CHAPTER FOUR

Exploring Shifts in Pre-Service Teachers' Perceptions of Mathematicians: From First Year to The End of Second Year

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Abstract: This study examines how pre-service mathematics teachers perceive mathematicians' roles and practices, emphasising the shifts in their epistemic and social understandings during their teacher education programme. Using Legitimation Code Theory (LCT) and variation theory, data were collected from 60 pre-service teachers at a South African university, comparing their views from their first to second year. Initial perceptions often conflated mathematicians with educators, focusing on societal roles and pedagogical functions. However, by their second year, after engaging in a mathematics methodology course rooted in variation theory, participants demonstrated an increased appreciation for mathematicians' epistemic practices, including problem-solving, abstraction, and theoretical application. Despite these advancements, many participants continued to frame their understanding through a pedagogical lens, with limited attention to the social dimensions of mathematical work, such as collaboration and interdisciplinarity. These findings underscore the need for teacher education programmes to balance epistemic and social dimensions, enabling future educators to view mathematics as both an intellectual discipline and a collaborative, dynamic field. This study contributes to understanding how theoretical frameworks can inform and transform pre-service teachers' conceptions of mathematicians' roles, bridging the gap between classroom teaching and authentic mathematical practices.

Keywords: Legitimation Code Theory, mathematical practices, mathematicians' perceptions, pre-service teachers, variation theory.

1. Introduction

Pre-service mathematics teachers (PSTs) face the challenge of mastering the complexities of teaching mathematics within a four-year programme. Although they have spent years observing mathematics teachers, they have not observed mathematicians engaged in authentic mathematical practices. Moreover, PSTs are only formally introduced to theoretical perspectives on teaching and learning mathematics during their second year of teacher education. This delay, coupled with the well-documented gap between theory and practice in teacher education, often results in novice teachers abandoning theoretical insights. Instead, they conform to established school norms or resist integrating theory into their teaching practices (Mårtensson & Ekdahl, 2021). Past school experiences heavily shape teaching decisions (Lortie, 1975; Sugrue, 1997). Without encountering practical challenges during training, PSTs may perceive theories as irrelevant, reverting to familiar methods of teaching (Mårtensson & Ekdahl, 2021; Olawale, 2024). While traditional instruction from mathematicians may not directly support research-based, student-centred teaching (Wagner, Speer & Rossa, 2007), robust mathematical knowledge acquired through university training is considered essential for successful teaching (Leikin, Zazkis & Meller, 2018; Hofman & Even, 2018). Therefore, mathematicians' work must expand PSTs' knowledge beyond the school curriculum, exposing them to deeper mathematical thinking. Understanding mathematicians' practices helps PSTs integrate these approaches into their teaching. Mathematicians solve challenging problems—those that are "perplexing or difficult" (Schoenfeld, 2016). Pólya (1954, in Schoenfeld, 2016) describes problem-solving as engaging with unfamiliar tasks where solutions are not immediately known. In contrast, emphasising procedural drills can inhibit students' ability to develop broader problem-solving skills. This practice risks portraying mathematics as a set of rigid facts and procedures (Boaler et al., 2022). Students often internalise the misconception that mathematics lacks creativity or reasoning, reducing it to rote memorisation. When PSTs adopt such classroom routines, they risk perpetuating this limited view of mathematics, conflating the roles of mathematics teachers and mathematicians.

1.2 Problem statement and research questions

PSTs must develop both mathematical content knowledge and mathematical practices to address these challenges. Content knowledge encompasses concepts, skills, and applications, while practices involve disciplinary habits such as argumentation, precision, and problem-solving (Lai & Ahrens, 2022). However, applying theoretical learning to practice remains a significant challenge (Korthagen, 2010; Sugrue, 1997). Treating theory as a "solution" to bridge the gap between research and practice is insufficient. Instead, Lampert (2010) argues that theory should deepen reasoning about practice, integrating with practical knowledge to generate new insights. This study examines how integrating theoretical frameworks with lesson planning in a mathematics education course transforms PSTs' understanding of their roles as mathematicians. At the outset, first-year PSTs primarily identified personal attributes in their teacher identities.

The following are the research questions that guided the study:

- How do pre-service mathematics teachers' perceptions of mathematicians change throughout their teacher education programme?
- What changes occur in pre-service mathematics teachers' understanding of the epistemic dimensions of a mathematician's work?
- How do pre-service mathematics teachers' perspectives on the social and relational aspects of mathematicians evolve over time?

Through iterative processes such as lesson planning, critical analysis, and revisions rooted in variation theory (Marton, 2015) and mathematical proficiency theory, second-year PSTs developed a deeper appreciation for specialised knowledge. By the conclusion of the course, their epistemic understanding had notably advanced, highlighting the critical role of linking mathematical theory to teaching practice (Beswick, 2011).

2. **Literature Review**

Mathematics, unlike fields such as Biology, History, or English, presents a unique challenge in education due to the inherent nature of the subject. Taylor (2018) argues that the limited technical scope of the school mathematics curriculum is a direct result of the complexity and abstractness of mathematics. While other disciplines engage learners with sophisticated questions and creative works that interest professionals, school mathematics often falls short of providing such experiences. This disconnection represents a fundamental difference between mathematics and other fields, where learners can directly relate to and engage with professional-level inquiries.

The discrepancy between learners' and mathematicians' perceptions of mathematics is well documented. Cirillo and Herbel-Eisenmann (2011) explore how classroom practices shape learners' views of mathematicians, revealing that teachers' mathematical backgrounds and portrayals significantly influence these perceptions. They emphasise the need for preservice teachers to reflect on their beliefs about mathematicians' roles and behaviours to present a more accurate and inspiring image of the profession. Latterell and Wilson (2012) reinforce this perspective by noting that mathematicians create new theorems and apply existing results to practical problems, a viewpoint rarely conveyed in traditional mathematics education. Building on this, researchers (Boaler, 2016; Schoenfeld, 2016) highlight the contrast between learners' view of mathematics as a collection of calculations, procedures, and rules and mathematicians' view of it as the study of problems, emphasising creativity and elegance. Together, these studies suggest the need for classroom practices that cultivate a deeper understanding and appreciation of the true nature of mathematics.

Most learners' mathematical experiences occur in the classroom, making these experiences critical in forming their images of mathematics and mathematicians (Schoenfeld, 1985). For example, Rock and Shaw (2000) found that learners primarily believed mathematicians solved "hard problems that other people don't know," reflecting a limited understanding of the profession's scope. Similarly, Picker and Berry (2000) identified various stereotypes in learners' drawings of mathematicians, such as the mathematician as coercive or foolish, indicating a need for more accurate representations in education. Studies by Burton (2004) and Nardi (2008) delve into the perspectives of mathematicians on mathematics education, revealing insights that could bridge the gap between university mathematics and school teaching.

2.1 Collaboration between mathematicians and educators

Blanton and Stylianou (2009) advocate for collaboration between mathematicians and mathematics educators to develop effective teaching strategies, a sentiment echoed by Wagner et al. (2007), who found traditional instructional practices insufficient for supporting learner-centred teaching. Research by Hagenkötter et al. (2022) investigates the role of professional mathematicians in training secondary school mathematics teachers. The study highlights the need for integrating meta-mathematical skills into core coursework and fostering collaboration between mathematicians and educators to improve teacher education. Overcoming barriers to collaboration is crucial for bridging the gap between university-level mathematics and school teaching.

Hagenkötter et al. (2022) found that learners commonly believe mathematical scientists primarily engage in advanced calculations, reflecting their classroom experiences. This limited perception overlooks critical activities such as reasoning, modelling, and problem-solving. While many studies explore learner conceptions of mathematicians, few implement interventions explicitly aimed at demonstrating the diverse and creative nature of mathematical work, leaving a gap in effectively addressing these misconceptions.

Pre-service teachers, like other social actors, become "knowers" by adopting a new perspective that allows them to identify and understand what defines "authentic reality" (Bernstein, 2000, p. 164 in Langsford & Rusznyak, 2024). This perspective, as Maton (2014) explains, is shaped through exposure to new concepts, relevant experiences, and interactions with key individuals. For pre-service teachers aspiring to become mathematicians, developing this perspective is a more intricate process that extends beyond simply "noticing and understanding" the "intentions and actions of teachers" (Langsford & Rusznyak, 2024, p. 5). It requires immersion in the intellectual work of mathematicians, which is often abstract, complex, and not typically visible in traditional classroom settings. This process demands that pre-service teachers go beyond reflecting on "the study of teachers' practices through live or recorded lessons" and begin to "practice the theory in real situations, as they conduct lessons based on theoretical concepts" (Mårtensson & Ekdahl, 2021).

The variation theory of learning emphasises structuring critical aspects in patterns of variation and invariance, allowing students to discern them rather than being explicitly told (Pang & Marton, 2003; Marton, 2015). Watson and Mason (2006) suggest that this approach helps teachers design tasks that encourage students to identify these critical aspects. This approach is closely linked to problem-solving, as the curriculum aims to foster learners' critical and creative thinking skills through tasks that reflect the practices of mathematicians. For pre-service teachers, engaging with variation theory cultivates a "mathematical gaze," helping them develop a nuanced understanding of mathematical practices. This enhances their ability to think about and teach mathematics in ways that go beyond basic pedagogical strategies (Maton, 2014).

While it is well-documented that pre-service teachers' ability to interpret teachers' intentions and actions is crucial for pedagogical learning and understanding effective teaching choices (Langsford & Rusznyak, 2024), this alone is insufficient for pre-service teachers to become true mathematics educators. Developing a "mathematical gaze" through variation theory is essential for their growth into effective mathematics teachers, enabling them to engage deeply with the complexities of mathematical practices.

3. Methodology

This study employed a qualitative research design, specifically a basic interpretive approach, as described by Merriam (2002). This approach seeks to understand how participants create meaning from their experiences and to identify common themes or patterns within the data. The analytical framework used was Legitimation Code Theory (LCT), which focuses on codes and legitimation to analyse curriculum, pedagogy, and assessment practices. The study population comprised 60 pre-service teachers (PSTs) enrolled in a Bachelor of Education (BEd) programme, majoring in mathematics, at a South African university. These PSTs were in their first and second years of study. Participants were selected based on their enrolment in the programme and their willingness to participate in the research. A purposive sampling technique was used to select the participants. This approach was appropriate as it allowed the researcher to focus on PSTs who were directly engaged in the mathematics education courses relevant to the study's objectives. Data were collected through written responses to the prompt, "What do you think it will be like to work as a qualified mathematician?" The responses were gathered at two points: at the beginning of the first year and at the end of the second year. These responses provided insights into the participants' evolving conceptions of mathematicians. Additionally, lesson plans and reflections uploaded to the university's learning management system were analysed to understand how PSTs applied theoretical concepts to their teaching practices.

Legitimation Code Theory (LCT) offers sophisticated tools for analysing curriculum, pedagogy, and assessment practices, focusing on codes and legitimation. Codes signify the principles that guide a practice's legitimacy, but implicit codes may exclude certain practices or perspectives. LCT reveals these underlying codes, enhancing comprehension of their influence on educational practices. Two crucial codes within LCT—knowledge and knower codes—are founded on the notion that "practices and beliefs are about or oriented towards something and by someone" (Maton, 2014, p. 29). This enables an analytical distinction between two sets of relations.: epistemic and social. Epistemic relations pertain to "what can be legitimately described as knowledge," while social relations concern "who can claim to be a legitimate knower" (p. 29). These relations indicate "what counts" in each practice (Luckett & Hunma, 2014, p. 183).

Epistemic relations in the work of mathematicians encompass practices such as investigation, problem-solving, critical thinking, generalisation, and justification, with PSTs leveraging this knowledge to tackle complex problems (author, 2024). Social relations, on the other hand, relate to teachers' perceived legitimacy as knowers within the work of mathematicians. These relations involve traits associated with analytical, creative, and critical thinking, as well as persistence, mathematical proficiency, collaboration, adaptability, confidence, curiosity, and reflection. Enhanced social relations correlate with PSTs demonstrating expertise and confidence in the work of mathematicians. According to Maton (2014), knowers can be differentiated based on both their identity (kinds of knowers) and their method of knowing (ways of knowing). When legitimacy in becoming mathematicians derives from how students know, it mandates specific gazes:

- Cultivated gaze — recognising what is valued in the work of a mathematician.
- Social gaze — emphasises the type of knower based on their social position, resulting in a social gaze. Born gaze — a specific type of knower and a particular way of knowing.
- Trained gaze — practices that require neither a specialised knower nor a specific way of knowing cultivate knowers with a trained gaze.

If the work of a mathematician forms part of the training for mathematics teachers, then the conceptions that PSTs hold should reveal a trained gaze. This study offers insights into this issue by examining the roles of both knowledge and knower codes in facilitating epistemological access to the work of mathematicians within the BEd programme at one university in South Africa. Table 1 outlines the translation device utilised to analyse the PSTs' conceptions.

Table 1: *Translation device to analyse the Epistemic and Social relations of the PSTs conceptions*

KNOWLEDGE	ER + Knowledge of the work of mathematicians	ELITE
SR – No Attributes of a mathematician		SR + Some Attributes of a mathematician
RELATIVIST	ER – No Knowledge of the work of mathematicians	KNOWER

Adapted from Jina Asvat (2024)

According to Maton and Chen (2020), four specialisation codes illustrate the interplay between epistemic relations (ER) and social relations (SR):

Élite Code: Characterised by strong ER and SR, this code emphasises both the knowledge of mathematicians' work and the attributes of a mathematician as a knower. It represents the ideal balance for understanding mathematical practices.
Knowledge Code: Defined by stronger ER and weaker SR, this code highlights knowledge of mathematicians' work over personal attributes. Pre-service teachers (PSTs) focus more on expertise than on the social identity of mathematicians.
Knower Code: Marked by weaker ER and stronger SR, this code prioritises the social attributes and personal characteristics of mathematicians over their knowledge and work.
Relativist Code: With both weak ER and SR, this code reflects general or non-specific responses, showing little emphasis on either the knowledge or the personal attributes of mathematicians.

Through coding, the strengths of epistemic and social relations were assigned, allowing for a comparison of specialisation among first- and second-year PST data. Extracts from the interviews were then utilised to delineate overarching patterns.

Ethical clearance was obtained from the University of the Witwatersrand Human Research Ethics Committee (Education) under approval number H23/11/09. The research was classified as minimal risk. Informed consent was obtained from all participants to ensure their voluntary participation and the confidentiality of their responses. Data were anonymised to protect participants' identities. The dataset is restricted to one university and includes the written conceptions provided by PSTs at the start of their first year and at the end of their second year. It is acknowledged that they may have observed additional aspects that were not included in their writings. Nonetheless, their submitted responses can be regarded as the most significant elements of their understanding of the work of mathematicians, which they deemed important enough to express in writing. Additionally, interviews with the PSTs were not conducted, so further probing was not possible.

4. Presentation of Results

The students' perceptions of a mathematician's work in the first year of the BEd programme, in March 2023, revealed that their views were heavily skewed towards ER- (42%) and SR- (39%), with much less emphasis on ER+ (13%) and SR+ (7%). However, in the second year of the BEd programme, after completing the teaching mathematics methods course, a shift in their perceptions occurred. The data show a more balanced distribution, with ER- at 27%, ER+ at 23%, and SR+ at 11%. Notably, SR- remained consistent across both years at 39%. These changes represent a shift from weaker to stronger forms of epistemic and social relations in students' descriptions of what they imagine the work of a qualified mathematician to be like.

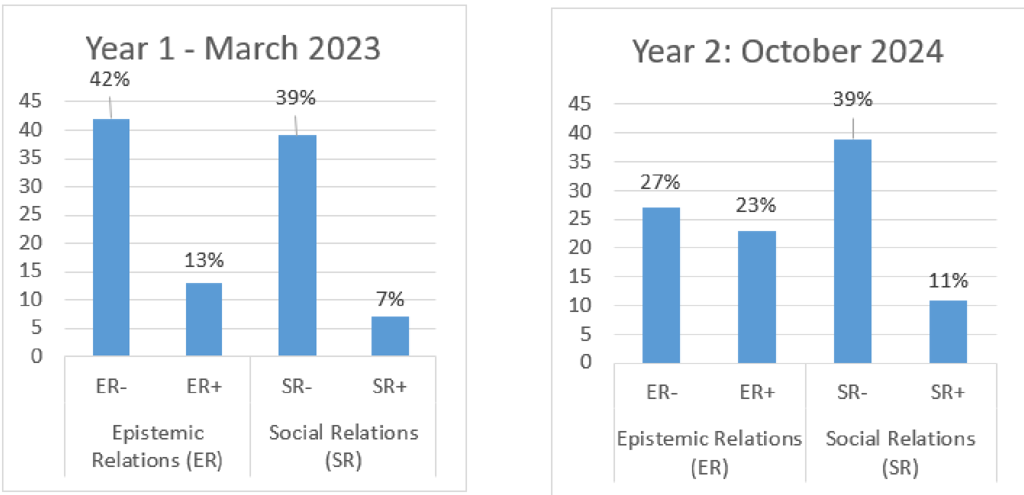

Figure 1 Shifts in the epistemic and social relations of the conceptions of the work of mathematicians

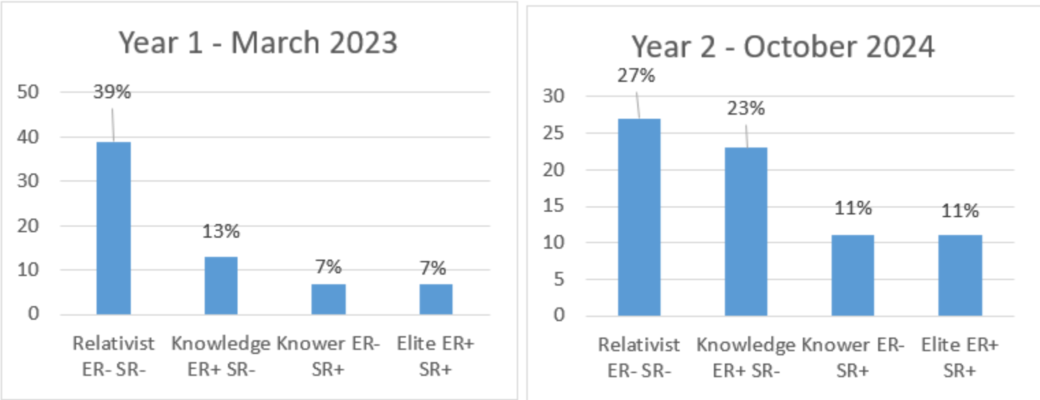

Figure 2 Shifts in the specialisation codes of the conceptions of the work of mathematicians

The graphs reveal a shift in pre-service teachers' (PSTs) perceptions of mathematicians from Year 1 (March 2023) to Year 2 (October 2024). Initially, the Relativist Code (ER- SR-) dominated at 39%, reflecting general, non-specific views on mathematicians, but this decreased to 27% by Year 2. Meanwhile, the Knowledge Code (ER+ SR-), which emphasises expertise over personal attributes, increased from 13% to 23%, indicating a growing focus on mathematicians' work and content mastery. The Knower Code (ER- SR+), prioritising personal and social attributes, rose slightly from 7% to 11%, suggesting increased recognition of identity and relatability. Similarly, the Elite Code (ER+ SR+), which balances expertise and personal traits, grew from 7% to 11%, pointing to a developing appreciation for holistic representations of mathematicians.

This study explored how pre-service mathematics teachers' (PSTs) perceptions of mathematicians evolved during their Bachelor of Education (BEd) programme, particularly between the first and second years. The findings reveal a significant shift in PSTs' understanding of mathematicians, reflecting changes in both epistemic relations (ER) and social relations (SR) as framed by Maton's Legitimation Code Theory (LCT).

4.1 A relativist lens on mathematicians

In Year 1, the dominant relativist perspective (ER- SR-) is reflected in statements that lack specificity about the work of mathematicians. For instance:

> *"I think it would be quite interesting and tedious at the same time."*
>
> *"It will be a good experience."*
>
> *"Obviously, it will be nice."*

These comments suggest a generalised view, often devoid of a concrete understanding of mathematicians' roles.

By Year 2, while ER- SR- responses have decreased from 42% to 27%, similar sentiments persist in comments such as:

"It will be a dream come true... because mathematics is a challenging subject in my community."

"I think it will be fun and sometimes challenging because mathematics is not an easy subject for the majority of learners in schools."

The decrease in relativist responses indicates a gradual move toward more nuanced perspectives but highlights the initial lack of specific knowledge about mathematicians' work.

4.2 Increasing recognition of Epistemic Relations

The rise in ER+ responses (from 13% in Year 1 to 23% in Year 2) illustrates the growing acknowledgement of mathematicians' specialised knowledge and contributions.

This shift is evident in Year 1 comments like:

"It will constantly train me to approach concepts differently and creatively, while still using the correct principles of mathematics."

"I should be able to know how to solve problems and attend to learners when they ask questions."

By Year 2, this theme becomes more pronounced with statements such as:

"Working as a qualified mathematician involves conducting research, solving complex problems, and applying mathematical principles in various fields such as science, engineering, finance, and technology."

"Theories learned can be applied to real-world problems, contributing to fields like artificial intelligence and finance."

"It will involve problem-solving, engaging in complex issues requiring deep analytical thinking and creativity."

These responses showcase students' evolving awareness of mathematics' intellectual rigour and its real-world applications.

4.3 Persistent pedagogical framing

Throughout both years, many responses frame mathematicians within a teaching context, reflecting pre-service teachers' roles. In Year 1, this is evident in statements such as:

"It will be fascinating because I will be able to teach my students ways to achieve their mathematics dreams."

"It will be interesting to teach children how to solve problems."

"I think it would be a fun experience, as once I know what I am doing, I become excited to help others understand as well."

By Year 2, similar views persist:

"I'll be helping learners understand and enjoy math. My day will be filled with sharing my passion for numbers and problem-solving."

"You would create lessons that make math interesting and relatable, showing students how it applies to their everyday lives."

"I believe teaching what you know will exactly help the student to understand."

These responses highlight a strong pedagogical framing, often centred on learners' needs and engagement.

4.4 Limited emphasis on Social Relations

Social relations (SR+) see a modest increase from 7% in Year 1 to 11% in Year 2, reflecting a slight shift toward considering mathematicians' personal and relational attributes. In Year 1, these are observed in comments such as:

> *"It would be an inspiration to others that being a mathematician is possible."*
>
> *"I love mathematics, but having to use it every day might not be something I enjoy."*

In Year 2, the focus expands slightly:

> *"I believe the experience will be fulfilling as it allows mathematicians to share their passion for mathematics with students and inspire the next generation."*
>
> *"Being a qualified mathematician will mean a big achievement, and people around me will surely get motivated."*
>
> *"It will involve collaboration with other professionals, which fosters teamwork and creativity."*

While social relations remain secondary to epistemic considerations, these responses reflect an increasing recognition of mathematicians' relational and societal impacts.

4.5 Evolution toward a balanced understanding

The growth in Elite Code responses (ER+ SR+), from 7% in Year 1 to 11% in Year 2, marks a shift toward a holistic understanding of mathematicians. This balance is reflected in Year 1 statements such as:

> *"I think it will be mentally challenging and fulfilling at the same time, as it constantly trains me to approach concepts differently."*

By Year 2, such balanced views are more evident:

> *"Being a qualified mathematician requires strong analytical skills, curiosity, and passion for continuous discovery."*
>
> *"The work would involve conducting research and applying knowledge while collaborating with others to solve complex challenges."*
>
> *"It involves using mathematical theories to solve real-world problems, with opportunities to explore and innovate in diverse fields."*
>
> *"It would be rewarding to see learners grow and develop their math skills over time, while also engaging in lifelong learning."*

These comments demonstrate the students' increasing appreciation for both the intellectual and relational aspects of mathematics.

5. Discussion of Findings

This study sought to explore shifts in pre-service mathematics teachers' (PSTs) perceptions of mathematicians from their first year to the end of their second year in a Bachelor of Education

(BEd) programme. The findings reveal nuanced changes in how PSTs understand the epistemic and social dimensions of a mathematician's work. Below, the findings are discussed in relation to the research questions, supported by previous studies, and presented in prose form.

Research Question 1: How do pre-service mathematics teachers' perceptions of mathematicians change throughout their teacher education programme?

The study found that PSTs' perceptions evolved significantly over the course of the programme. In their first year, PSTs predominantly exhibited a relativist perspective (ER- SR-), characterised by general and non-specific views of mathematicians. For instance, many described mathematicians' work as "showing young people different perspectives in mathematics" or "helping students learn maths better." These perceptions reflect a societal framing of mathematicians as educators rather than practitioners of mathematics, aligning with Cirillo and Herbel-Eisenmann's (2011) findings that classroom experiences shape learners' views of mathematicians, emphasising teaching roles over authentic mathematical practices.

By the second year, PSTs' descriptions reflected a growing recognition of mathematicians' epistemic practices (ER+). The study reveals that PSTs began articulating deeper insights into mathematicians' roles, such as engaging in "abstract thinking" and "applying mathematical theories to solve real-world problems." This shift aligns with Boaler's (2016) argument that mathematics education should extend beyond procedural drills to highlight creativity and problem-solving.

Pre-service teachers, like other social actors, become "knowers" by adopting a new perspective that allows them to identify and understand what defines "authentic reality" (Bernstein, 2000, p. 164 in Langsford & Rusznyak, 2024). As Maton (2014) explains, this perspective is shaped through exposure to new concepts, relevant experiences, and interactions with key individuals. However, many PSTs continued to view mathematicians primarily through a pedagogical lens, indicating an enduring influence of their teaching-focused experiences, consistent with Lortie's (1975) concept of the "apprenticeship of observation."

These findings align with prior research highlighting the challenges pre-service teachers (PSTs) face in bridging the theoretical and practical aspects of mathematics education (Mårtensson & Ekdahl, 2021). For instance, Lampert (2010) emphasises the importance of integrating theoretical knowledge with practical experiences to deepen reasoning about teaching practices. This study supports such approaches, as the integration of variation theory proved instrumental in reshaping PSTs' epistemic perceptions. By engaging with variation theory, PSTs began to develop a "mathematical gaze" that allowed them to identify critical aspects of mathematical practices and consider teaching mathematics in more sophisticated ways (Maton, 2014; Pang & Marton, 2003). However, the persistent framing of mathematicians' work through a pedagogical lens reflects Korthagen's (2010) observation that, in the absence of robust theoretical integration, PSTs often revert to familiar teaching narratives, as also noted by Mårtensson and Ekdahl (2021).

Research Question 2: What changes occur in pre-service mathematics teachers' understanding of the epistemic dimensions of a mathematician's work?

The study reveals that PSTs developed a stronger epistemic gaze, shifting from weaker to stronger epistemic relations (ER+). Initially, only 13% of PSTs demonstrated knowledge of the epistemic practices of mathematicians, such as abstraction, proof, and problem-solving. By the end of their second year, this increased to 23%, reflecting a growing appreciation for the intellectual rigour and theoretical aspects of mathematics.

This transformation was attributed to the mathematics methodology course, which used variation theory to structure learning around critical aspects of mathematical practices (Pang & Martone, 2003). For example, one PST described mathematicians as "innovators who develop new principles to address complex problems," illustrating a deeper engagement with the epistemic dimensions of mathematics. Such responses resonate with Schoenfeld's (2016) argument that problem-solving and abstract thinking are central to mathematicians' work and should be emphasised in mathematics education.

Developing a mathematical gaze requires immersion in the intellectual work of mathematicians, which is often abstract, complex, and not typically visible in traditional classroom settings (Langsford & Rusznyak, 2024). This process demands that PSTs go beyond reflecting on "the study of teachers' practices through live or recorded lessons" to "practise the theory in real situations" (Mårtensson & Ekdahl, 2021). Variation theory facilitated this by emphasising patterns of variation and invariance, enabling PSTs to discern critical aspects without explicit instruction (Pang & Marton, 2003; Marton, 2015). This method mirrors Watson and Mason's (2006) suggestion for task design that highlights problem-solving and fosters critical and creative thinking.

Research Question 3: How do pre-service mathematics teachers' perspectives on the social and relational aspects of mathematicians evolve over time?

The study found limited progress in pre-service teachers' (PSTs) recognition of the social and relational aspects of mathematicians' work (SR+). While some PSTs acknowledged collaboration and interdisciplinarity, their understanding often lacked depth. For example, responses such as "collaborating with professionals in various fields" reflect a superficial appreciation of teamwork and fail to fully capture the dynamic and relational nature of mathematical work, as emphasised by Burton (2004) and Hagenkötter et al. (2022).

By the end of their second year, only 11% of PSTs demonstrated a balanced understanding (ER+ SR+), compared to 7% in their first year. This marginal improvement highlights the persistent gap in PSTs' appreciation of the social dimensions of mathematics, underscoring the need for teacher education programmes to explicitly address this aspect. Fostering a holistic view of the discipline requires integrating opportunities for PSTs to engage with the collaborative and interdisciplinary practices that define authentic mathematical work. Additionally, this limited focus on social dimensions aligns with Taylor's (2018) critique of the narrow scope of school mathematics curricula, which often prioritise procedural knowledge at the expense of relational and collaborative skills. The findings reinforce the importance of exposing PSTs to the broader contexts of mathematics, encouraging them to move beyond individualistic or teaching-focused narratives. Blanton and Stylianou (2009) similarly stressed the value of collaboration between mathematicians and educators to bridge the gap between university-level mathematics and school teaching. By doing so, teacher education programmes can equip PSTs with a more comprehensive understanding of mathematicians' roles, fostering both epistemic and social engagement.

6. Conclusion

This study underscores the evolving perceptions of pre-service mathematics teachers (PSTs) regarding the work of mathematicians, revealing significant shifts in their epistemic understanding while highlighting enduring challenges in integrating social and relational dimensions. PSTs progressed from a relativist perspective to a more nuanced epistemic gaze, recognising the abstract and applied practices of mathematicians. However, their views remained strongly influenced by prior teaching experiences, reflecting persistent pedagogical biases and limited engagement with the collaborative and interdisciplinary aspects of mathematical work. The integration of variation theory in the curriculum played a pivotal role in fostering a deeper understanding of critical mathematical concepts and practices. By enabling PSTs to discern key aspects of mathematicians' intellectual work, this approach contributed to the development of a "mathematical gaze." Nonetheless, the findings highlight a need for teacher education programmes to place greater emphasis on the social and creative dimensions of mathematics, encouraging PSTs to view mathematicians not only as problem-solvers but also as collaborators engaged in dynamic and relational practices. Balancing the epistemic and social dimensions of mathematics education is essential for bridging the gap between theory and practice. Teacher education programmes must expose PSTs to authentic mathematical practices, fostering a comprehensive understanding of the discipline. By doing so, future educators will be better equipped to inspire a more dynamic, creative, and holistic appreciation of mathematics, advancing both their teaching strategies and the discipline as a whole.

7. Declarations

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Data Availability: The datasets generated and/or analysed during the current study are available from the author upon reasonable request.

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- is learning that is interdisciplinary;
- is learning experiences that are tailored to the local audience;
- is learning that is grounded in and supports the development of a love for one's place; and
- is local learning that serves as the foundation for understanding and participating appropriately in regional and global issues.

These principles not only give PBE its unique character, but it also define PBE, and it makes it possible for PBE to realise its aims and objectives in a unique and a distinctive way

3.6 Aims and objectives of place-based education

In principle, PBE aims to amplify 'place' by locating learning within the local context or the community. This it aims to do, firstly by overcoming the often hegemonic and ubiquitous educational agendas developed elsewhere in favour of an approach that uses the local as a starting point (Lyle, 2015), and secondly by partnering teachers, students and community members in the process of teaching and learning using the local context (Coughlin & Kirch, 2010). In this way, learners and teachers become participants in meaning-making and knowledge production and so contribute in various ways to their place or their community (Starrett et al., 2020). These aims correspond with what is expected from Mathematics education in SA. According to the Curriculum and Assessment Policy Statement, Grades R-12 (CAPS) (2011, p. 8) teaching and learning of Mathematics aims to develop:

a) a critical awareness of how mathematical relationships are used in social, environmental, cultural and economic relations;
b) confidence and competence to deal with any mathematical situation without being hindered by a fear of mathematics
c) a spirit of curiosity and a love for mathematics
d) an appreciation for the beauty and elegance of mathematics
e) recognition that mathematics is a creative part of human activity
f) deep conceptual understanding in order to make sense of mathematics
g) acquisition of specific knowledge and skills necessary for:

- the application of mathematics to physical, social and mathematical problems
- the study of related subject matter (e.g. other subjects)

PBE also aims at creating better citizens (Carter-Guyette, 2019). These citizens will not only have developed learning skills, but they will also be able to use these skills to solve problems, to address inequality, to challenge inequity and to create a better world for all. This is also evident from the CAPS (DBE, 2011, p. 5) which expect that also Mathematics learners be equipped to:

a) identify and solve problems and make decisions using critical and creative thinking;
b) work effectively as individuals and with others as members of a team;
c) organise and manage themselves and their activities responsibly and effectively;