who go to schools and assist teachers in integrating Haida knowledge and perspective with the school curriculum. The Role Models provide a vital connection between the school district community and the Haida community. The outcomes of the instructional design show that the mathematical practices in the community life of Haida Gwaii are unique to its people, land and context. The culture-based intervention was useful in integrating students' experiential mathematics with their school mathematics, for the purpose of helping them to be motivated and make new connections to improve achievement. This disposition that mathematics is useful and meaningful for the Indigenous students is demonstrated by showing them how traditional and contemporary cultural activities have many mathematical concepts embedded in them. Neel (2010) reports that broadly, the ability to learn mathematics increases when the students are taught skills that are useful for their daily functioning in the home, the workplace, and the community. A similar approach was used by Francoise, Mafra, Fantinato and Vandendriessche (2018) to design culture-based mathematics instruction involving string figure making and handcrafted calabash gourds, with the outcome affirming that out-of-school practices are dynamic in nature and they are performed along an informal-formal learning continuum.

6. Digital Content Management in Instructional Environments

Today's learners have changed incrementally from those of the past due to the arrival and rapid dissemination of digital technology. Connected devices, social networks, educational cloud services and other innovations have essentially inverted relationships between learners and schools (Elliot, Kay & Laplante, 2016; Iji, Abah & Anyor, 2017; Iji, Abah & Anyor, 2018).

Today’s learners represent the first generations to grow up with this new technology, spending their entire lives surrounded by and using computers, video games, digital music players, video cameras, smartphones and all other toys and tools of the digital age (Prensky, 2001). Learners today are native speakers of the digital language of computers, video games and the Internet.

These disruptive technological changes imply that teachers have to learn to communicate in the language and style of their students. There is also an enforced redefinition of instructional content. In this sense, Prensky (2001) observes that there are now two kinds of content: “Legacy” content and “Future” content. Legacy content includes reading, writing, arithmetic, logical thinking, and understanding the writings and ideas of the past and the traditional curriculum. Future content is to a large extent digital and technological. Evidently, present-day educators need to think about how to teach both legacy and future content in the language of the Digital Natives (Prensky, 2001).

Digital content is referred to by one or more several terms including reusable learning objects, electronic records, digital assets, datasets and media assets. Basically, the term digital content is a general one that encompasses text, image, audio and multimedia digital files, and datasets that are used for the purposes of instruction, research and study (Cywin *et al.*, 2011). These digital files are referred to as assets because a significant amount of time, effort and expenses goes into creating content, thus making the files a valuable resource. However, the value of these assets can only be truly realised if they are accessible to everyone who needs them, when they need them (Extensis, 2018). Digital materials provide many teaching and learning benefits to educators and students. They can be updated more quickly than traditional print materials, may be adapted to address students' learning differences and styles, and can offer interactive functions that pique learners’ interests (State Educational Technology Directors Association - SETDA, 2015). These advantages are particularly true of digital Open Educational Resources (OER), which offer a lawful pathway for tailoring and adapting content to meet a student’s unique learning needs.

Digital Content Management (DCM) has been defined as a series of tasks and decisions surrounding the annotation, cataloguing, storage, retrieval, and distribution of digital content (Cywin *et al.*, 2011). However, in light of recent technological developments, the phrase “Digital Content Management” has become more synonymous with software applications and enterprise solutions. A digital content management system is built upon a central repository that facilitates digital file storage, organisation, retrieval, utilisation, and reuse. Such a system is a “filing cabinet” containing individual files that are stored with detailed information or metadata about a digital asset (Frey, Williams-Allen, Vogl & Chandra, 2005). Metadata can be wrapped around information as a sort of digital data container. The data container is a set of categories, such as creation date, creator, additional versions, related files, and copyrights (Frey *et al.*, 2005). Digital Content Management Systems tend to be very robust and address a many-to-many relationship with database objects, particularly within systems designed to aid mathematics instruction.

The Web-based Ethnomathematics Instructional Content Repository reported by Abah, Iji, Abakpa, and Anyagh (2021) and Abah (2024) is managed through WordPress. WordPress (WordPress.org) is a free and open-source content management system (CMS) based on PHP and MySQL. *PHP*is a server-side scripting language for creating dynamic web pages. When a visitor opens a page built in PHP, the server processes the PHP commands and then sends the results to the visitor's browser. *MySQL*is an open-source relational database management system (RDBMS) that uses*Structured Query Language (SQL)*, the most popular language for adding, accessing, and processing data in a database. MySQL is a big filing cabinet where all the content on a site is stored. Every time visitors go to https://villagemath.net to interact with ethnomathematics instructional content, they make a request that is sent to a host server (Abah, Iji, Abakpa, & Anyagh, 2021; Abah, 2024). The PHP programming language receives that request, makes a call to the MySQL database, obtains the requested information from the database, and then presents the requested information to the visitors through their web browsers (See Figure 1).

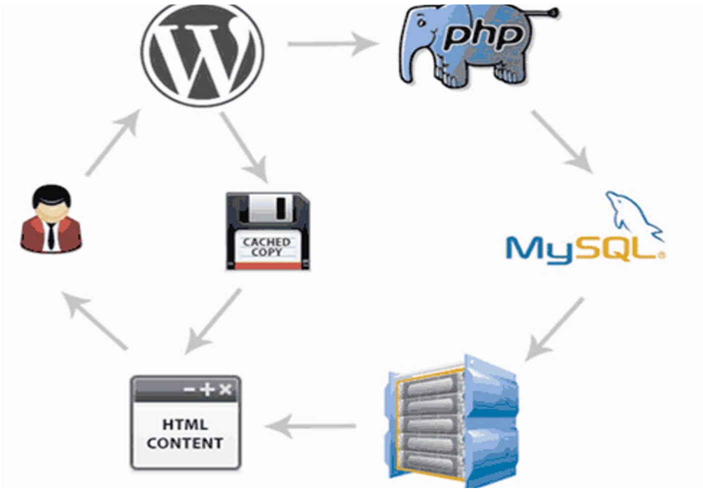


Figure 1: *User Pathway in WordPress*

Digital Content Management Systems are integrated processes, which can be broken down into three parts by function (Frey *et al*., 2005):

i. *Collection:* During collection, raw information is transformed into a set of content components. Information is created or acquired, converted into a master format (e.g. XML) if necessary, edited, segmented into components, and has metadata added.

ii. *Management:* The management function is part of the administrative infrastructure of a DCM System. In general, the administrative infrastructure provides a way to keep track of digital assets and associated status. It is a repository used for long-term storage and other administrative resources. The repository is made up of database records and/or files that hold content and other administrative data (such as a system's users).

iii. *Publishing:* The publishing function can combine content components and other resources from the repository to create publications. Publications can be created in a variety of formats, printable or electronic (e.g. web, e-mail). They consist of components, functionality, metadata, and navigation information. Content can also be published simultaneously in multiple formats.

With the robustness of digital content management, delivering experiences – rather than merely publishing content – is an aspirational goal for educational initiatives. Elliot, Kay, and Laplante (2006) state that the availability of proven technologies for content and experience management is not a key challenge, as the technology landscape is mature and populated with choices that meet most intervention needs, with some categories offering digital capabilities well beyond web publishing. In relation to culture-based content, such systems aim to preserve cultural heritage and collections; popularise fine cultural landmarks; encourage information and knowledge sharing; invigorate cultural content and value-added services; and improve literacy, creativity, and quality of life (Hsu, Ke, & Yang, 2006). A knowledge management framework not only aims to manage knowledge assets but also to manage the processes that act upon the assets. These processes include developing, preserving, using, and sharing knowledge, as shown in the system framework in Figure 2.

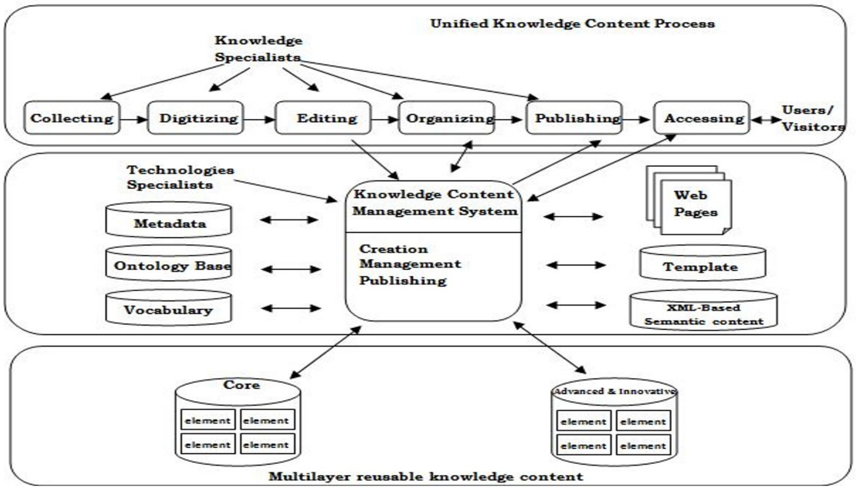


Figure 2: *Unified Knowledge-based Content Management System Framework* (Source: Hsu, Ke & Yang, 2006)

The three components of this framework are unified content processes, an integrated knowledge-based content management system, and a multilayer reusable knowledge content structure. The first component functions as a common workflow among participants and projects that include knowledge content collection, digitisation, editing, organising, publishing, and accessing stages (Hsu, Ke & Yang, 2006). The multilayer reusable knowledge content structures define the spectrum of knowledge content for all participants to follow, from core knowledge elements to advanced and innovative elements. A core knowledge element is the basis of knowledge content and comprises a multimedia object and semantic metadata. Advanced and innovative elements are further manually authored or automatically inferred from existing content. The integrated knowledge-based content management system comprises the creation subsystem for constructing vocabulary, metadata, content, and the classification hierarchy, the management subsystem, and the publishing subsystem to transfer the authored content into the publishing structure and web pages for all users (Hsu, Ke & Yang, 2006).

In an instructional environment, Ghirardini (2011) relates that e-learning approaches can combine four (4) broad types of e-learning components.

i. E-Learning content

E-learning content can include simple learning resources, interactive e-lessons, electronic simulations, and job aids. Simple learning resources are non-interactive resources such as documents (Word and PDF), PowerPoint presentations, videos, or audio files. These materials are non-interactive in the sense that learners can only read or watch content without performing any other action. These resources can be quickly developed and, when they match defined learning objectives and are designed in a structural way, they can be a valuable learning resource even though they do not provide any interactivity.

The most common approach for self-paced e-learning is web-based training consisting of a set of interactive e-lessons. An e-lesson is a linear sequence of screens that can include text, graphics, animations, audio, video, and interactivity in the form of questions and feedback. E-lessons can also include recommended reading and links to online resources, as well as additional information on specific topics.

Simulations are also highly interactive forms of e-learning. The term "simulation" basically means creating a learning environment that mimics the real world, allowing the learner to learn by doing. Simulations are a specific form of web-based training that immerse the learner in a real-world situation and respond in a dynamic way to their behaviour.

Job aids provide just-in-time knowledge. They usually provide immediate answers to specific questions, thus helping users to accomplish job tasks. Technical glossaries and checklists are a few examples of simple job aids, but they can also assist workers in complex decision-making.

ii. E-tutoring, e-coaching, e-mentoring

These services add human and social dimensions to learners, supporting them throughout the learning experience. E-tutoring, e-coaching, and e-mentoring offer individual support and feedback to learners using online tools and facilitation techniques.

iii. Collaborative learning

Collaborative activities encompass discussions and knowledge sharing, as well as working together on a common project. Social software, such as chats, discussion forums, and blogs, is utilised for online collaboration among learners. Synchronous and asynchronous online discussions allow learners to comment and exchange ideas about course activities, as well as contribute to group learning by sharing their knowledge. Collaborative project work entails cooperation among learners to complete a task. Collaborative activities may include project work and scenario-based assignments.

iv. Virtual classroom

A virtual classroom is the instructional method most similar to traditional classroom training, as it is entirely led by an instructor. It is an e-learning event where an instructor teaches remotely and in real time to a group of learners using a combination of materials, such as PowerPoint slides and audio or video content. This format is also referred to as synchronous learning.

7. Empirical Case Studies

The conceptual frameworks for instructional design presented thus far have been practically deployed in various studies under different conditions for a wide range of teacher preparation purposes. Wang, Zhou, and Zou (2004) conducted a study on the Web-based Mathematics Education Framework (WME), which aims to create a web for mathematics education, empowering mathematics teachers, learning content developers, and dynamic mathematics computation and education service providers to deliver an unprecedented mathematics learning environment to students and educators. WME provides an authoring language—Mathematics Education Markup Language (MeML)—which works with regular browsers, simplifies page creation, allows systematic access to supporting WME services, and enables independently developed components to interoperate seamlessly. The designed MeML aims to provide an effective and expressive means for authoring and delivering both static and dynamic content on the web. The MeML Document Type Definition (DTD) defines the syntax of MeML elements. The prototype client-side WME Page Processor, Woodpecker, includes an XSLT processor, XSLT templates, and a collection of web browser plug-ins called MeML plug-ins. Translation is performed by an XSLT processor. The translated XHTML page may also include embedded event handlers (in Javascript) and references to MeML plug-ins. The WME/MeML represents an ambitious vision that, when fully realised, will revolutionise computing in mathematics education, making the presentation of mathematical symbols, equations, styles, and fonts easy on the web.

In a related study, Changiz, Haghani, and Masoomi (2012) provided a report on the design and implementation of a web directory for medical education, creating a tool to facilitate research in this field. The objective was to design and implement a comprehensive, subject-specific web directory to ease access to medical resources online. The categories to be included in the directory were defined through a review of related directories and consultations with medical education experts in a focus group. Sources such as search engines, subject directories, databases, and library catalogues were searched to select and collect high-quality resources. The main categories indexed are journals, organisations, best evidence, and textbooks. Sub-categories and related resources for each category are described and linked. Despite the success of the implemented directory, limitations related to the inherent constraints of subject directories were reported. The database size of subject directories is very small compared to search engines, indexed sites and resources in subject directories are typically updated with delays, there is a risk of overlooking potentially useful sites and resources, and the costs of maintenance and updating are high.

In a study implementing a game to support learning in mathematics, Katmada, Mavridis, and Tsiatsos (2014) focused on the design, implementation, and evaluation of an online game for elementary and middle school mathematics. The two-fold aim of the effort was the development of a prototype for a flexible and adaptable computer game, and the evaluation of this prototype regarding its usability and technical aspects. In addition to the game, an administration website was constructed to enable educators to configure the game by altering various parameters, such as the content and total number of questions. The game was evaluated in real school settings through a pilot study involving 12 students, followed by a long-term intervention with 37 students lasting 14 weeks. The results indicated that students had a positive opinion of the game and suggested that, with some enhancements, it could serve as an effective learning tool. Although the study suggests that game-based learning activities were well accepted and appreciated by the students, some encountered minor usability challenges. Specifically, "five students stated in their comments that they would prefer the game to be in Greek rather than English" (p.238), indicating a missing ethnomathematical dimension.

A Nigerian study by Charles and Babatunde (2014) presented the design and implementation of a virtual classroom system focused on collaborative learning between students and tutors at remote locations. The various components of the system allow students to engage in group activities and collaborate with instructors in commonly shared windows where text, audio, or video objects can be added and shared online. The system was developed and hosted on the web using Moodle, Elluminate, WAMP server, JavaScript, MySQL, PHP, and Dreamweaver. A practical demonstration was conducted using an undergraduate course in the Department of Computer Science at the Federal University of Technology, Akure, Nigeria. Results from the demonstration exhibited the suitability and adequacy of the virtual classroom system. The views and responses of 100 students and 10 instructors from FUTA were gathered through a questionnaire featuring indices such as accessibility, provision of an appropriate level of interaction, efficiency of support for learning methods, convenience of use, usefulness for active learning, and aid to teaching. These indices broadly align with the criteria for evaluating the quality of web-based instructional environments.

The study by Abah, Iji, Abakpa, and Anyagh (2021) adopted a developmental research design to systematically build and evaluate an educational intervention as a solution to complex problems in mathematics education practice. This development was achieved using WordPress Version 5.4, hosted online at https://villagemath.net on a Linux OS server running cPanel v80.p (Build 20), Apache Version 2.4.39, PHP Version 5.6.40, and MySQL Version 5.7.26. The study examined common web metrics, key performance indices, and quality assessment in terms of content, navigation, structure, appearance, and uniqueness of the designed web tool. The results yielded a positive pattern of common web metrics for the designed web tool, indicating that the platform appeals to a wide range of users. Key performance indicators such as speed index, page size, and last painted hero affirmed that the platform is robust, elegantly designed, and fast. The outcomes from Abah, Iji, Abakpa, and Anyagh (2021) demonstrated that culture can indeed become an integral part of every aspect of instructional design, making it important to consider social and cultural peculiarities in planning and delivering mathematics instruction. The Web-based Ethnomathematics Instructional Content Repository has humanised mathematics for users and provided a reservoir of resources for training students in conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition.

The context of presenting a culture-based mathematics education repository is expected to be structured in culturally appropriate ways, making learning meaningful and relevant through culturally grounded content and assessment. The system must also gather and maintain data to ensure progress in culturally responsible ways. On these premises, Abah (2024) undertakes a re-examination of an earlier designed web tool to present the outcomes of the structural quality assessment of the VillageMath educational intervention for mathematics teachers. The analysis of the results affirmed the structural quality of VillageMath, indicating that mathematics teacher educators in institutions of higher learning can continue to use the platform as a dependable tool for voicing narratives across the field of ethnomathematics. The forum available on the platform can be utilised by these experts in mathematics education to communicate their development of state-of-the-art pedagogies for the field, particularly in extending narratives of African indigenous knowledge systems.

8. Conclusion

The deliberations of this chapter have highlighted the existing gap in the literature regarding the incorporation of cultural competence and diversity training in teacher preparation programmes. The review has adequately showcased the potential of culture-based mathematics instructional design in integrating culturally responsive teaching practices, addressing implicit biases, and promoting equity and social justice in mathematics education. The diverse case studies cited in the discussion underscore growing efforts in the instructional design community to project the cultural significance of mathematical abstractions as they emerge and are used in the everyday lives of learners. Teacher preparation programmes in higher education institutions must inculcate the intricacies and richness of the culture-based approach in the delivery of the mathematics education curriculum. Early exposure of trainee teachers to available digital platforms for mathematics instruction delivery is key. Projects that seek to integrate cultural dimensions into mathematics problem-solving must be encouraged during mathematics teacher preparation, thereby laying a practical foundation for pre-service mathematics teachers to translate adopted culture-based paradigms into actionable instructional experiences for learners when they enter the field of practice.

This chapter has advocated for mathematics education instructional design that is rooted in the everyday lives of learners, with the hope of demystifying the subject of mathematics across all school levels. To achieve seamless technology-based mathematics teaching and learning, mathematics teachers must be properly trained in the content, pedagogy, and technology of mathematics instructional delivery that considers the culture of the learners. This approach serves not only to improve academic outcomes in mathematics but also to preserve the cultural heritage of the learners. The body of research reviewed in this chapter has affirmed the feasibility of using digital technology to blend culture-based mathematics education within mathematics teacher preparation programmes.

Despite the elegance of the conceptualisation discussed in this chapter, there must be a concerted effort by practitioners, in-service and pre-service mathematics teachers, educational policymakers, and the indigenous communities served by schools to translate tested templates into fruitful realities. Further research into understanding how the elements of ethnomathematics instructional design affect the effectiveness of pre-service teachers in diverse classrooms is of utmost importance. Future studies may explore the prospects of integrating specific folklore of indigenous communities into mathematics instructional design. Additional studies may test unique culture-based interventions within quasi-experimental setups to interpret the efficacy of the conceptual foundation documented in this chapter.

9. Declarations

Funding: This research did not receive any external funding.

Conflicts of Interest: The author declares no conflict of interest.

References

Abah, J. A. (2024). Structural quality assessment of the villagemath instructional content platform. *Journal of Advanced Sciences and Mathematics Education*, *4*(1), 1-14. https://doi.org/10.58524/jasme.v4i1.349

Abah, J. A., & Chinaka, T. W. (2024). Conceptualization and Development of a Web-based Platform for Multicultural Mathematics Resources. *Research in Educational Policy and Management*, *6*(2), 256-280. https://doi.org/10.46303/repam.2024.34

Abah, J. A., Iji, C. O. & Abakpa, B. O. (2018). Blown Away in the Wind of Change: Can Extinct School Folktales be awakened through Mathematics Storytelling in Nigerian Basic Education? *International Journal on Emerging Mathematics Education (IJEME), 2(2),* 165-178. http://journal.uad.ac.id/index.php/IJEME/article/download/10561/pdf_28

Abah, J. A., Iji, C. O., Abakpa, B. O. & Anyagh, P. I. (2021). Design, Implementation and Assessment of a Web-Based Ethnomathematics Instructional Content Repository for Pre-Service and In-Service Mathematics Teachers in Benue State, Nigeria. *Journal of Mathematics and Science Teacher*, 1(1), em002. https://doi.org/10.29333/mathsciteacher/11075

Barkatsas, A. (2004). *A new scale for monitoring students' attitude for learning mathematics with technology*. Melbourne: St. Joseph's College. pp 1-8.

Best, J. & Dunlap, A. (2013). Native culture and language education: Addressing the interests of special populations within US Federal policy. *Forum Briefing Materials: Culturally Based education* *for Indigenous Language and Culture.* pp 5-8. Rapid City, South Dakota: REL Project, US Department of Education Institute of Educational Sciences.

Beswick, K. & Goss, M. (2018). Mathematics teacher educator knowledge: What do we know and where to from here? *Journal of Mathematics Teacher Education, 21,* 417-427.

Changiz, T., Haghani, F. & Masoomi, R. (2012). Design and implementation of web directory for medical education (WDME): A tool to facilitate research in medical education. *MED ARH, 66(2)*, 133-136. https://doi.org/10.5455/medarh.2012.66.133-136

Charles, A. O. & Babatunde, I. G. (2014). Design and implementation of a web-based virtual classroom system. *IOSR Journal of Research & Method in Education, 4(3), VII,* 68-77.

CUNY (2003). *Final report: Handbook on contextualized learning.* New York, NY: Division of Adult and Continuing Education, Office of Academic Affairs, City University of New York.

Cywin, A. J., Ferrar, J., Russo, T., Maucione, J. C., Confar, D., McNeil, C., Sylvain, M., Pacheco, S., & Gatenby, D. (2011). *Digital asset management team report.* Dartmouth: University of Massachusetts.

D'Ambrosio, U. (1987). Reflections on ethnomathematics. *International Study Group on Ethnomathematics Newsletter, 3(1).*

D'Ambrosio, U. (1994). Cultural framing of mathematics teaching and learning. In R. Biehler, R. W. Scholz, R. Straber & B. Winkelmann (Eds.), *Didactics of Mathematics as a Scientific Discipline*, 443-455.

D'Ambrosio, U. (2001a). *Ethnomathematics: Link between traditions and modernity.* Rotterdam: Sense Publisher Publishers.

D'Ambrosio, U. (2001b). What is ethnomathematics, and how can it help children in schools? *Teaching Children Mathematics, 7(6)*, 308.

Dede, C. (2014). The role of digital technologies in deeper learning. *Student at the Center: Deeper Learning Research Series.* Jobs for the Future.

Easterday, M. W., Lewis, D. R. & Gerber, E. M. (2014). Design-based research process: Problems, phases, and applications. *Learning and Becoming in Practice. The International Conference of the Learning Science (ICLS) 2014, Proceedings Volume 1*, 317-324.

Ebby, C., Lim, V., Reinke, L., Remillard, J., Magee, E., Hoe, N., & Cyrus, M. (2011). Community based mathematics project: Conceptualizing access through locally relevant mathematics curricula. *Perspectives on Urban Education, Spring 2011*, 11-18.

Elliot, T., Kay, M. & LaPlante, M. (2016). *Digital transformation in higher education: How content management technologies and practices are evolving in the era of experience management.* New York, NY: Digital Clarity Group. pp. 1-35.

Epper, R. M., & Baker, E. D. (2009). Technology solutions for developmental math: An overview of current and emerging practices. *Journal of Developmental Education, 26(2)*, 4-23.

Extensis (2018). *Digital asset management best practices guide.* Extensis.

Ezeife, A. N. (2011). A cultural and environmental spin to mathematics education: Research implementation experience in a Canadian aboriginal community. *First Nations Perspectives*, *4*(1), 2-39.

Fenyvesi, K., Koskimaa, R., & Lavicza, Z. (2014). Experiential education of mathematics: Art and games for digital natives. *Kasvatus & Aika, 9(1)*, 107-134.

Fenyvesi, K., Koskimaa, R. & Lavicza, Z. (2014). Experiential education of mathematics: Art and games for digital natives. *Kasvatus & Aika, 9(1)*, 107-134.

Fouze, A. Q., & Amit, M. (2018). Development of mathematical thinking through integration of ethnomathematics folklore game in math instruction. *Eurasia Journal of Mathematics, Science and Technology Education, 14(2)*, 617-630.

Francois, K., Mafra, J. R., Fantinato, M. C., & Vandendriessche, E. (2018). Local mathematics education: The implementation of local mathematics practices into the mathematics curriculum. *Philosophy of Mathematics Education Journal, 33,* 1–18.

Freudenthal, H. (1968). Why to teach mathematics so as to be useful. *Education Studies in Mathematics, 1,* 3–8.

Freudenthal, H. (1971). Geometry between the devil and the deep sea. *Educational Studies in Mathematics, 3,* 413–435.

Freudenthal, H. (1973). *Mathematics as an educational task*. Dordrecht: Reidel Publishing.

Freudenthal, H. (1991). Revisiting mathematics education. *China lectures*. Dordrecht: Kluwer.

Frey, F., Williams-Allen, S., Vogl, H., & Chandra, L. (2005). *Digital asset management: A closer look at the literature*. Rochester, NY: Printing Industry Center, RIT. pp. 1–44.

Ghirardini, B. (2011). *E-learning methodologies: A guide for designing and developing e-learning courses*. Rome: Food and Agricultural Organization of the United Nations. pp. 1–138.

Gravemeijer, K. (2008). RME theory and mathematics teacher education. In D. Tirosh & T. Wood (Eds.), *Tools and processes in mathematics teacher education* (pp. 283–302). Boston, CT: Sense Publishers.

Gravemeijer, K., Stephan, M., Julie, C., Lin, F.-L., & Ohtani, M. (2017). What mathematics education may prepare students for the society of the future? *International Journal of Science and Mathematics Education, 15 (Suppl. 1),* S105–S123.

Hsu, T.-Y., Ke, H.-R., & Yang, W.-P. (2006). Unified knowledge-based content management for digital archives in museums. *The Electronic Library, 24*(1), 38–50. https://doi.org/10.1108/02640470610649236

Iji, C. O., & Abah, J. A. (2019). Internet skills as a measure of digital inclusion among mathematics education students: Implications for sustainable human capital development in Nigeria. *International Journal of Education and Knowledge Management (IJEKM), 2*(1), 1–16. https://rpajournals.com/wp-content/uploads/2019/02/IJEKM-01-2019-06.pdf

Iji, C. O., Abah, J. A., & Anyor, J. W. (2018). Educational cloud services and the mathematics confidence, affective engagement, and behavioural engagement of mathematics education students in public universities in Benue State, Nigeria. *International Journal of Teaching and Learning in Higher Education, 30*(1), 47–60. http://www.isetl.org/ijtlhe/articleView.cfm?id=2825

Iji, C. O., Abah, J. A., & Anyor, J. W. (2017). Impact of cloud services on students' attitudes towards mathematics education in public universities in Benue State, Nigeria. *International Journal of Research in Education and Science (IJRES), 3*(1), 228–244. http://dergipark.ulakbim.gov.tr/ijres/article/download/5000202010/5000179858

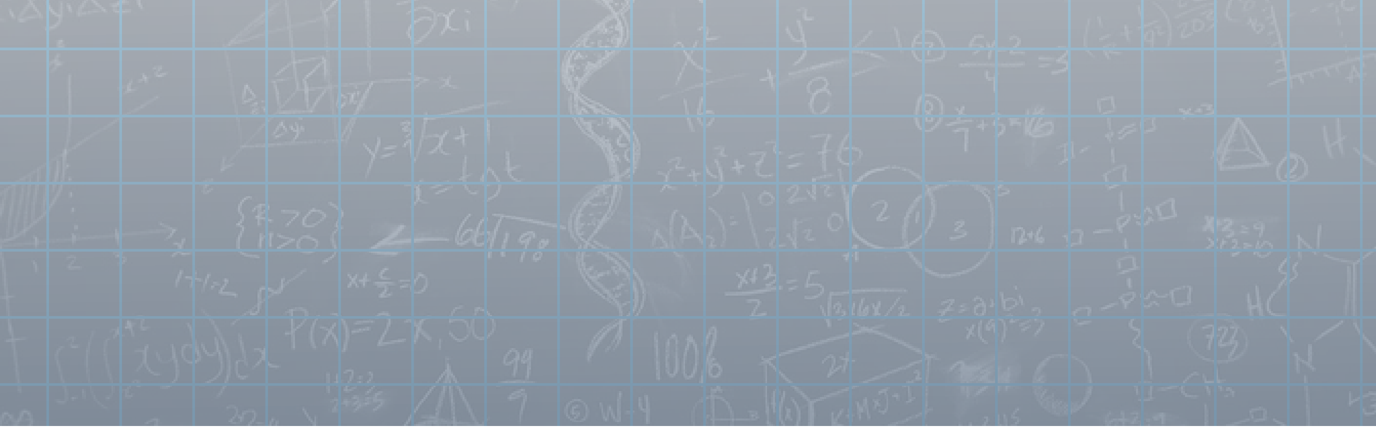
Iversen, E., & Jonsdottir, G. (2018). A bit more than a fly on the wall: Roles and responsibilities in design-based research. *Designs for Learning, 10*(1), 18–28. https://doi.org/10.16993/dfl.79

Jacob, M. M., Sabzalian, L., Jansen, J., Tobin, T. J., Vincent, C. G., & Lachance, K. M. (2018). The gift of education: How indigenous knowledges can transform the future of public education. *International Journal of Multicultural Education, 20*(1), 157–185.

Kanaiaupuni, S. (2007). *A brief overview of culture-based education: An annotated bibliography*. Kamehameha Schools Research and Evaluation Division.

Katmada, A., Mavridis, A., & Tsiatsos, T. (2014). Implementing a game for supporting learning in mathematics. *Electronic Journal of e-Learning, 12*(3), 230–242.
Kilpatrick, J., & Findell, B. (2001). *Adding it up: Helping children learn mathematics*. Washington, D. C.: National Academy Press.
Longview Foundation. (2008). *Teacher preparation for the global age: The imperative for change*. Longview Foundation for Education in World Affairs and International Understanding, Inc.
Masoumi, D., & Lindstrom, B. (2009). Foundations of cultural design in e-learning. *International Journal of Internet and Enterprise Management, 6*, 124–142. https://doi.org/10.1504/IJIEM.2009.023926
Mncube, V., & Olawale, E. (2020). Exploring readiness of South African universities for the Fourth Industrial Revolution. In*EDULEARN20 Proceedings*(pp. 8989–8995). IATED.
Mosimege, M. (2004). Indigenous mathematical knowledge at South African cultural villages: Opportunities for integration in mathematics classrooms. Retrieved 1 November 2018 from http://citeseerx.ist.psu/viewdoc/download?doi=10.1.1.577.1920&rep=rep1&type=pdf
Neel, K. S. (2010). *Ability to learn mathematics for Aboriginal students increases if the context, personal and cultural relevance are meaningful*. Paper presented at the AARE Annual Conference, Melbourne, Australia. Paper Code 1779.
Nouri, J., Spikol, D., & Cerratto-Pargman, T. (2016). A learning activity design framework for supporting mobile learning. *Designs for Learning, 8*(1), 1–12. https://doi.org/10.16993/dfl.67
Prensky, M. (2001). Digital natives: Digital immigrants – part 1. *On the Horizon, 9*(5), 1–6. https://doi.org/10.1108/10748120110424816
Richey, R. C., Klein, J. D., & Nelson, W. A. (2004). Developmental research: Studies of instructional design and development. In D. Jonassen (Ed.), *Handbook of research for educational communications and technology* (2nd ed., pp. 1099–1130). Mahwah, NJ: Lawrence Erlbaum Associates Inc.
Rosa, M., & Orey, D. (2010). Ethnomodeling as a pedagogical tool for the ethnomathematics program. *Revista Latinoamericana de Ethnomathematica, 3*(2), 14–23.
Rosa, M., & Shirley, L. (2016). Introduction. In G. Kaiser (Ed.), *Current and future perspectives of ethnomathematics as a program*. ICME-13 Topical Surveys (pp. 1–4). Switzerland: Springer. https://doi.org/10.1007/978-3-319-30120-4_3
State Educational Technology Directors Association (SETDA). (2015). *Ensuring the quality of digital content for learning: Recommendations for K–12 education*. Retrieved 21 October 2018, from https://www.setda.org/wp-content/uploads/2015/03/Digital_brief_3.10.15c.pdf
The Design-Based Research Collective. (2003). Design-based research: An emerging paradigm for educational inquiry. *Educational Researcher, 32*(1), 5–8.
Vainio, T., Walsh, T., & Varsaluoma, J. (2014). Cross-cultural design of mobile mathematics learning for South African schools. *IADIS International Journal on WWW/Internet, 12*(1), 81–93.
Van den Heuvel-Panhuizen, M. (2003). The didactical use of models in realistic mathematics education: An example from a longitudinal trajectory on percentage. *Educational Studies in Mathematics, 54*, 9–35.
Van den Heuvel-Panhuizen, M., & Drijvers, P. (2014). Realistic mathematics education. In S. Lerman (Ed.), *Encyclopedia of mathematics education*. Dordrecht: Springer Science and Business Media. https://doi.org/10.1007/978-94-007-4978-8

Wang, P. S., Zhou, Y., & Zou, X. (2004). Web-based mathematics education: MeML design and implementation. *Proceedings of the Conference on Information Technology: Coding and Computing, 1,* 169–175.



The field of mathematics teacher preparation is at a pivotal moment in its evolution. As educational landscapes continue to shift, there is an urgent need for research and pedagogical strategies that embrace inclusivity, social justice, and innovation in teacher training. This edited volume presents a comprehensive exploration of contemporary challenges and solutions in preparing mathematics educators for diverse and dynamic classrooms. Incorporating perspectives from scholars across various contexts, this book provides invaluable resources for educators, policymakers, and researchers committed to improving mathematics instruction at all levels.

This book comprises ten chapters, each addressing a critical aspect of mathematics teacher education, including equity and social justice, self-efficacy, culturally relevant pedagogy, technological integration, and sustainable educational practices. Through a blend of empirical research and theoretical discourse, the contributors highlight the evolving nature of teacher preparation and the necessity of equipping future educators with the skills and mindsets required to navigate complex learning environments. Therefore, foregrounding issues of diversity and inclusion, this book underscores the transformative potential of effective teacher preparation in fostering equitable educational outcomes.

Hence, the book serves as both a catalyst for meaningful change and a reference point for future research in mathematics teacher education. As readers engage with the insights presented in these chapters, they are invited to reflect on how these perspectives can inform their own practices and policies. The collective responsibility is to ensure that mathematics education remains a space where every learner can thrive, and it is inspired that this book contributes meaningfully to that mission.

9781037053894