CHAPTER EIGHT

Aftermath of Overlooking Foundation Pre-Service Mathematics Teachers' Beliefs: A Self-Study in Post-Apartheid South Africa

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Abstract: Mathematics is often described as a "high status" discipline and in primary schools, mathematics is compulsory. However, in educational research, myriad performance issues in primary mathematics teaching and learning have been emphasised. The foundations of future mathematics learning are laid for young learners by their teachers who are often the learners' main resource, and foundation mathematics problems snowball as learners progress through schooling levels. Teachers are certified by higher education institutions so it is thus worthwhile for primary mathematics teacher educators to explore constructive, reflexive, optimistic, starting-with-ourselves approaches to reflect on what is offered in mathematics education. This looking inward approach contrasts with research in which recommendations are proposed to encourage improvements in professional practice. The research question for this study was: "What primary mathematics education professional learning can I (re)construct by reflecting on teacher education modules offered at my higher education institution, using a self-reflexive approach?" My primary study source was the foundation phase mathematics education modules detailed in University of KwaZulu-Natal's Handbook for 2024. This qualitative study explored the competences offered by these modules in the university's foundation phase mathematics teacher education programme, and included a set of objects used as a metaphor for self-reflexivity and to represent the importance of attending to the often-overlooked pre-service teachers' beliefs about mathematics education in post-apartheid South Africa.

Keywords: Beliefs, post-apartheid South Africa, pre-service mathematics education, self-reflexive metaphor, starting with ourselves.

1. Introduction

Mathematics is often described as a "high status" discipline and as Valero (2013, p. 2) noted, in a 21st-century school curriculum, it is "unthinkable" to omit mathematics. In South African primary schools, mathematics is compulsory with foundation phase (FP) learners required to spend approximately 32% of school time doing mathematics and intermediate phase (IP) learners, 22% of school time (Department of Basic Education, 2011). Reasons usually given for why mathematics is an essential subject are that mathematics knowledge and skills are valuable in many aspects of everyday life and positively influence life opportunities.

Myriad performance issues in South African primary mathematics teaching and learning are constantly emphasised in education research. Some of these were summarised by economist, Servaas van der Berg, who asserted that poverty and financial inequality are directly related to the unsatisfactorily poor quality of South African education systems. Van der Berg further

contended: "If you want to change the economy, you have to start with the education system" (as quoted in Thompson, 2024, para. 7).

There is no doubt that even in post-apartheid South Africa, school learner education is generally unequal and differentiated according to family income (Beckmann, 2021). Three decades after the birth of South African democracy, most schools in low-income areas, where learners are mainly of African origin, still experience apartheid after-effects. Hlatshwayo (2023) acknowledged that although there are limited improvements in learners' numeracy levels, many have not benefitted significantly from being schooled in the post-apartheid context.

The foundations of future mathematics learning are laid for young learners by their teachers who are often learners' main resource in South Africa. Thus learners depend on their teachers' mathematics subject content knowledge (SCK), pedagogical content knowledge (PCK), and beliefs about mathematics teaching and learning for their development of "appropriate knowledge, skills and values" (Department of Education, 2006, p. 7). In the early grades, foundation mathematics difficulties commence and snowball as learners progress through schooling levels (Lénárt et al., 2022). Given that pre-service teachers are certified by higher education institutions, their education programmes impact on primary, secondary, and tertiary levels of learners' mathematics learning.

Some of the most important role players in school education are the teachers, who "are central to the success of education outcomes" Hlatshwayo (2023, p. 389). However, if the teachers believe that mathematics is only for the gifted few, then that attitude towards mathematics achievement could be internalised by their learners. Teachers' attitudes and the general psycho-social environment in mathematics classrooms play a significant role in mathematics learning; learners' achievement is influenced by learners' mathematics attitudes and beliefs (Mazana et al., 2019). Those scholars further noted that suitable pedagogical strategies and appropriate social psychological environments promote enjoyment and positive attitudes towards mathematics, which in turn, benefit learners' performance.

On reviewing South African educational research on teaching and learning for a World Bank report, van der Berg and Hofmeyr (2018, p. 16) concluded that teacher education programmes are "not succeeding in adequately preparing students for the teaching profession." They bluntly pointed out that mathematics teachers are underprepared in teacher education programmes, certified teachers have insufficient mathematics SCK and PCK, pre-service teachers enrolled for teacher education are not top-performing candidates, and teacher development in higher education institutions is unsuccessful in building pedagogical skills for teaching young learners mathematics in African languages. Van der Berg and Hofmeyr (2018, p. 23) further declared that teacher development programmes have failed and hence require "independent evaluation as to whether they are working and why," and that "rigorous evaluation of the effectiveness of such pedagogical training must be done regularly."

After reading findings, conclusions, and recommendations in mathematics educational research, I realised that many disheartening, uncomfortable, and disconcerting issues have been raised. Furthermore, in mathematics education literature there are worthy (but sometimes complex) recommendations offered for improving mathematics teaching and learning. However, there is no point in dwelling on these saddening, problematic issues without doing something to make a difference in my professional practice as a South African primary mathematics teacher educator.

The *National Qualifications Framework Act* (Department of Higher Education and Training, 2015), which specifies expertise required by South African teachers in fundamental disciplines and subject areas, states that standards to define competence for specific subjects or specialisation are not defined in that government policy. Rather, specific subjects, such as primary mathematics education, need to develop their own standards within relevant teacher education communities of practice. In other words, it is important that mathematics teacher education community members take up the opportunity to communicate and make public improvement possibilities. Thus, instead of thinking, "there is nothing I can do about transformation in primary mathematics teacher education," it was preferable to explore constructive, reflexive, optimistic "starting with ourselves" (van Manen, 1990, p. 43) approaches to reflect on what is offered in mathematics teacher education at my own higher education institution. The knowledge gained through looking inward could then be used to improve my professional practice and hopefully inspire others in my educator community to also commit to using improvement-orientated self-study approaches to address the criticisms levelled at mathematics teacher education.

An approach where self-improvement considerations are key, contrasts with research where recommendations are presented to encourage changes in professional practice. The research question for this starting-with-ourselves study was: "What primary mathematics education professional learning can I (re)construct by reflecting on teacher education modules offered at my higher education institution, using a self-reflexive approach?"

2. Theoretical Framework for Mathematics Teacher Education Programmes

Many mathematics teacher educator researchers (for example, Shulman, 1987; Tatto, 2012; 2018) have suggested competence requirements for primary mathematics education curricula. Manouchehri (1995) addressed various areas of mathematical competence including SCK, PCK, pedagogical reasoning, and beliefs. Manouchehri (1995, p. 1) further defined SCK as "key facts, concepts, principles and explanatory framework in the discipline," PCK as the "knowledge of students and learning, knowledge of curriculum and school context, and knowledge of teaching," pedagogical reasoning as the "process of transforming content knowledge into forms that are pedagogically powerful and adaptive to particular groups of students," and beliefs as the

"attitudes and assumptions of entering preservice teachers, beginning teachers, and experienced teachers."

2.1 Affect in mathematics and mathematics teacher education

Tatto (2018, p. 210), in her international (13 countries, including Botswana) study of primary mathematics teacher education programmes, pointed out that teachers' beliefs are an "essential component of what makes a teacher effective and thus have been an important feature of teacher education." And Manouchehri (1995) indicated that often, pre-service teachers' beliefs about pedagogy were developed prior to their enrolment in teacher education programmes, thereby having a profound influence on their learning in those programmes. Additionally, Tatto (2018) highlighted the significance of pre-service teachers' beliefs about learning mathematics—how they perceived the nature/vision of mathematics, how they experienced learning mathematics in schools, and what their approach to teaching mathematics might be.

In the South African post-apartheid context, the past experiences of pre-service teachers' mathematics school learning may not have been ideal because of the distressing and haunting influences of 37 years of apartheid government that ended in 1994. The government enforced racism, using separate and unequal social, political, and educational opportunities to further its apartheid ideals. It employed pervasive ideology systems that imposed perverse "symbols, images, beliefs, feelings, thoughts, and attitudes" (Wills, 2011, p. 14). The watered-down government school curriculum for most South African learners (generally, of African origin) was a political strategy, and the dumbing-down of mathematics could regretfully result in teachers and learners developing a vision of mathematics as being only for gifted learners, too difficult to achieve, and reserved for other scholars.

From Manouchehri's (1995, p. 1) definition of beliefs, it appears that the author considered "beliefs" to be synonymous with "attitudes and assumptions." Research in mathematics education related to affect (see for example, Di Martino & Zan, 2011; Zhang & Morselli, 2016) has noted that in the affect field, there is no agreement on the definitions of constructs such as beliefs, attitudes, emotions, and values because of the changing nature of teaching and learning environments. However, this does not detract from the need to make pre-service teachers aware of beliefs that influence practice. Zhang and Morselli (2016) pointed out that teacher education programmes should stress exploring belief awareness rather than only belief changing.

Di Martino and Zan (2011) emphasised the relationship between beliefs and emotions because emotions bring physiological reactions into play, which interfere with cognitive processes and may stifle thinking. Furthermore, emotions play a significant role in "coping, adapting and decision making" (Di Martino & Zan, 2011, p. 472). After analysing the narrative and autobiographic descriptions of well over a thousand research participants' relationships with mathematics, these authors proposed a model that explained the relationships between affect

constructs and concluded that attitude acts as a "bridge between beliefs and emotions" (Di Martino & Zan, 2011, p. 484).

Di Martino (2024) then developed a theoretical framework to study the attitudes of pre-service primary teachers required to teach mathematics in their Italian schooling context. His study "confirmed how negative attitudes towards mathematics and towards the idea [of] having to teach mathematics are very common" (Di Martino, 2024, p. 434) among prospective primary school teachers.

2.2 Mathematics education literature addressing pre-service teachers' beliefs

Di Martino and Zan (2011) questioned assessment approaches that focus mainly on accuracy. They promoted the notion that the mathematics vision in solving mathematics problems should be focused on the processes rather than just the accuracy of the final solution. In other words, possible mathematics assessment shifts are required to promote the "idea of success from the production of correct answers to the enactment of meaningful thinking processes" (Di Martino & Zan, 2011, p. 487). Accuracy emphasis, along with other classroom experiences, influence learners' attitude to mathematics because some learners consequently perceive themselves as innately unable to succeed in mathematics. The authors acknowledged the difficulties of teachers and teacher educators in accepting the emphasis on thinking process required for this vision, but emphasised the importance of overcoming the "distorted vision of mathematics and possibly low perceived competence" of learners in order to address their affect in mathematics education (Di Martino & Zan, 2011, p. 487).

Tatto's (2018, p. 244) study indicated that lower levels of performance of newly qualified teachers were associated with those teachers believing that mathematics problem solving required "memorizing a collection of rules and procedures," prescribing methods, and applying "definitions, formulas, mathematical facts and procedures." This means that these teachers subscribed to the belief that mathematics teaching mainly involved encouraging instrumental understanding with a "rule-based . . . focus on how to do something" as opposed to relational understanding (Herheim, 2023, p. 390), which relates to "understanding structures, searching for patterns, and relating new concepts to previous understanding" (Herheim, 2023, p. 391). According to Herheim (2023, p. 391), "both instrumental and relational understanding have their advantages, but the instrumental one is more short-term, rigid, and context-dependent."

Furthermore, Manouchehri (1995, p. 12) encouraged teacher education programmes to "address the prospective teachers' pre-existing beliefs," assist pre-service teachers in making their "implicit beliefs about teaching, learning, subject matter and learning to teach explicit, . . . challenge the adequacy of those beliefs," and facilitate the integration of new ideas and information into their existing beliefs. Tatto et al. (2012, p. 172) noted that belief "change is unlikely to occur unless teacher-preparation programs explicitly address beliefs about mathematics and mathematics learning." And Manouchehri (1995) recommended the use of

reflection for challenging pre-service teachers' beliefs. In answering his question: "Can reflective practice be taught?" Russell (2005, p. 202) concluded that reflective practice can and should be taught. He noted that explicit, direct, thoughtful, and patient self-reflection-in-action interpretations improved reflective practice teaching.

In addition, Tatto et al. (2012) found that future teachers' belief patterns matched their mathematics teacher educators' beliefs, which implies that altering the teacher education curriculum is unlikely to change beliefs of pre-service teachers. Tatto et al. (2012, p. 172) contended that a change in the beliefs of pre-service teachers "if it is to occur, would probably require a significant investment in professional development for practicing teachers as well as for teacher educators."

Various mathematics education researchers consider pre-service teacher belief reflections to be purposeful in mathematics education. This means that in mathematics teacher education, attending to affect should be an integral part of teacher education programmes because mathematics is the subject that "triggers the strongest negative emotions" (Di Martino & Zan, 2011, p. 471). Moreover, there appears to be a need to inspire mathematics teacher educators to make use of reflexive self-study strategies for professional learning in order to seek opportunities to improve their professional practice in sustainable, doable, and context appropriate ways (Manouchehri, 1995; Tatto, 2018). The aims of my study were to contribute to the development of my own usable self-learning related to social justice issues, to improve my own professional practice as a primary mathematics teacher educator, and to make public my self-study for scrutiny by the mathematics education community.

3. Self-Study Research

3.1. Situating myself as a mathematics teacher educator

I have been a mathematics teacher educator for almost four decades, teaching at South African higher education institutions in both the apartheid and post-apartheid contexts. I spent most of my mathematics teacher educator career at a higher education institution in Durban, South Africa. I am a White South African woman and prior to 1994, was a primary mathematics teacher educator in an institution where only White pre-service teachers were enrolled. After 1994, primary pre-service teachers were predominantly of African origin, and I realised that the context in which they had been schooled could influence the way they experienced mathematics education studies. In other words, the contexts in which most pre-service teachers were educated at high schools could be substantially different from what White privileged pre-service teachers experienced. The interplay between their cognitive and emotional aspects in mathematics education (affect) could be somewhat different from what White pre-service teachers and their teacher educators experienced. Thus, I saw the need to (re)construct my professional learning as a post-apartheid South African mathematics teacher educator. I subscribe to Griffiths' (2003, p. 167) slogan, "social justice is a verb," and align my views with her approach because social

justice is "always action oriented, [and] always unfinished," and for social justice and self-improvement reasons, chose to examine FP teacher mathematics education at my higher education institution using a self-reflective approach.

3.2. Characteristics of self-study

Although Tatto (2017, p. 622) advocated self-study by mathematics teacher educators, she cautioned that "reflective researchers are especially vulnerable to criticisms by policymakers" and if a bottom-up approach is pursued, then "to be taken seriously, their research needs to demonstrate rigour." In self-study, researchers consider the term "vigour" to be more appropriate than rigour to gauge research quality (Faulkner, 2019). This means that self-study research should rather be assessed in terms of, for example, worthiness, competence, enthusiasm, and robustness.

A criticism levelled at self-study is that it is naval-grazing, meaning that the researcher is self-absorbed and has a narrow outlook with a "limited desire to move [and] change" or relate to worldwide perspectives (Darian-Smith, 2023, p. 226). However, a defining characteristic of self-study methodology is that it is improvement-aimed (LaBoskey, 2004), which requires the researcher to actively pursue professional practice changes. Other self-study characteristics include being self-initiated and self-focused, being interactive, using mainly qualitative methods, and using authentically supported teaching practices (LaBoskey, 2004). Furthermore, self-study for improvement is usually facilitated through relearning, rethinking, and reimagining, using self-reflexivity within "discourses of the social construction of knowledge, reflective practice and action for social change" (Guðjónsdóttiri & Dalmau, 2006, p. 108).

3.3. Data sources and methods

University of KwaZulu-Natal's (UKZN, n.d.) *College of Humanities: Handbook for 2024* detailed information about the five mathematics education modules specified for pre-service teachers enrolled in the FP specialisation for the Bachelor of Education (BEd) curriculum. I copied information dispersed within the handbook, and tabulated the aims and contents of these modules to facilitate easy, aggregated data access. Thereafter, I used an interpretive method of close data reading to gain information about what is offered across these modules in relation to SCK, PCK, and beliefs. That was the first qualitative method utilised.

My second qualitative method was an arts-inspired strategy to provoke self-reflexivity. I selected a set of objects as a means of making sense of my learnt knowledge about the competences addressed in the five modules. These objects facilitated reflexivity to convey a "new perspective on a situation" by viewing a "concept from a target domain [primary concept] in terms of another, apparently dissimilar concept from a source domain [secondary concept]" (Akula et al., 2023, p. 23202). The target domain was the competences indicated in UKZN's primary teacher education curriculum and the source domain was my selected object set. Furthermore, an

object's visual image serving as a metaphor is powerful in "conveying the metaphorical message" (Akula et al., 2023, p. 23202). Thus, I used the metaphor to convey my understanding of the importance of teaching and learning beliefs in mathematics education.

I adapted Samaras' (2011, p. 105) writing prompts to explain how my self-selected objects and associated metaphor expressed my learning in order to improve my professional practice as a primary mathematics teacher educator in our post-apartheid context. Thus, to elaborate on my metaphor reflection, I used the following four self-asked prompts to capture my professional learning after interpreting the offerings in UKZN's FP primary mathematics education modules.

- Prompt 1: Describe what metaphor you chose to capture the main idea of your research interest linked to the FP mathematics education curriculum at UKZN. Explain why you chose this metaphor. Give it a title that signifies the core meaning of your metaphor.
- Prompt 2: Indicate what in the metaphor represents you, the teacher educator, and what represents your professional learning.
- Prompt 3: Does culture play a role your metaphor?
- Prompt 4: Are there others involved in your metaphor memories? What roles do they play? What is their influence on your thinking? Do they see things the way you do? (van Laren & Mudaly, 2024).

3.4. FP mathematics education modules in UKZN's *Handbook for 2024*

UKZN's 4-year bachelor's degrees in education offer three specialised curricula: FP (Grades R–3), IP (Grades 4–6), and a combined senior phase (Grades 7–9) and further education and training (Grades 10–12). For each BEd specialisation, the required credit minimum is 512. An academic performance score (APS) of 31 points is the minimum for acceptance into Bed specialisations. For pre-service teachers specialising in the FP, UKZN offers two language options: English Home Language and isiZulu First Additional Language, or isiZulu Home Language and English First Additional Language.

Because of the importance of mathematics foundations, my interest lay in the primary mathematics education modules. I drew up a summary (see Figure 1) of the FP specialisation, indicating the minimum mathematics APS required for registration; number of credits; module study year, title, aim, content; and page number of the information in the UKZN (n.d.) handbook (see Figure 1).

Figure 1: *UKZN FP primary mathematics education*

FOUNDATION PHASE
For admission to specialization: Minimum Academic Performance Score on NSC for Mathematical Literacy Level 4 (50% – 59%) or Mathematics Level 3 (40% - 49%)
First year - Numeracy in the Early Years (16 Credits): **Aim:** •To introduce students to fundamental concepts of early mathematics to strengthen their basic mathematical knowledge. •To provide students with knowledge of number sense and how to deal with numbers in number relationships and in operations in the early years of schooling. To explore early geometric thinking, geometric concepts and to develop students' reasoning about space and shape in the early years of schooling. •To provide students with an understanding of the indigenous concepts of measurement; the measuring process and the use of mathematics vocabulary in the early years of schooling. • To equip students with knowledge of early data handling including collection, organisation, representing and interpreting data. **Content:** • Fundamental concepts of mathematics to strengthen students' mathematical knowledge. • Number sense and how to deal with numbers in number relationships and in operations. • Geometric thinking, geometric concepts and reasoning about space and shape. • The concepts of measurement; the measuring process and the use of mathematics vocabulary. • Data handling including collection, organisation, representing and interpreting of data. (UKZN, 2024, p. 427)
Second year - Mathematics Education for Foundation and Intermediate Phase 1 (16 Credits): **Aim:** The aim of this module is to provide a sound basis of the educational theories that underpin mathematics teaching in the Foundation Phase and Intermediate Phases, and in depth understanding of the mathematics content areas taught in the Foundation Phase and Intermediate Phases. **Content:** Introduction to theoretical basis of mathematics education: • Views on the nature of mathematics • Views on how children learn mathematics • Mathematical proficiency • Development of algebraic thinking Study of important and foundational mathematics education ideas applicable to Foundation and Intermediate Phase teaching of • Number • Operations • Patterns • Early algebra • Proportional reasoning (UKZN, 2024, pp. 408, 409)
Second year - Mathematics Education for Foundation and Intermediate Phase 2 (16 Credits): **Aim:** The aim of this module is to provide a sound basis of the educational theories that underpin mathematics teaching in the Foundation and Intermediate phases, and in depth understanding of the mathematics content areas taught in the Foundation and Intermediate phases. **Content:** Introduction to theoretical basis of mathematics education: • Development of geometric thinking, and theories of learning geometry • Views on how children learn measurement concepts • Development of statistical literacy • Proportional reasoning • Study of important and foundational mathematics education ideas applicable to Foundation and Intermediate Phase teaching of • Shape and Space • Measurement. (UKZN, 2024, p. 409)
Third year - Mathematics for Foundation Phase Method 1 (16 Credits): **Aim:** • To provide students with theoretical understanding of how Foundation Phase Learners learn Mathematics • To empower students to plan and teach the Mathematics curriculum in the Foundation Phase (Grades R-3) • To introduce students to various approaches for teaching Foundation Phase learners: - Number sense and number relationships - Patterns and Algebra - Geometry (Space and Shape) - Data handling. • To equip students with knowledge and strategies of assessing Mathematics in the Foundation Phase. • To provide students with knowledge and skills to develop Foundation Phase Mathematics resources. **Content:** •Theoretical understanding of how Foundation Phase Learners learn Mathematics • Planning and teaching Mathematics curriculum in the Foundation Phase (Grades R-3) • A variety of approaches for teaching: - Number sense and relationships - Patterns and Algebra - Geometry (Space and Shape) - Data Handling. • Knowledge and strategies of assessing Mathematics in the Foundation Phase. • Knowledge and skills of developing Foundation Phase Mathematics resources. (UKZN, 2024, p. 430)
Third year - Mathematics for Foundation Phase Method 2 (16 Credits): **Aim:** • To develop students' comprehensive theoretical understanding of how Foundation Phase learners learn Mathematics. • To empower students to plan and teach the Mathematics curriculum in the Foundation Phase (Grades R-3). • To equip students with knowledge of critiquing observed mathematics through theoretical lenses • To provide students with a variety of strategies for teaching: - Number relationships and place values - Geometry (Space and Shape) - Measurement. • To equip students with knowledge to identify and use relevant assessment strategies for Mathematics in the Foundation Phase. • To develop students' in-depth understanding of preparing and designing resources for Mathematics in the Foundation Phase. • To empower students with knowledge of identifying barriers and applying intervention strategies to learning Mathematics in the Foundation Phase. **Content:** • Theoretical concepts about how Foundation Phase learners learn Mathematics. • Mathematics CAPS curriculum in the Foundation Phase (Grades R-3). • A variety of approaches for teaching: - Number relationships and place values - Geometry (Space and Shape) - Measurement. • Knowledge of identifying and using relevant assessment strategies for Mathematics in the Foundation Phase. • An in-depth understanding of preparing and designing resources for Mathematics in the Foundation Phase. • Knowledge of identifying barriers and applying intervention strategies to learning Mathematics in the Foundation Phase. (UKZN, 2024, p. 431)

3.5. Analysis of FP primary mathematics education

This qualitative analysis focuses on competences offered across the modules in the FP. Thereafter I explain the self-selected set of objects' metaphor I chose to express my professional learning gained by exploring FP UKZN primary mathematics education. Then I used this metaphor to assist in answering my research question.

The South African National Senior Certificate achievement levels determine the FP BEd admission. To register for this phase, pre-service teachers require a minimum of 40–49% in mathematics or 50–59% in mathematical literacy. The total number of credits allocated to mathematics education modules is 80.

The first-year Numeracy in the Early Years module concentrates on FP SCK to "strengthen their basic mathematical knowledge" (UKZN, n.d., p. 427). The study topics to reinforce pre-service teachers' mathematics knowledge include number sense development, number relationships, number operations, geometric thinking, mathematics vocabulary, indigenous measurement concepts, measurement, and data handling. The emphasis is on developing pre-service teachers' SCK essential for teaching young mathematics learners.

The two second-year modules Mathematics Education for Foundation and Intermediate Phase 1, and Mathematics Education for Foundation and Intermediate Phase 2, attend to PCK using educational theories that pertain to "mathematics content areas" (UKZN, n.d., pp. 408–409) taught in the FP and IP. In the former module, FP and IP PCK for the teaching of early algebraic thinking, number operations, patterns, and proportional reasoning are included. In the latter module, FP and IP PCK for development of geometric thinking, measurement concepts, statistical literacy, and proportional reasoning are covered.

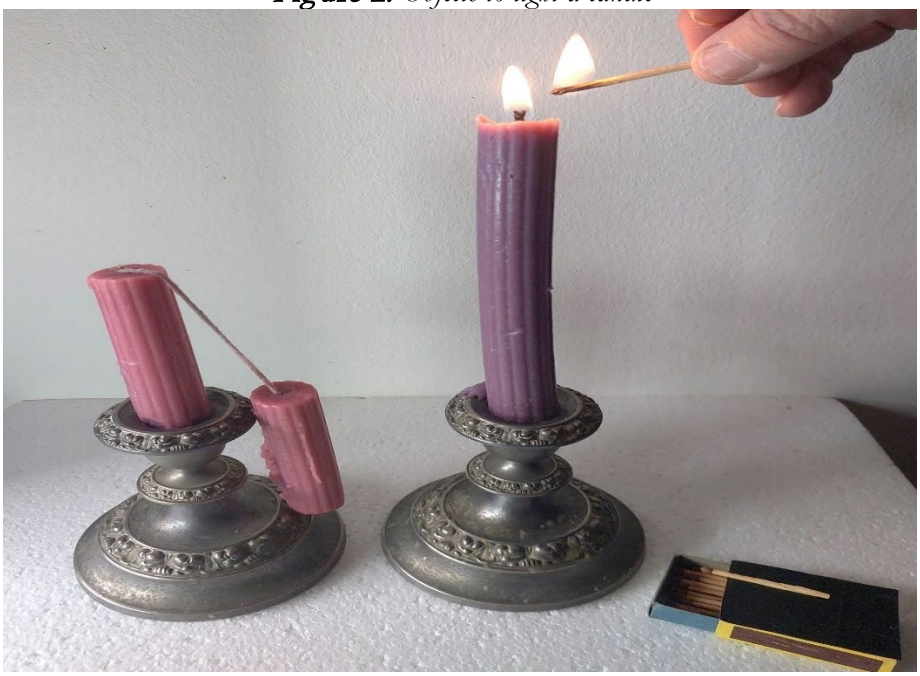
The two third-year modules, Mathematics for Foundation Phase Method 1 and Mathematics for Foundation Phase Method 2, focus on FP PCK utilising theoretical understanding of how FP "learners learn mathematics" for planning to teach the curriculum using a variety of approaches (UKZN, n.d., pp. 430–431). In the former module, FP PCK for teaching number sense and relationships, patterns and algebra, geometry, and data handling are offered. Other topics in this module include strategies for assessment and resource development. In the latter module, FP PCK focuses on teaching number relationships and place, value, geometry, and measurement. Additional topics explored are assessment methods, resource development, and identification of barriers to learning mathematics together with intervention strategies.

My analysis showed that 20% of this total number of credits is allocated to FP SCK, 40% to combined FP and IP PCK, and 40% to FP PCK. The approximate credit percentage of mathematics education modules in the FP specialisation is 16% of the total number of credits (512) required for the BEd degree.

4. My Professional Learning Gained: A Self-Reflexive Metaphor

The professional learning I gleaned from my analysis of the competencies offered in primary mathematics education modules led me to recognise possibilities for improving my teacher education practice in the post-apartheid context. I selected objects related to "lighting a candle" (see Figure 2) to illustrate my learning, to convey improvement possibilities for UKZN, and to answer my research question. I used self-asked writing prompts to interrogate my metaphor.

Figure 2: *Objects to light a candle*



4.1 Prompt 1

I chose candleholders, wax candles, and a box of matches as objects for my metaphor because these are familiar items found in most South African homes. The main function of this object set is to shed light in the darkness. The two candleholders are identical and made of sturdy, substantial, opaque metal. For stability, the base of the holder is wider than its apex where the candle is secured. The holder keeps the candle vertical to prevent it from tipping, thus ensuring that it can safely and effectively provide light without being a fire hazard. The candle on the left-hand side of the image is broken into two pieces. The wick—a usually unseen yet a vital component of a candle— is embedded within it, running through the entire centre of the cylindrical wax column connecting the two broken pieces. This candle will not be effective when the wick at the candle tip is set alight. The intact candle will be effective when the candlewick is set alight. The box of matches is an essential requirement for lighting the wick to make a flame to illuminate darkness.

Although a candle's primary function is to shed light in darkness, there are many other symbolisms associated with a candle and candlelight. Candle flames are a symbol of enlightenment, clarity, knowledge, wisdom, optimism, hope, truth searching, anticipating new beginnings, and joy (Axiom Home, 2023). For example, the birthday tradition of making a wish after blowing out all birthday cake candles facilitates positive, joyful, personal reflection. Furthermore, the proverb "It is better to light a candle than to curse the darkness" emphasises the need for constructive action when faced with bleak circumstances. An appropriate title for my object set was thus "Igniting Positive Inner Illumination."

4.2 Prompt 2

I consider the candles to be the competences of a FP pre-service mathematics education teacher. In the damaged candle, the bottom piece of candle is the mathematics SCK offered in mathematics education modules and the top piece is the mathematics PCK covered in these modules. The wick, which is an essential yet almost invisible, overlooked element of the candle, is the pre-service teacher's mathematics teaching and learning beliefs. These beliefs form an integral part of their competence because it connects, and is central to, their SCK and PCK. Although beliefs and affect, in general, are abstract and difficult to pinpoint or physically observe, it is possible to interrogate and reflect on beliefs.

The ignited match is me, the mathematics teacher educator, and the box contains my current professional beliefs.

The opaque candleholder is the SCK the pre-service acquired as a school learner. The opaque, dense colour is fitting because the mathematics teaching and learning in many post-apartheid school classrooms may still feel the influences of unequal, unjust apartheid circumstances that may not always be observable and transparent. The shape of the candleholder shows the importance of having a stable, firm foundation to successfully support the candle—pre-service teachers' competences required for effective mathematics teaching.

The invisible oxygen surrounding the objects is the pre-service teacher's personal experiences of teaching and learning in post-apartheid primary and secondary school mathematics classrooms.

The flame is the pre-service teacher who enlightens by effective teaching.

To make the intact candle's flame effective, there are three essential components: fuel, oxygen, and heat. The fuel is the pre-service teacher's mathematics SCK, mathematics PCK, and belief systems influenced by the modules presented in the BEd programme that makes up the candle wax. The oxygen is the pre-service teacher's own relationships with mathematics influenced by 12 years of schooling in a post-apartheid context, which is not visible. The heat to ignite the flame is the mathematics teacher educator who presents and develops appropriate knowledge, skills, and beliefs in university lecture halls and during practice teaching. In other words, the candle's flame illuminates because of action taken by me, the mathematics teacher educator lecturing mathematics education content to merge mathematics PCK, SCK, and mathematics beliefs. Therefore, it is important that I attend to the pre-service teacher's hidden, yet significant, beliefs to benefit their prospective mathematics teaching.

4.3 Prompt 3

My culture is my way of life. As a South African who has lived and taught as a mathematics teacher and mathematics teacher educator in both apartheid and post-apartheid contexts it is important that I embrace the proverb: "It is better to light a candle than to curse the darkness."

In terms of lighting a candle, I realise that it is possible to take constructive action through positive, hopeful, optimistic, self-improvement endeavours to confront social justice issues using a starting-with-ourselves approach.

However, it is impossible and imprudent to turn a blind eye to social injustices underlying teacher development in mathematics education when taking action to support and encourage appropriate mathematics education belief reflections. Balfour (quoted in Naidu, 2024, para. 26) noted that the consequences of apartheid still impact and influence one's "experience of how you learn, how you teach, and what your chances are of succeeding." In other words, lingering social, political, and educational inequalities related to, for example, "access to resources, historical legacies of advantage, [and] historical legacies of disadvantage" (Balfour, quoted in Naidu, 2024, para. 26) may still haunt many South African learners, pre-service teachers, teachers, teacher educators, and others.

Looking inwards, by analysing my own higher education institution's modules, gave me opportunity to explore, reflect, and look forward to envisaging my professional practice improvements as a primary mathematics teacher educator in a post-apartheid context, but has also made me mindful of looking backwards. I need to take cognisance of pre-service teachers' past schooling experiences, where they may have developed discouraging beliefs about mathematics learning and teaching. Past beliefs are of particular importance for prospective FP schoolteachers because these generalists may not have anticipated having to teach mathematics for approximately one third of their practice.

Furthermore, looking inwards enabled looking outwards. This outward looking facilitated consciousness of possible alternative pre-service teacher experiences vastly different from my privileged opportunities. In addition to opening possibilities of modelling reflective practice (Sweeney et al., 2023) in teacher education, thinking outward after self-reflection made me realise how my mathematics teacher educator practice can make a difference in the teaching and learning of pre-service teacher education. Being a life-long learner and a reflective teacher is an educator requirement but as a mathematics teacher educator, these approaches support my professional practice by understanding my own PCK and beliefs, which are grounded in my post-apartheid way of life.

4.4 Prompt 4

The many primary pre-service teachers enrolled in mathematics education modules are in my metaphor memories. FP pre-service teachers are destined to become generalist teachers with vital roles to play in laying mathematics foundations in primary schools. Some generalist teachers may not be passionate about, or have personal confidence in, mathematics.

Looking back, I remember a tutorial activity that we, a team of UKZN primary mathematics educators, once used. We invited primary pre-service mathematics teachers, enrolled in their first mathematics education module, to share their mathematics school learning and teaching memories. I recall being perturbed and shocked by the openness of some who declared that they "hated" mathematics. As a mathematics education team, we did not anticipate this disadvantageous response and did not address the issue adequately because we were unsure how to support those pre-service teachers in overcoming their strong emotions. However, if prospective teachers harbour such intense unfavourable feelings about mathematics, then it is possible that they could adversely influence their young impressionable learners' relationship with mathematics whilst laying vital, influential mathematics foundations.

4.5 Found poem

To summarise and condense my prompt reflections, and to move towards answering my research question, I created a found poem from pertinent reflection words and phrases (Butler-Kisber, 2010) in a shape that visually resembled a flaming candle (see Figure 3). I chose this shape because of the many everyday symbolisms associated with a flaming candle and because, in my metaphor, the candle flame is the pre-service teacher.

Figure 3: *Summary of pertinent prompt reflections*

fuel, oxygen, heat
Igniting positive inner illumination
better to light a candle than to curse the darkness
heat to ignite flame is mathematics teacher educator
fuel is pre-service teacher's SCK, PCK and belief systems
flame is pre-service teacher that enlightens by effective teaching
oxygen is pre-service teacher's relationships with mathematics
influenced by 12 years schooling in post-apartheid context
may harbor intense unfavorable feelings
beliefs difficult to physically observe
confronting social justice issues
invisible overlooked
constructive action
looking backwards
looking outwards
looking inwards
look forward

5. Answering Research Question, Gazing Forward and Outward

By analysing UKZN's FP enrolment requirements, language focus, and specialisation subject packages I have learnt that UKZN is addressing unfavourable comments (van der Berg & Hofmeyr, 2018) aimed at South African teacher education institutions. The UKZN School of Education enrols pre-service teachers with an APS higher than the minimum UKZN

requirement of 28 points. Moreover, in the FP specialisation the importance of mathematics is recognised because a pass in NSC mathematics or mathematical literacy are prerequisites for enrolment. This indicates that school mathematics SCK achievement is an important consideration for improving FP teacher education. However, school SCK achievement does not necessarily provide information about learners' mathematics beliefs developed during their 12 years of schooling.

In addition, I realised that substantial progress is being made in building and strengthening language and pedagogical skills in isiZulu. UKZN School of Education's efforts in providing teacher education in English and isiZulu at university level is, according to Hlongwa (quoted in Greenleaf Walker, 2021, p. 1) a "linguistic revolution," particularly in the post-apartheid teacher education context.

My module analysis showed that at least three quarters of the UKZN module content is devoted to educational theory-based PCK in the FP specialisation. In the mathematics education modules, the SCK and PCK development appear to be taught separately, which according to Manouchehri (1995), influences the pre-service teachers' beliefs because they may consider these knowledges to be unrelated and disconnected. It is thus necessary to model what is advocated in PCK modules in mathematics SCK modules. For example, emphasise in mathematics SCK assessment that awarding process marks for meaningful use of problem-solving processes deserves acknowledgement.

Furthermore, in FP primary mathematics education modules, no mention or exploration of the central influence of affect is explicitly stated. This omission suggests that the significance of the vital links between cognition, emotions, and beliefs in mathematics may have been overlooked. In other words, disregard for prior beliefs of primary mathematics pre-service teachers is an aspect that is not given sufficient consideration (Olawale, 2024). Perhaps affect is explored in other UKZN teacher education disciplines but beliefs and emotions are noteworthy in relation to mathematics education. Affect is relevant in mathematics teaching and learning because of possible strong, adverse emotional reactions associated with mathematics that have far reaching cognitive influences.

However, there are possibilities within the existing mathematics education modules to allow for inclusion of mathematics teaching and learning beliefs. For example, exploration of pre-service teachers' beliefs could be comfortably situated in the third- year FP module where "knowledge of identifying barriers" (UKZN, n.d., p. 431) is covered. Here the pre-service teachers could explore their beliefs as possible barriers. However, this means that pre-service teachers' beliefs would only be addressed in their last year of mathematics education. It would be beneficial to include these beliefs in an earlier mathematics education module so that they can benefit from their learning over an extended period to develop refreshed, new, additional, or alternate beliefs. Other possibilities for inclusion of personal mathematics teaching and learning beliefs could be

effectively addressed during pre-service teachers' annual practice sessions when they teach mathematics in schools.

Whilst exploring personal mathematics teaching and learning beliefs, pre-service teachers would benefit from using metaphor drawings to assist in their belief reflection and development in primary mathematics education. This arts-inspired method of metaphor drawing is well documented in mathematics education teaching and learning (see, for example, Hobden, 1999; van Laren, 2007). By making use of personal hand-drawn metaphors to explore their own beliefs, where there are no correct or incorrect responses, primary pre-service teachers can be offered opportunities to incorporate additional, novel, creative self-reflexive self-study methods. Reflection possibilities for rethinking, reinventing, or revising personal beliefs about teaching and learning foundational mathematics provides agency and insight into this key yet overlooked component of mathematics education.

Mathematics teacher educators may also enjoy and benefit from self-reflexive methods, and model what is required of pre-service teachers who need to explore their mathematics teaching and learning beliefs. Discussions and dialogue amongst mathematics teacher educators and pre-service teachers could benefit this central influence on the learning of mathematics SCK and PCK. As Olawale (2024, para. 6) noted, "limited effectiveness of training [pre-service teachers] might be attributed" to the beliefs about mathematics teaching and learning that are inculcated during their 12 years of foundational school mathematics, post-apartheid experiences.

5.1 What is the Relevance of my Self-Study?

As a mathematics teacher educator, I acknowledge my prior privileged learning influences but, as a self-study researcher, I am committed to positively lighting the candle. My use of a metaphor to reflect on the central importance of pre-service teachers' personal mathematics teaching and learning beliefs (candlewick) facilitated my professional learning as a teacher educator (igniting match) by highlighting my role in preparing pre-service teachers (flame) for effective mathematics teaching (illuminating flame). This study made me more aware of the daunting responsibility of being a primary mathematics teacher educator in the post-apartheid context, how influential my professional beliefs (matchbox) are, and how these beliefs could inadvertently be conveyed to pre-service mathematics teachers. The challenging task of remedying the overlooking of mathematics beliefs provided an opportunity to do something about the social injustices many pre-service teachers may have endured during their mathematics schooling.

I used a self-study approach to learn more about what FP pre-service teachers are offered in mathematics education at my higher education institution because these teachers will lay the foundations for mathematics learning in their prospective generalist teaching. My inward-looking professional learning facilitated the envisioning of positive, constructive, workable improvement possibilities for attending to the influential, under-explored, mathematics teaching

and learning beliefs of pre-service primary mathematics teachers in our post-apartheid context. I offer and make public my findings for scrutiny by the mathematics teacher education community, and hope to pave the way for other starting-with-ourselves reflective studies that envisage sustainable self-improvement possibilities in primary mathematics teacher education.

6. Conclusion

My study explored what is offered to generalist FP pre-service teachers in mathematics education modules at the higher education institution where I am a mathematics teacher educator. Module content presented in UKZ-N's *Handbook for 2024*, suggested that pre-service teachers' belief awareness in mathematics teaching and learning was not covered. Given that they are prospective teachers in post-apartheid South Africa, overlooking pre-service teachers' beliefs ignores how they might perceive the nature/vision of mathematics, how they experienced learning mathematics in schools, and what their approach to teaching mathematics may be. Furthermore, this means that the important links between cognition and beliefs in mathematics and mathematics education were underestimated.

Using a qualitative, reflexive, constructive, optimistic starting-with-ourselves approach, I sought possibilities to improve what is offered to FP pre-service teachers because they will influence young learners' beliefs, which are significant for their subsequent mathematics learning. The study thus offers self-improvement possibilities for the integration and incorporation of belief awareness in existing modules offered to FP mathematics pre-service teachers at UKZN. It concludes that self-reflexive re-examination and re-evaluation of what is offered in one's own mathematics education programmes allows for constructive rethinking and self-improvement possibilities to make a difference in mathematics pre-service teacher education.

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