CHAPTER THREE

Visualising Mathematical Concepts through Dual Digital and Non-Digital Teaching Tools on Preservice Teachers' Pedagogical Content Knowledge

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Abstract: This chapter explores how dual digital and non-digital visualisation tools contribute to the formation of robust pedagogical content knowledge (PCK) in PSTs, enhancing their PCK and ability to teach mathematics effectively in diverse classroom settings. Grounded in the TPACK framework, the study examines the influence of these tools on PSTs' PCK. An interpretive qualitative approach was adopted, focusing on a cohort of 20 third-year PSTs from one class. Initially, their PCK was assessed using observational tools and focus group discussions during their microteaching sessions. Thereafter, design-based interventions were implemented during lectures, allowing PSTs to explore, develop, and integrate digital and non-digital tools in teaching Grade 10 mathematics topics (functions, measurements, and analytical geometry) over a semester. In the post-intervention phase, their PCK was re-evaluated as they integrated digital and non-digital tools into their microteaching. The same observational tools and focus group discussions were utilised to assess any changes in their PCK. Finally, semi-structured interviews were conducted to gather their reflections. Data were collected through observational tools, focus group discussions, and semi-structured interviews. The data were analysed using the TPACK framework as the analytical tool, intertwined with thematic analysis. The findings show that integrating digital and non-digital teaching tools to visualise mathematics concepts can significantly enhance PSTs' PCK and their ability to teach mathematics effectively. Therefore, this chapter recommends that mathematics teacher education programmes highlight the need for a balanced integration of diverse instructional tools to better prepare teachers for the challenges of contemporary mathematics education.

Keywords: Digital teaching tools, mathematics education, mathematics - 31 -aption- 31 -ing- 31 -o, non-digital teaching tools, pedagogical content knowledge.

1. Introduction

Mathematics teaching and learning remain critical areas of focus in education, with the persistent challenge of effectively communicating abstract concepts across educational levels. According toOlawale(2024), pre-service teacher (PST) education programmes significantly influence mathematics teachers' ability to instruct mathematical content in accordance with curriculum standards, augment their subject knowledge, and bolster their confidence. Nevertheless, the programmes exert negligible impact on their capacity to execute differentiated instruction within their instructional methodologies.For pre-service mathematics teachers, developing Pedagogical Content Knowledge (PCK), which integrates subject matter knowledge and pedagogical strategies, is essential for presenting complex ideas in ways that students can understand

(Shulman, 1986). Microteaching serves as a vital component of teacher education, offering pre-service teachers (PSTs) structured opportunities to practice and refine their teaching skills in a controlled environment (Fki, 2023; García-Esteban et al., 2016). Through cycles of teaching, feedback, and reflection, this approach enhances instructional techniques and fosters the development of PCK (Setyaningrahayu et al., 2019; Ramadhanti & Yanda, 2021).

Visualisation, as a pedagogical strategy that includes self-made visual representations, visual arts, and technology-based tools (An et al., 2023), further supports this development by bridging the gap between abstract mathematical theory and practical application, thereby improving PCK (Shulman, 1986). Digital tools, such as GeoGebra and augmented reality (AR), have been shown to facilitate the visualisation of complex mathematical concepts through dynamic and interactive features (Muslim, 2023). GeoGebra, for example, supports learning in calculus and geometry by providing dynamic representations that enhance conceptual clarity (Muslim, 2023). Similarly, AR enriches the learning experience by enabling students to manipulate and visualise mathematical objects in three dimensions, fostering deeper comprehension (Cirneanu & Moldoveanu, 2024). In contrast, non-digital tools, including manipulatives and visual aids, remain indispensable in resource-limited contexts, providing concrete experiences that aid in transitioning from tangible to abstract understanding (Urrutia et al., 2019).

While these tools offer unique benefits, there is limited research on their combined use in improving PSTs' PCK. Existing studies predominantly focus on either digital tools or traditional methods without examining their synergistic effects (Sangwin, 2021). Moreover, research tends to emphasise the impact of these tools on learners' understanding, leaving a significant gap in understanding their effects on PSTs' instructional capabilities, particularly during microteaching sessions. Furthermore, studies often overlook how PSTs' attitudes toward technology and pedagogical beliefs influence the integration of these tools into teaching practices (Voogt et al., 2012). Against this background, this chapter investigates how the combined use of digital and non-digital tools influences PSTs' PCK during microteaching sessions. Specifically, it explores how PSTs visualise and teach mathematical concepts before and after an intervention programme utilising tools like GeoGebra alongside manipulatives and visual aids. This research seeks to inform teacher education programmes about the optimal integration of these tools to enhance mathematics teaching and learning, equipping future teachers to navigate diverse classroom contexts effectively.

1.1 Research questions

This chapter, therefore, seeks to answer the following questions:

- In what ways do visualising mathematical concepts using both digital and non-digital tools influence preservice teachers' PCK during their microteaching sessions?
- What are preservice teachers' experiences of using dual digital and non-digital tool approaches during their microteaching sessions?

2. Literature

2.1 Pedagogical content knowledge in mathematics teacher education

Pedagogical Content Knowledge (PCK) is a vital construct in mathematics teacher education, enabling teachers to integrate mathematical content and pedagogical strategies effectively (Marshman & Porter, 2013). It plays a critical role in fostering student understanding and addressing misconceptions, yet, research highlights that pre-service teachers (PSTs) often struggle to develop robust PCK (Ekiz-Kiran et al., 2021; Wakhata et al., 2022). Studies emphasise that combining general pedagogical knowledge with mathematics-specific approaches enhances teaching efficacy and student outcomes (Charalambous et al., 2020; Sarama et al., 2021). This suggests that effective teacher training programmes should offer targeted experiences that cultivate both content knowledge and pedagogical knowledge, as these are essential for impactful instruction, as confirmed by Ball et al. (2008) and Tröbst et al. (2018). As noted by Olawale (2023), teachers' experience, academic qualifications, and pedagogical content knowledge are integral to teacher quality, which significantly influences learners' academic achievement by enhancing the teacher's capacity to engage students, deliver instruction effectively, and facilitate meaningful learning experiences. This highlights the need to explore how tools like visualisation technologies influence PSTs' PCK development.

2.2 Digital and non-digital tools in mathematics education

A wide range of concepts is utilised to delineate digitisation in relation to the investment, adoption, and application of advanced technologies in educational practices and research. These concepts include digital instruments, digital technologies, information technology (IT), information and communication technology (ICT), and educational technology. Typically, these terms are employed interchangeably, as a clear distinction between them is absent (Salavati, 2016). As noted by Griffin (2003), these technologies hold considerable promise for educational purposes, with the effectiveness and applicability of digital technology being dependent on the teacher's proficiency and passion. The educator ultimately plays a pivotal role in enhancing the learning environment, contingent upon their adept utilisation of technology to their advantage (Griffin, 2003). Tondeur et al. (2008) contend that teachers are more predisposed to adopt innovations that align with their individual philosophies and beliefs concerning teaching and learning.

The integration of digital and non-digital visualisation tools, such as GeoGebra, is critical in developing PSTs' PCK by blending content knowledge with effective teaching strategies (Shulman, 1986). GeoGebra, in particular, offers dynamic and interactive learning opportunities, enabling preservice teachers to explore and teach complex mathematical concepts effectively (Dockendorff & Solar, 2018; Bakar et al., 2020; Marange & Tatira, 2023). Non-digital tools, such as physical models and diagrams, complement this by fostering hands-on engagement and enhancing conceptual understanding (Samuel, 2019; Vladušić et al., 2020). Together, these tools encourage diverse instructional strategies tailored to learners' needs (Rice & Kitchel, 2015; Wooditch et al., 2018).

GeoGebra's integration into teacher training programmes significantly enhances teachers' ability to employ technology in the classroom (Buchori & Puspitasari, 2023). Research indicates that pre-service teachers (PSTs) who use GeoGebra improve their instructional skills and gain a better understanding of the intersection between pedagogy and technology (Marange & Tatira, 2023; Bueno et al., 2021). Additionally, GeoGebra supports inquiry-based learning, which fosters student self-efficacy and motivation, enabling more engaging, student-centred teaching approaches (Zakariya, 2022; Barçin & Yenmez, 2023). Studies have also shown its potential in addressing misconceptions in mathematics and enhancing PSTs' professional growth (Dağlı & Elif, 2021; Horzum & Ünlü, 2017; Putra et al., 2021). Beyond its impact on teaching, GeoGebra positively influences student learning by making abstract mathematical concepts accessible and enjoyable. Students demonstrate improved attitudes towards mathematics and greater conceptual understanding when GeoGebra is used in lessons (Uwurukundo et al., 2022; Muslim, 2023). The tool's ability to foster visualisation and simplify complex ideas supports deeper engagement with mathematical content, particularly in areas such as geometry and three-dimensional models (Dahal et al., 2022).

GeoGebra's transformative role in modern mathematics education is evident in its capacity to foster innovative instructional practices and deepen understanding among both teachers and students. For PSTs, it offers a robust platform to develop TPACK and equips them to create active, learner-centred environments (Kuzu, 2021; Nzaramyimana et al., 2021). These findings underscore GeoGebra's crucial role in preparing pre-service teachers for effective mathematics instruction in technology-rich educational contexts.

In contrast, non-digital tools such as manipulatives, visual aids, and traditional paper-and-pencil methods significantly enhance students' understanding of mathematical concepts. These tools facilitate a tactile and visual approach to learning, which can be particularly beneficial for students who struggle with abstract mathematical ideas. Traditional resources, such as textbooks and physical manipulatives, provide tangible experiences that can enhance understanding and retention of information (Clark-Wilson, 2020). For instance, the use of physical objects in mathematics education has been shown to improve students' conceptual understanding by allowing them to visualise and manipulate abstract concepts (Clark-Wilson, 2020). Research has indicated that the use of physical manipulatives can lead to a deeper conceptual understanding and improved problem-solving skills among students (Hussein & Khoiruzzadittaqwa, 2024; Ng & Tsang, 2021).

One of the primary advantages of non-digital tools is their ability to promote active learning. When students engage with physical objects, they can explore mathematical concepts in a hands-on manner, fostering engagement and motivation. For example, using blocks to teach addition and subtraction allows students to visualise the process, making it more concrete and understandable. This approach aligns with the principles of Realistic Mathematics Education (RME), which emphasises the importance of context and real-life applications in learning mathematics (Agustina et al., 2018; Sari & Mutmainah, 2018).

Moreover, non-digital tools encourage collaborative learning. In a classroom setting, students can work together to manipulate objects, discuss their strategies, and share their findings. This collaborative approach enhances mathematical understanding and develops critical social skills such as communication and teamwork (Ersozlu et al., 2022). Studies have indicated that cooperative learning strategies, which often utilise non-digital tools, can significantly improve students' mathematical performance and attitudes towards the subject (Shah, 2023; Bhagwonparsadh, 2024). The dual-tool approach enhances content knowledge and promotes a constructivist mindset, encouraging pre-service teachers to view mathematics as a dynamic and interconnected discipline rather than a series of isolated facts (Carbonneau et al., 2018). The integration of both tool types can create a balanced learning environment that caters to diverse learning styles and preferences.

2.3 Theoretical frameworks

2.3.1 Technological pedagogical content knowledge

TPACK is a framework that integrates three fundamental domains of knowledge: technological knowledge (TK), pedagogical knowledge (PK), and content knowledge (CK). This integration is essential for effective instruction, particularly as teachers grapple with the complexities of incorporating both digital and non-digital tools into their pedagogical methodologies. The framework was originally introduced by Mishra and Koehler in 2006, highlighting the interaction among these knowledge domains to improve teaching and learning outcomes (Koehler et al., 2013). Furthermore, the TPACK framework comprises various components that further elucidate the interactions between technology, pedagogy, and content. These include Technological Content Knowledge (TCK), Technological Pedagogical Knowledge (TPK), and Pedagogical Content Knowledge (PCK) (Saubern et al., 2020; Raihanah, 2024). Technological Content Knowledge (TCK) pertains to understanding how technology can be employed to effectively represent specific content. In contrast, Technological Pedagogical Knowledge (TPK) encompasses the understanding of how technology can be leveraged to enhance pedagogical approaches. Conversely, Pedagogical Content Knowledge (PCK) emphasises the adaptation of pedagogical strategies to facilitate the effective instruction of specific content areas (Stapf & Martin, 2019; Padmavathi, 2017). The interaction of these elements is crucial for PSTs as they acquire the skills necessary to navigate the intricacies of teaching mathematics within a technology-enhanced environment.

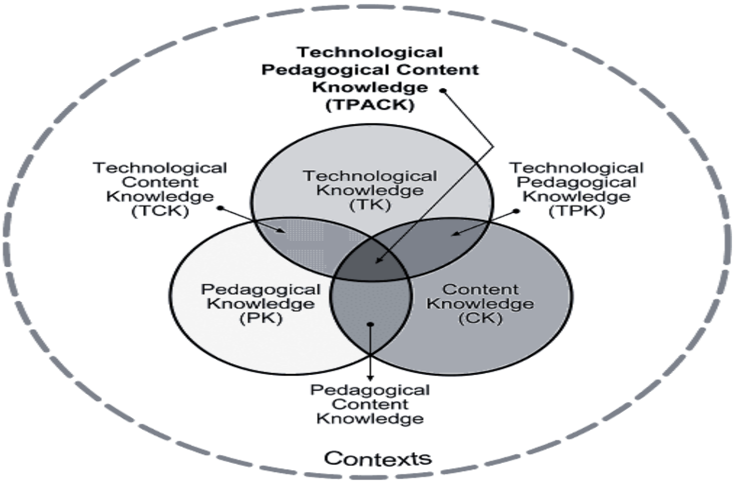

Figure 1: *The TPACK Framework* (adopted from Koehler & Mishra, 2009)

The TPACK framework, as shown in Figure 1 above, is particularly relevant in the education of pre-service teachers (PSTs), as these student teachers must cultivate competencies to integrate technology effectively into their pedagogical practices. Research indicates that engaging PSTs in TPACK-focused training can significantly improve their capacity to devise and execute technology-integrated teaching (Chai et al., 2020; Valtonen et al., 2020). For instance, studies have shown that PSTs participating in TPACK-based professional development programmes demonstrate increased confidence and competence in using technology to facilitate student learning (Admiraal et al., 2016; Chai et al., 2018). This is especially important in mathematics education, where visualising mathematical concepts through various teaching tools can lead to deeper understanding and engagement among students (Rohmitawati, 2018).

3. Materials and Methods

The study adopts an interpretivist qualitative approach, characterised by its focus on understanding the subjective experiences and perspectives of individuals within their specific contexts. Interpretivism highlights the significance of context and the meanings attributed by participants to their experiences, making it particularly suitable for examining the intricacies of PSTs' perceptions and practices regarding the utilisation of digital and non-digital tools in teaching mathematics (Mohajan, 2018). Participants were selected through purposeful sampling, a method frequently employed in qualitative research to identify individuals who possess specific characteristics relevant to the research question. Six PSTs from a cohort of twenty were sampled to take part in the study. Data collection was conducted through observations, focus group discussions, and semi-structured interviews, guided by the analytical framework.

Data collection occurred in three phases:

- Pre-Intervention Phase: PSTs' baseline PCK was assessed during microteaching sessions using observational tools and focus group discussions.

- Intervention Phase: A design-based intervention was conducted during lectures, exposing PSTs to digital tools like GeoGebra and opportunities to create non-digital solid objects. The intervention focused on teaching Grade 10 mathematics topics, including functions, measurements, and analytical geometry, over a semester.
- Post-Intervention Phase: PSTs' PCK was re-evaluated through microteaching sessions, utilising the same observational tools and focus group interviews. Lastly, we conducted semi-structured interviews to gather reflections on the tools' impact on their teaching practices and their ability to visualise mathematical concepts.

Observation data were analysed using a Likert scale checklist to assess PSTs' PCK, with scores converted into qualitative criteria based on established intervals (Widyoko, 2014; Sugiyono, 2013). The TPACK framework was applied to code and analyse data, categorising it into CK, PK, PCK, and TPACK. Emerging themes from the semi-structured interviews were integrated into the framework to enrich the analysis. This dual approach ensured a comprehensive understanding of how digital and non-digital tools influenced PSTs' instructional strategies and PCK.

The data analysis procedure is based on research instruments utilising a Likert scale presented as a checklist, as indicated in Table 1. Sugiyono (2019) asserts that Likert scales are employed to assess the attitudes, views, and perceptions of individuals or groups regarding social issues. The questionnaire employs the Likert scale presented as a checklist. The obtained data are then converted into qualitative criteria displayed in Table 2. The data gathered from the pre-and post-observations of the PSTs were analysed using the TPACK framework. The framework was also utilised to systematically analyse the data collected from observations by categorising it into the following domains: CK, PK, PCK, and TPACK.

Table 1: *Likert scale categories*

Interval	Criteria
1.00 < score < 1.75	Very low
1.75 < score < 2.50	Low
2.50 < score < 3.25	Good
3.25 < score < 4.00	Very good

Source: (Adopted from Widyoko, 2014)

Table 2: *The Percentage range values and qualitative criteria*

Value	Range	Qualitative Criteria
1	0 — 20	Very low
2	21 — 40	Low
3	41 — 60	Fair
4	61 — 80	Good
5	81 — 100	Very good

Source: (Adopted from Sugiyono, 2013)

Emerging themes from the interview data were identified and integrated into the analytical framework to comprehensively understand how digital and non-digital tools influenced preservice teachers' PCK. This approach ensured a nuanced analysis by linking the participants' observed and expressed experiences to the theoretical components of the TPACK framework.

This chapter establishes the study's trustworthiness through credibility, transferability, dependability, and confirmability. The study's credibility was augmented in multiple ways. We employed an effective approach involving the implementation of member checking (Lincoln & Guba, 1985). In this study, following the preliminary analysis of interviews and observations, the PSTs were solicited to reflect on the findings and verify the accurate representation of their experiences. This technique authenticates the data and enables participants to rectify any misconceptions, thus augmenting the study's credibility. We utilised various data collection methods, including interviews, lesson observations, focus group discussions, and video recordings of micro-teaching sessions. This facilitated the triangulation of results (Creswell & Poth, 2018). Observations of the micro-teaching sessions supplemented the interview data, enabling the researchers to discern how the PSTs implemented their knowledge of GeoGebra and other tools in practice. This comprehensive approach ensures that the findings are not derived from a singular perspective but instead represent a more holistic understanding of the participants' experiences.

To improve transferability, we included comprehensive descriptions of the research context, the training programme, the tools used (GeoGebra and non-digital instruments), and the instructional strategies implemented during micro-teaching (Merriam & Tisdell, 2016). This enabled other researchers to assess the applicability of the findings to their contexts.

The researchers upheld a detailed audit trail during the study to guarantee confirmability (Shenton, 2004). This comprises documentation of the research process, the decisions undertaken, and the modifications to the technique as the study advanced. By ensuring transparency in the research design and execution, the researchers enable others to replicate the techniques and evaluate the trustworthiness of the results. Lastly, we involved colleagues in mathematics education to evaluate the research methodology and offer insights on methodological coherence and data analysis (Creswell & Creswell, 2017). This provided an external viewpoint that aids in recognising potential biases or preconceptions that may have impacted the analysis. This collaborative method enhances the thoroughness of data analysis and bolsters the study's overall trustworthiness.

4. Presentation of Results

4.1 Biographic profile of pre-service teachers

The study involved six PSTs specialising in mathematics teaching in their third year. These participants, with an average age of 21 years, were part of a cohort of 21 PSTs, which included 6 males and 15 females from one class. For the sake of anonymity, they were assigned the following codes: PSTB, PSTD, PSTE, PSTI, PSTK, and PSTO. Each participant demonstrated a strong commitment to advancing their pedagogical and content knowledge in mathematics, aspiring to become effective teachers in diverse classroom settings. Their academic training included theoretical coursework and practical teaching components, equipping them with the necessary skills to integrate digital and non-digital teaching tools into their practice. This group was selected to represent various perspectives within the programme.

4.2 Pre-service teachers PCK on visualisation of mathematics concepts before and after the intervention

Interview transcripts and observation codes have been analysed to provide a narrative version of pre-service teachers' visualisation and teaching of mathematics concepts before and after the interventions.

4.2.1 Content Knowledge (CK)

CK is an understanding of the subject matter that does not consider the pedagogical aspects of teaching the subject (Chai et al., 2013). CK is essential as it shapes the distinctive cognitive approach to the discipline within each field of study. The analysis of CK competencies among PSTs prior to the interventions yielded an average score of 42, categorising their performance as fair. Table 3 indicates that PSTB, with a score of 54, and PSTO, with a score of 58, are classified within the satisfactory category. Nonetheless, PSTD (scoring 22), PSTE (scoring 39), PSTE (scoring 46), and PSTK (scoring 34) are all classified within the low category. The results suggest that the PSTs have not yet attained proficiency in the mathematical concepts necessary to effectively visualise and teach students (see Table 3). Moreover, the ratings for each component within the CK section for all PSTs are distinctly presented in Table 4. The data indicates that the performance of each component is classified as low, with an average score of 2.13 within the low category (refer to Table 4).

This was evident during the interview. When pre-service teachers were asked how they visualise and teach mathematics concepts, most indicated that they used traditional, low-tech methods to visualise mathematical concepts and found it difficult to explain some complex concepts in mathematics. Two pre-service teachers indicated the following:

> *I explained verbally and wrote on the board to visualize concepts." I wrote on the board to explain concepts and described visualizing a triangle to help illustrate the topic being taught. I also struggled to explain some concepts and hence I confused my students* (PSTO).
>
> *"I used the traditional method of teaching...I focused on the distance, gradient, and midpoint." I used only verbal explanations to teach the concept of calculating the distance between two points and the midpoint without any visualisation tool* (PSTK).

Freehand drawings, although a common substitute, were often imprecise and prone to causing misconceptions. PSTs recognised that this hindered their ability to accurately demonstrate mathematical associations. PSTK expressed his frustration:

> *The main challenge was that my drawings were freehand, not to scale, which could easily cause misconceptions. I could not demonstrate the parameters' impact visually, especially if they were negative.*

These challenges highlight the need for access to both digital and non-digital tools to improve clarity, accuracy, and engagement in mathematical teaching.

This chapter presents an analysis of content knowledge abilities among pre-service teachers post-intervention, revealing an average score of 79, categorised as good. Table 3 indicates that PSTB, with a score of 82, and PSTO, with a score of 86, are classified in the very good category. PSTD (score: 78), PSTE (score: 80), PSTI (score: 78), and PSTK (score: 72) are all classified within the good category. The results demonstrate that the PSTs have effectively mastered the mathematical content, facilitating their ability to visualise and teach mathematics to students (Table 3). Furthermore, Table 4 presents the scores for each component in the CK section post-intervention for all PSTs. The data indicate that each component's ability is classified as efficient, with an average score of 3.44 in the very good category (Table 4). Tables 3 and 4 provide an overview of the preparedness of PSTs in developing professional competencies pertinent to the mastery of the material intended for student instruction.

Table 3: *Pre-service teachers; CK abilities*

No	Pre-service teacher Code	CK Score before intervention	Qualitative Criteria	CK Score after intervention	Qualitative Criteria
1	PSTB	54	Fair	82	Very good
2	PSTD	22	Low	78	Good
3	PSTE	39	Low	80	Good
4	PSTI	46	Fair	78	Good
5	PSTK	34	Low	72	Good
6	PSTO	58	Fair	86	Very good
Average		42	Fair	79	Good

Table 4: *Pre-service teachers' average scores of CK abilities*

No	CK Component	Average score before intervention	The average score after intervention
CK1	Accuracy in explaining mathematical concepts using digital and non-digital visualisation tools.	2.00	3.4
CK2	Depth of understanding of the mathematical content demonstrated through their use of tools.	2.00	3.3
CK3	Sufficient knowledge of mathematics concepts	1.75	3.2
CK4	Use mathematical way of thinking	2.50	3.9
CK5	Use various ways and strategies of developing understanding of mathematics	2.40	3.4
	Average	2.13	3.44

The results of the data analysis indicate that the CK component of PSTs in preparing learning tools to assist students in visualising and learning mathematical concepts falls into the very good category. Therefore, it can be concluded that PSTs with strong content knowledge positively influence their competencies in other areas. PSTE correctly explained concepts related to 2D and 3D shapes using both digital and non-digital tools, demonstrating robust content knowledge in his explanation of the concepts.

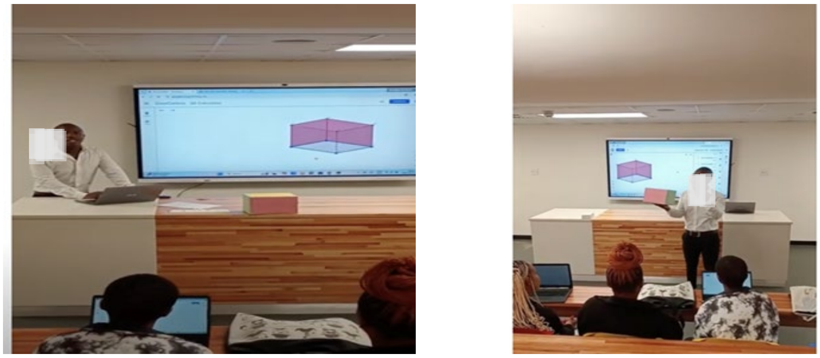

Figure 2: *Authors own illustrations of PSTE*

The field notes indicated that the digital tools available to PSTE were relevant to each mathematical concept he explained throughout his lesson. He seamlessly transitioned from discussing the properties of basic rectangular paper to a rectangular box and further performed dynamic visual demonstrations using GeoGebra software.

This observation was supported during the interview, where it was found that most PSTs (e.g., PSTE, PSTB) emphasised that using both digital and non-digital tools deepened their understanding of the content and enhanced their ability to teach effectively. This demonstrates that integrating these tools positively impacts both content mastery and pedagogical practices. Teachers such as PSTB, PSTO, and PSTK highlighted that tools like GeoGebra allowed for dynamic visualisation of abstract concepts, enriching their ability to connect mathematical ideas to the world as they explained algebraic functions and their graphical representations using both digital and non-digital methods.

The responses reveal significant improvements in the PSTs' content knowledge for teaching mathematics, emphasising the dynamic and interactive capabilities provided by digital tools like GeoGebra. Two pre-service teachers shared their joy by narrating:

> *I used points with coordinates to illustrate concepts dynamically, such as showing faces on a 3D net, which unfolded and closed dynamically* (PSTK).
>
> *With GeoGebra, I could click on a tool for the gradient, select the line, and immediately see the gradient displayed. I displayed triangles on the screen and used the formula for calculating distance directly within the GeoGebra window, which dynamically showed how the distance values changed* (PSTD).

Pre-service teachers noted that the interactive nature of the tools contributed to a deeper connection with the mathematical content. This finding aligns with a growing body of literature that emphasises the positive impact of digital tools on content knowledge (CK) development. This observation is supported by Rabi et al. (2021), who highlight that GeoGebra improves students' mathematical representation skills, enabling them to see and manipulate mathematical concepts in dynamic ways. Such interaction deepens the pre-service teachers' understanding of the content and equips them with the skills to present abstract mathematical ideas in more

Table 5: Pre-service teachers' PK ability

No	Pre-service teacher Code	PK Score before intervention	Qualitative Criteria	PK Score after intervention	Qualitative Criteria
1	PSTB	20	Very less	68	Good
2	PSTD	32	Less	80	Good
3	PSTE	28	Less	78	Good
4	PSTI	30	Less	76	Good
5	PSTK	32	Less	75	Good
6	PSTO	26	Less	70	Good
Average		28	Less	75	Good

Table 6: Pre-service teachers' average scores for PK ability

No	CK Component	Average score before intervention	The average score after intervention
PK1	Ability to implement effective teaching strategies that integrate visualisation tools.	2.00	3.25
PK2	Use of formative assessment strategies through the integration of visualisation tools.	1.75	3.50
PK3	I know how to organise and maintain classroom management.	2.50	3.72
PK4	I am familiar with common student understandings and misconceptions.	1.25	3.00
PK5	I can adapt my teaching based-upon what students currently understand or do not understand.	2.00	3.25

This finding was evident during the interviews. PSTs indicated that the intervention programme had improved their efficiency and accuracy in concept representation, enhanced their pedagogical knowledge, and simplified the demonstration of complex mathematical ideas. This reduced the effort required for explanations while improving the accuracy of visualisations. This marked a significant improvement over their previous reliance on freehand drawings or static visuals. PSTD shared her joy:

> *I could immediately see the gradient displayed with GeoGebra, which was a big improvement. Extending and reducing a triangle's vertices, I could dynamically show how the distance values changed as the points were moved.*

PSTs diversified their instructional approaches by combining technology tools like GeoGebra with interactive elements like Mentimeter. This broadened their pedagogical repertoire and allowed for more engaging lessons. PSTB concurred with PSTD:

> *I used GeoGebra software to visualise concepts and an interactive tool, Mentimeter, at the start to engage learners. In addition, I also incorporate physical non-digital objects as well in my instructions.*

The intervention programme significantly transformed the PSTs' approaches to teaching mathematics. Integrating GeoGebra and other digital tools enabled dynamic and precise visualisations, improved student engagement, and enhanced the clarity of explanations. The shift from static or traditional methods to interactive and technology-based approaches marked a critical development in their teaching practices. The literature supports this finding. Research has consistently demonstrated that the use of GeoGebra in mathematics education facilitates a deeper understanding of mathematical concepts. For instance, Putra et al. (2021) highlight that prospective elementary teachers who integrated GeoGebra into their training were able to construct their mathematical knowledge of three-dimensional shapes, thereby enhancing their understanding of complex mathematical theories and practices. This aligns with findings from Barçin and Yenmez (2023), who noted that GeoGebra software aids in developing mathematical language and self-efficacy among students, suggesting that teachers trained in such technologies can better facilitate communication in mathematics classrooms.

4.2.3 Pedagogical Content Knowledge (PCK)

PCK refers to the integration of content knowledge with teaching approaches. It encompasses more than mere content expertise or familiarity with general pedagogical principles; it involves comprehending the specific interactions between content and pedagogy (Koehler et al., 2013). The analysis of PCK abilities among PSTs in visualising and teaching mathematical concepts, both digital and non-digital, prior to the interventions yielded an average score of 27, categorising it as low. Table 7 indicates that PSTB, with a score of 17, and PSTO, with a score of 19, are classified in the very low category. Nonetheless, PSTD (score of 28), PSTE (score of 24), PSTI (score of 35), and PSTK (score of 38) are all classified within the low category. Furthermore, Table 8 presents a clear overview of the scores for each component in the PCK aspects across all PSTs. The data indicates that the performance of each component is classified as very low, with an average score of 1.85 falling within the low category (Table 8).

The low scores across all components of PCK, as indicated by the results, suggest that PSTs are inadequately prepared to effectively deliver subject content to their students. This finding highlights the urgent need for targeted intervention programmes that focus on building PCK. This conclusion was evident during the interviews, where PSTs indicated that they relied on verbal explanations to communicate their ideas to students. Two PSTs lamented:

> *I relied solely on the traditional method of teaching without incorporating any visual aids or technological tools and that made me struggle to teach the concept. When teaching Analytical Geometry, I focused on explaining concepts like distance, gradient, and midpoint through verbal explanations and writing on the board. I didn't explore using diagrams, graphs, or software to help students visualise these concepts, which might have limited their understanding* (PSTI).

PSTO concurred with PSTI:

> *"In my lesson on Analytical Geometry, I focused on calculating the distance between two points and finding the midpoint of a line. I didn't use any technology or interactive tools to demonstrate these concepts. My approach was primarily verbal, and I didn't provide any visual representations or practical examples that could help students relate to the material more effectively.*

This finding indicates that PSTs' reliance on traditional, verbal-only teaching methods suggests insufficient preparation for modern classrooms where technology integration is essential. This may limit their ability to meet diverse learner needs and leverage digital tools to enhance engagement and comprehension.

The analysis of PCK abilities among the PSTs following the interventions yielded an average score of 77, categorising it as good. Table 7 indicates that PSTB (score of 69), PSTO (score of 68), PSTE (score of 72), and PSTI (score of 78) are all classified within the good category. Both PSTK, with a score of 83, and PSTD, with a score of 89, are classified within the good category. The results demonstrate that the PSTs effectively mastered the material, enabling them to visualise and teach mathematics using technology proficiently (Table 8). Additionally, Table 7 presents a clear overview of the scores for each component in the PCK aspects following the intervention for all PSTs. The data indicates that each component's ability is classified as efficient, with an average score of 3.40 falling within the very good category (Table 7). Tables 7 and 8 provide an overview of the PSTs' preparedness to visualise and instruct mathematics concepts, demonstrating robust PCK competencies pertinent to the mastery of the material intended for students.

Table 7: *Pre-service teachers' PCK ability*

No	**Pre-service teacher Code**	**PCK Score before intervention**	**Qualitative Criteria**	**PCK Score after intervention**	**Qualitative Criteria**
1	PSTB	17		69	Good
2	PSTD	28		89	Very good
3	PSTE	24		72	Good
4	PSTI	35		78	Good
5	PSTK	38		83	Very Good
6	PSTO	19		68	Good
Average		27		77	Good

Table 8*: Pre-service teachers' average score for PCK ability*

No	CK Component	Average score before intervention	Average score after intervention
PCK1	Ability to explain complex mathematical ideas using both digital and non-digital visualisation tools to enhance learners' understanding.	2	3.45
PCK2	Adaptability in using visual tools to address common learner misconceptions.	2.20	3.25
PCK3	Ability to select effective teaching approaches to guide student thinking and learning in mathematics.	1.34	3.50

The data from the field notes concurred with the data from the observation schedules. This finding is also consistent with the results from the interviews. PSTs indicated that the intervention enhanced their PCK, enabling them to explain complex mathematical concepts using both digital and non-digital visualization tools to improve learners' understanding.

PSTB narrated:

> *I used GeoGebra to illustrate how algebraic changes affect graphical outputs and employ physical models to reinforce the same concept. The GeoGebra and 2D/3D shapes visually and dynamically helped me to explain complex concepts such as the total surface area. Then again, I managed also to recognize if my learner struggles, then I would switch to a different visualization tool to clarify and target any misconceptions. For instance, when learners struggled to understand the surfaces of a rectangular prism, I would shift from using a non-digital box to a GeoGebra window, which dynamically unpacked/unfolded the box into 6 faces.*

This finding suggests that the intervention programme has significantly impacted the development of PSTs' PCK. It is consistent with studies by Dağlı and Elif (2021), which found that GeoGebra helps PSTs better identify and address common misconceptions in mathematics. Misconceptions are a frequent barrier to student learning in mathematics, and the ability to diagnose and correct them requires a deep understanding of both the content being taught and the pedagogical methods that can make that content more accessible. By using GeoGebra, PSTs are not only enhancing their own understanding of mathematical concepts but also developing strategies to help students overcome difficulties in learning these concepts. Additionally, the findings from Tröbst et al. (2019) reinforce the idea that instruction focused on PCK improves content knowledge and pedagogical strategies. This suggests that the intervention aimed at enhancing PCK through GeoGebra is effective not only in developing technical proficiency with the tool but also in deepening PSTs' overall understanding of how to teach mathematics.

4.2.4 Technological Pedagogical and Content Knowledge (TPACK)

TPACK is a framework that delineates the knowledge required by teachers to enhance pedagogical practices and conceptual understanding through the integration of technology within the learning environment (Koehler & Mishra, 2009). The analysis of TPACKabilities among Pre-Service Teachers (PSTs) in visualising and teaching mathematical concepts, both digital and non-digital, prior to the interventions yielded an average score of 25, categorising it as low. Table 9 indicates that PSTB, with a score of 38, is classified in the low category. PSTD (21), PSTE (24), PSTI (20), PSTK (28), and PSTO (18) are categorised as very low scores. Furthermore, Table 9 presents a clear overview of the scores for each component within the TPACK framework for all PSTs. The data indicates that the performance of each component is classified as low, with an average score of 1.61, falling within the very low category (Table 10).

The analysis of TPACK abilities among PSTs post-intervention yielded an average score of 75, categorising the results as good. Table 9 indicates that PSTB (score: 84), PSTO (score: 69), PSTD (score: 72), PSTE (score: 70), PSTI (score: 74), and PSTK (score: 78) are all classified within the good category. The results indicate that the PSTs effectively mastered the material, enabling them to utilise technological tools for visualising and teaching mathematical concepts.

Additionally, Table 10 presents a clear overview of the scores for each component in the TPACK aspect following the intervention for all PSTs. The data indicates that each component's ability is classified as good, with an average score of 3.24, falling within the very good category (Table 10). Tables 9 and 10 present an overview of the preparedness of PSTs in utilising technology to visualise and teach mathematical concepts.

***Table 9:** Pre-service teachers' TPACK ability*

No	Pre-service teacher Code	TPACK Score before intervention	Qualitative Criteria	TPACK Score after intervention	Qualitative Criteria
1	PSTB	38		84	Very good
2	PSTD	21		72	Good
3	PSTE	24		70	Good
4	PSTI	20		74	Good
5	PSTK	28		78	Good
6	PSTO	18		69	Good
Average		25		75	Good

***Table 10:** Pre-service teacher's average score for TPACK ability*

No	TPACK Component	Average score before intervention	Average score after intervention
TPACK1	Mastery in integrating technology and pedagogy to enhance content delivery and learner engagement.	1,48	3.20
TPACK2	Ability to teach lessons that appropriately combine algebraic functions, technologies and teaching approaches	1.38	3.50
TPACK3	Ability to teach lessons that appropriately combine analytical geometry, technologies and teaching approaches	1.27	3.25
TPACK4	Ability to teach lessons that appropriately combine measurements, technologies and teaching approaches	2.32	3.00

ACK4 ility to teach lessons that appropriately com2 0
asurements, technologies and teaching approaches

This finding was also evident during the interviews. PSTs indicated that they selected technologies for use in their microteaching classrooms that enhance what they teach, how they teach, and what students learn. The findings resonate with studies showing that preservice teachers who participate in TPACK-based professional development programmes demonstrate increased confidence and competence in using technology to support student learning (Admiraal et al., 2016; Chai et al., 2018). This is particularly important in mathematics education, where visualising mathematical concepts through various teaching tools can lead to deeper understanding and engagement among students (Rohmitawati, 2018). This finding further aligns with the study by Bueno et al. (2021), which discussed how an online course incorporating

GeoGebra facilitated the development of TPACK among mathematics teachers, emphasising the importance of blending technology with pedagogical strategies. This integration prepares PSTs to use digital tools effectively, enhancing their teaching efficacy.

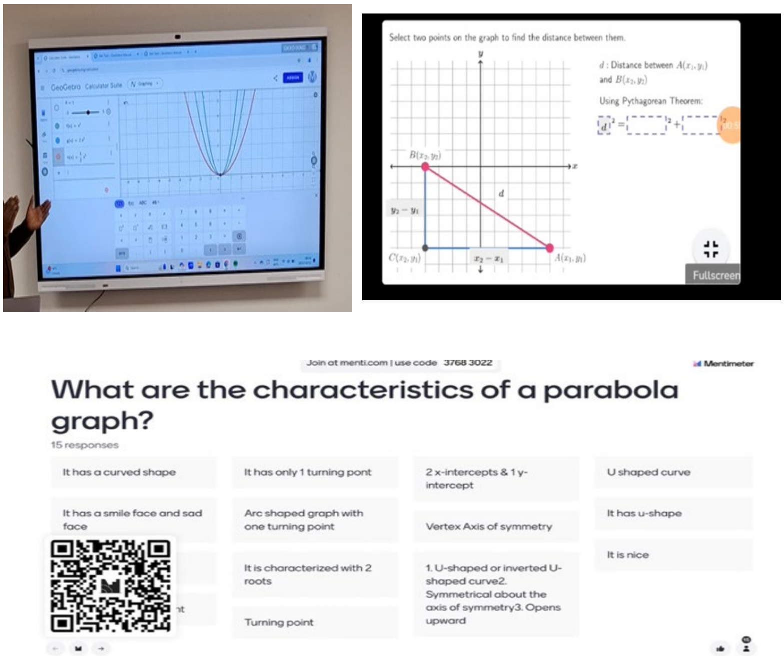

Figure 3: Pre-service use of digital tool in teaching (Author's own illustration)

Figure 3 shows that PSTs employed technologies such as GeoGebra and Mentimeter to enhance learners' participation and conceptual understanding, thereby influencing student learning outcomes. Studies have demonstrated that when PSTs effectively utilise GeoGebra in their teaching, their students exhibit improved understanding and engagement in mathematics (Putra et al., 2021). This reciprocal relationship between teacher preparation and student learning underscores the importance of equipping PSTs with the necessary tools and knowledge to foster positive learning experiences.

4.3 Pre-service teachers' experiences using dual digital and non-digital tools approaches during their microteaching

PSTs reflected on their experiences integrating digital and non-digital tools during microteaching sessions after the intervention. Their responses highlight a mix of positive outcomes, personal growth, and challenges encountered in balancing these approaches. Several participants (PSTE, PSTB, PSTD) described their overall experience as transformative, emphasising increased confidence and an appreciation for the importance of tool integration in teaching practice. Some specifically mentioned a shift in perspective, viewing the dual approach as essential for their future teaching. GeoGebra was repeatedly highlighted as a powerful digital tool for enhancing teaching and learning. PSTs noted its ability to provide precise, dynamic visuals, which helped explain abstract concepts such as distance and midpoint. Although digital tools were highly effective, some participants (PSTI, PSTO) emphasised the continued importance of non-digital tools for accessibility and foundational teaching, particularly for students who struggle with digital technologies.

Some PSTs highlighted how the dual approach boosted their teaching confidence, allowing them to engage students dynamically while maintaining clarity in explanations. This combination was seen as making their lessons more effective and adaptable. PSTD had this to say:

> *I employed both digital and non-digital tools during my microteaching. Both tools helped me reach diverse learners and encouraged interaction, as students could better relate to the practical, visual aspects. Using GeoGebra encouraged more questions from my students, as they could see changes immediately, making them curious and interactive and encouraging them to participate actively.*

This finding indicates that the approach benefits PSTs during teaching practice. Several studies align with these findings, emphasising the transformative impact of such an approach on PSTs' confidence and pedagogical effectiveness. Using digital tools like GeoGebra, which facilitates dynamic visualisation of mathematical concepts, enhances PSTs' teaching confidence. Research by Dintarini et al. (2024) supports this, noting that GeoGebra enables PSTs to present abstract concepts interactively, making lessons more engaging and student-centred. Similarly, Mentimeter allows real-time student feedback and interaction, creating a two-way communication channel that fosters engagement and responsiveness (Kohnke & Moorhouse, 2021). When combined with non-digital tools, such as physical shapes for tactile learning, PSTs reported feeling more in control and capable of delivering clear and comprehensive explanations.

Nevertheless, PSTs' responses revealed technical and practical challenges when integrating digital and non-digital tools into their microteaching sessions. These challenges were related to technical difficulties, time constraints, tool familiarity, and accuracy in teaching methods. Some PSTs faced connectivity issues with GeoGebra, which initially hindered their teaching ability. Once resolved, however, they felt more confident in using digital tools. Others also mentioned challenges related to students' familiarity with GeoGebra, which required additional effort to ensure all students could effectively engage with the tool. PSTs lamented:

> *My biggest challenge was ensuring that all students could interact with the digital tools, as not all students were familiar with using GeoGebra at first. That took most of my time to navigate through with the students.*

Some PSTs found that switching between tools (digital and non-digital) required careful time management and planning, making it difficult to focus on one method fully. Moreover, the time needed to become comfortable with GeoGebra limited what they felt they could achieve with the tool. PSTO shared her frustration:

> *The main challenge was the time and effort to switch between tools that required planning. Also, becoming comfortable with GeoGebra took time, so I felt limited in what I could achieve without more practice.*

Some PSTs expressed challenges related to their lack of proficiency in using GeoGebra, as they had limited exposure to the tool during the microteaching preparation phase. PSTI narrated:

> *I was not really skilled in GeoGebra, probably because of my short time exposure to the tool.*

The challenges faced by PSTs when integrating digital and non-digital tools were primarily related to technical issues (e.g., connectivity and tool familiarity), the need for careful planning to balance both methods, and the lack of accuracy in traditional tools, such as freehand drawings.

5. Conclusions and Recommendations

In concluding this chapter, it is essential to reflect on the main arguments presented throughout the text, the key findings regarding the enhancement of pedagogical content knowledge (PCK) among preservice teachers (PSTs), and the broader implications of these findings for mathematics education. The chapter establishes that integrating digital and non-digital tools in mathematics education significantly enhances PSTs' PCK. This dual approach allows PSTs to experience a richer pedagogical repertoire, equipping them with diverse strategies to address various learning styles and needs in future classrooms. The findings indicate that PSTs who engage with various teaching tools develop a more profound understanding of mathematical concepts and their instructional implications, which is critical for effective teaching (Tröbst et al., 2019). Using manipulatives and visual aids, alongside digital resources, fosters a more comprehensive grasp of mathematical principles, enabling PSTs to visualise and convey complex ideas more effectively. Moreover, the chapter highlights the importance of mathematics teaching modules in shaping PSTs' beliefs, attitudes, and self-efficacy regarding mathematics instruction. The dual-tool approach enhances content knowledge and promotes a constructivist mindset, encouraging PSTs to view mathematics as a dynamic and interconnected discipline rather than a series of isolated facts (Carbonneau et al., 2018). This shift in perspective is crucial for developing future teachers who can inspire and engage their students in meaningful mathematical teaching and learning.

The findings also underscore the role of collaborative experiences in the development of PCK. Engaging preservice teachers in group activities that utilise both digital and non-digital tools fosters a community of practice where they can share insights, strategies, and challenges. This collaborative learning environment enhances their ability to reflect on their teaching practices and adapt their approaches based on peer feedback and shared experiences (Ersozlu et al., 2022). Such interactions are vital for building a supportive network that can sustain preservice teachers throughout their professional journeys. Furthermore, the chapter addresses the broader implications of these findings for teacher education programmes. In order to keep up with the changing nature of education, teacher preparation programmes need to incorporate hands-on experience with both digital and traditional teaching resources to prepare PSTs to meet the demands of contemporary classrooms and equip them with the skills necessary to foster critical thinking and problem-solving abilities in their students, as advised by Mouza et al. (2017).

In conclusion, the chapter has demonstrated that using both digital and non-digital teaching tools significantly enhances pre-service teachers' (PSTs) Pedagogical Content Knowledge (PCK), ultimately leading to improved mathematics instruction. Integrating these tools fosters a deeper understanding of mathematical concepts, promotes positive attitudes towards teaching mathematics, and encourages collaborative learning experiences. As teacher education programs continue to evolve, it is essential to prioritise the development of PSTs' PCK through the strategic use of diverse teaching tools, ensuring they are equipped to inspire and educate future generations of learners.

This chapter highlights the transformative potential of integrating digital tools, such as GeoGebra and Mentimeter, alongside non-digital tools like physical objects, to enhance pre-service teachers' PCK in mathematics. The findings emphasise the need for teacher education programs to prioritise blended pedagogical approaches that combine technology with hands-on materials. This integration equips pre-service teachers with diverse strategies to effectively explain abstract mathematical concepts and address various student learning styles.

The successful use of tools like GeoGebra facilitates dynamic visualisations, promoting a deeper conceptual understanding of mathematical ideas. In contrast, non-digital tools allow for tangible manipulation, which is especially beneficial for foundational learners. Intervention training programs that stress the complementary use of both types of tools can help pre-service teachers design engaging lessons, accommodate diverse learners, and build confidence in adopting innovative teaching practices.

Furthermore, this chapter underlines the importance of incorporating targeted professional development workshops into teacher education curricula. These workshops should provide hands-on experience with both digital and non-digital tools, ensuring that pre-service teachers can seamlessly integrate them into their lesson plans. Such training enhances PCK and prepares future teachers to create inclusive, interactive, and resource-rich mathematics classrooms. This, in turn, contributes to improved learning outcomes and addresses the demands of 21st-century education.

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