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Exploring Preservice Teachers' Development of Teaching Proficiency in Grade 10 Parabola Functions Through GeoGebra

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Abstract: This paper investigates the development of teaching proficiency among preservice teachers in Grade 10 parabola functions through the use of GeoGebra. A significant number of preservice teachers encounter challenges in effectively teaching parabola functions due to limited resources for illustrating complex mathematical relationships, which can impede both their teaching proficiency and the students' conceptual understanding. The study was conducted at a rural-based university and involved four third-year Bachelor of Education (FET & SP) preservice teachers (PSTs) who were purposively selected. These preservice teachers specialised in mathematics and instructed Grade 10 parabola functions during both microteaching sessions and their teaching practice in rural schools. Prior to their teaching experiences, the preservice teachers received training on the integration of GeoGebra into their lesson plans. Employing an interpretivist paradigm and a case study research design, the study utilised the Technological Pedagogical Content Knowledge (TPACK) framework along with Kilpatrick et al.'s five strands of mathematical proficiency as analytical perspectives. Data were gathered through video-recorded lesson observations and stimulus recall interviews. The findings indicate that GeoGebra significantly enhanced the preservice teachers' proficiency by supporting critical areas of mathematical understanding: conceptual understanding, fluency, strategic competence, and adaptive reasoning. The dynamic visualisations offered by GeoGebra enabled preservice teachers to engage learners more deeply, facilitating a better understanding of parabola functions. PSTs reported improved confidence in their teaching, particularly in using real-time graph manipulation to explain complex concepts.

Keywords: GeoGebra, preservice teachers, teaching proficiency, parabola functions.

1. Introduction

Research on the impact of GeoGebra on preservice teachers' proficiency in teaching Grade 10 parabola functions, particularly in the rural schools of the Eastern Cape province, reveals several notable gaps in the existing literature. These include (1) a lack of focus on preservice teachers as opposed to in-service teachers or learners; (2) limited attention to the specific algebraic topic of parabola functions, with most research centred on geometry; and (3) a scarcity of studies contextualised within rural education settings characterised by resource constraints and limited technological infrastructure (Manganyana et al., 2020; Mokotjo, 2023; Sousa et al., 2022). This context is particularly important because rural schools in the Eastern Cape often face systemic challenges such as poor infrastructure, lack of technological resources, and underqualified educators, which can hinder both teaching effectiveness and learner outcomes (DBE, 2021). Addressing preservice teacher development in such contexts is therefore essential for bridging educational inequalities and promoting equitable access to quality mathematics education. While there is a growing body of literature addressing the use of GeoGebra in various educational contexts (Bansilal, 2015; Kekana, 2016; Ndlovu, 2014; Ndlovu et al., 2020), specific studies focusing on preservice teachers in rural

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settings remain scarce. This lack of targeted research is significant, given the unique challenges faced by rural educators, such as limited access to resources and training opportunities (Khalo, 2020).

One relevant study by Manganyana et al. (2020) emphasises the positive effects of GeoGebra in disadvantaged rural classrooms, indicating that it can effectively enhance learning in geometry. However, this research does not specifically address the teaching of parabola functions or the proficiency of preservice teachers, leaving a gap in understanding how GeoGebra can be utilised to improve their instructional strategies in this particular area. Furthermore, while the study highlights the effectiveness of GeoGebra in geometry, it does not explore the broader implications for algebraic concepts, such as parabolas, which are critical in the Grade 10 curriculum.

Additionally, the research conducted by Villaroza (2023) on the impact of GeoGebra on mathematical proficiency and inductive reasoning in geometry students does not specifically focus on preservice teachers or the rural context of the Eastern Cape (Villaroza, 2023). This indicates a need for studies that specifically investigate how preservice teachers in rural areas perceive and utilise GeoGebra in teaching parabola functions, as their experiences and challenges may differ significantly from those in urban settings.

Moreover, the existing literature often emphasises the technological pedagogical content knowledge (TPACK) framework, which is crucial for understanding how preservice teachers integrate technology into their teaching practices. However, studies like that of Morales-López et al. (2021) highlight that while preservice teachers may have some proficiency with GeoGebra, this does not necessarily translate into effective integration within their teaching practices (Morales-López et al., 2021). This highlights an additional gap regarding the alignment between preservice teachers' technological skills and their actual classroom implementation strategies, particularly in under-resourced contexts.

In light of the above insights, the research gap in the impact of GeoGebra on preservice teachers' proficiency in teaching Grade 10 parabola functions in the rural Eastern Cape region is evident. Hence, this study seeks to explore how the use of GeoGebra enhances the development of preservice teachers' proficiency in teaching Grade 10 parabola functions and to gain preservice teachers' perspectives on how using GeoGebra has enhanced their content knowledge and teaching proficiency.

1.1 Literature review

Several studies have highlighted the importance of preparing teachers with appropriate technologies for mathematical teaching and learning (Žilinskiene & Demirbilek, 2015; Bowers & Stephens, 2011; Aiym et al., 2022; Atteh et al., 2023; Acharya, 2023; Arida et al., 2022). Zambak (2015) asserted that the use of electronic technologies is one potential approach to enhance teachers' development of mathematical content knowledge and conceptual understanding, thereby facilitating teaching proficiency. This assertion is supported by Sedega et al. (2018), who affirm that the effective use of ICTs in mathematics classrooms improves learners' academic performance and provides teachers with innovative teaching methods. However, learners do not necessarily experience improvements in performance solely due to the availability of technology in the classroom. As stated by Jackson (2017, p. 90), "it is the teacher's decisions on how to integrate ICT into the mathematics classroom that will either improve or hinder student outcomes." Thus, there is a need for professional development opportunities that support teachers in designing technologically based lessons effectively. Consequently, this study emphasises the necessity for preservice teachers to receive adequate training on how to effectively utilise and develop GeoGebra applets for teaching and learning. GeoGebra software is well-known for its visual representations of mathematical concepts, including algebraic functions, calculus, and geometry (Hohenwarter et al., 2008). It is regarded as an extremely supportive technological tool that elucidates mathematical concepts and procedures through visuals, graphics, images, and symbols (Zulnaidi & Zamri, 2017). GeoGebra enhances the teaching of parabola functions by enabling dynamic visualisation and real-time manipulation of quadratic equations, thereby assisting learners in understanding how changes in coefficients affect the vertex, axis of symmetry, and direction of the parabola (Sun, 2023; Hidayat et al., 2024). The integration of GeoGebra into mathematics education has been shown to significantly enhance preservice teachers' proficiency in teaching mathematics (Za'ba, 2020). Dedicated GeoGebra applets also promote independent exploration and inquiry-based learning, which enables preservice teachers to design lessons that encourage active student engagement and deeper conceptual understanding (Sudarsana et al., 2022).

Teaching proficiency refers to teachers' ability to employ multiple effective teaching methods and to work with a diverse range of students across various settings and mathematical content (Kilpatrick et al., 2001).Kilpatrick et al. (2001, p. 369) state that "Teachers must have a clear vision of the goals of instruction and what proficiency means for the specific mathematical content they are teaching." In this context, GeoGebra serves as a tool that supports preservice teachers in achieving instructional clarity and promoting visual and analytical reasoning aligned with lesson objectives (Sun, 2023; Hidayat et al., 2024). Thus, for this study, preservice teachers were required to have a fundamental understanding of the intended outcomes of their lessons and the nature of proficiency in teaching parabola functions. Therefore, the five interrelated components proposed by Kilpatrick et al. (2001) served as a comprehensive framework for preservice teachers' understanding of the knowledge and skills required for effective mathematics teaching. The following are brief descriptions of each strand of teaching for mathematical proficiency (Kilpatrick et al., 2001, p. 380):

- Conceptual understanding (CU) of the core knowledge required in the practice of teaching.
- Fluency (F) in carrying out basic instructional routines.
- Strategic competence (SC) in planning effective instruction and solving problems that arise during instruction.
- Adaptive reasoning (AR) in justifying and explaining one's instructional practices and in reflecting on those practices so as to improve them.
- Productive disposition (PD) toward mathematics, teaching, learning and the improvement of practice.

Öçal (2017) asserts that GeoGebra facilitates a deeper conceptual understanding of mathematical concepts among preservice teachers. Additionally, Öçal (2017) highlights that GeoGebra's integration of Computer Algebra System (CAS) and Dynamic Geometry Environment (DGE) positively influences students' conceptual knowledge, suggesting that preservice teachers who utilise GeoGebra are better equipped to convey complex mathematical ideas to their future students (Öçal, 2017). Furthermore, studies conducted by Mensah (2023) and Hedi (2023) have demonstrated that preservice teachers employing GeoGebra exhibit enhanced performance in comprehending topics such as circle theorems (Grade 11) and derivatives (Grade 12), which are essential components of the South African secondary school mathematics curriculum. Moreover, Mensah (2023) contends that the interactive nature of GeoGebra facilitates dynamic exploration of mathematical concepts, which can lead to increased interest and active participation in learning activities. Mensah's (2023) study emphasises that integrating mathematics instruction with ICT tools like GeoGebra creates meaningful learning experiences, rendering abstract concepts more accessible and engaging for preservice teachers. This finding is consistent with Rajagopal et al.'s (2015) study, which suggests that GeoGebra's interactive features promote discovery learning and foster a positive attitude towards mathematics. In addition, Muslim (2023) argues that employing GeoGebra enhances understanding and engagement, while also aiding preservice teachers in developing effective teaching strategies. The GeoGebra software encourages diversification of teaching methods, as it can be integrated into various pedagogical approaches such as problem-based learning and flipped classrooms (Muslim, 2023). This adaptability is critical for preservice teachers, as it prepares them to implement a range of instructional strategies in their future classrooms. Research indicates that preservice teachers who engage with GeoGebra report increased confidence in their ability to teach mathematics effectively, as they become more familiar with integrating technology into their teaching practices (Marange & Tatira, 2023).

2. Theoretical Framework

The study integrates the five strands of mathematics teaching proficiency as delineated by Kilpatrick et al. (2001) with the Technological Pedagogical Content Knowledge (TPACK) conceptual framework to advance the objectives of this research and contribute to the body of knowledge. The amalgamation of the TPACK framework with Kilpatrick et al.’s (2001) five strands of mathematical proficiency offers a robust lens for investigating the impact of GeoGebra on preservice teachers' proficiency in instructing Grade 10 parabola functions. This integration facilitates a comprehensive analysis of the interactions among technology, pedagogy, and content knowledge, thereby enhancing teaching effectiveness and improving learners’ learning outcomes. The TPACK framework underscores the interplay between three primary forms of knowledge: content knowledge (CK), pedagogical knowledge (PK), and technological knowledge (TK) (Koehler & Mishra, 2006). In the context of teaching parabola functions, preservice teachers ought to possess a deep understanding of the mathematical concepts related to parabolas (CK), effective teaching strategies to convey these concepts (PK), and the ability to integrate GeoGebra as a technological tool to facilitate learning (TK) (Morales-López et al., 2021). This framework supports the notion that the successful integration of technology in teaching depends on a teacher's ability to blend these knowledge domains effectively (Kanandjebo & Lampen, 2022).

On the other hand, Kilpatrick et al.'s (2001) five strands of mathematical proficiency—conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition—provide a detailed framework for evaluating the quality of mathematical teaching and learning (Alzubi, 2021). Each strand reflects a critical aspect of mathematical proficiency that preservice teachers should develop. For instance, using GeoGebra can enhance conceptual understanding by allowing students to visualise and manipulate parabolas dynamically, thereby deepening their comprehension of the properties and behaviours of quadratic functions (Iqbal, 2023). Additionally, GeoGebra can support procedural fluency by providing interactive tools for practising calculations and transformations related to parabolas. Thus, merging TPACK with Kilpatrick et al.'s strands enabled this study to investigate how preservice teachers' proficiency in teaching parabola functions is influenced by their ability to integrate technology effectively. For example, Ngware et al. (2015) and Stephanus (2018) have shown that preservice teachers who are well-versed in TPACK are more likely to employ teaching strategies that align with the five strands of mathematical proficiency, leading to improved learner outcomes. This interaction can be particularly beneficial in fostering strategic competence and adaptive reasoning, as preservice teachers learn to adapt their teaching methods based on student interactions with GeoGebra and their understanding of parabolas (Ngware et al., 2015).

3. Methodology

This qualitative study employed an interpretive paradigm and a case study research design to explore how the use of GeoGebra enhances preservice teachers' proficiency in teaching Grade 10 parabola functions. The interpretive paradigm, rooted in the philosophical works of Wilhelm Dilthey and Max Weber, emphasises understanding participants’ lived experiences and subjective meanings. It allowed the researcher to engage closely with participants, capturing their personal experiences and perspectives (Lincoln & Guba, 1985; Kawulich & Chilisa, 2012). The case study design was chosen to investigate a specific cohort of four third-year mathematics preservice teachers who received GeoGebra training and subsequently applied it in their teaching practice.

The study employed a non-probability purposive sampling method to facilitate the collection of rich and valid data. This sampling approach involves the deliberate selection of participants based on specific characteristics and a clear understanding of the population parameters (Gentles et al., 2015). Initially, all third-year Bachelor of Education (Further Education and Training & Senior Phase) mathematics majors—designated in this study as pre-service teachers (PSTs)—who were being instructed by the researchers were invited to participate in a GeoGebra training programme. From the approximately 25 students who attended, four participants were purposefully selected according to specific criteria. These criteria included consistent attendance at the GeoGebra training sessions, a demonstrated willingness and interest in utilising GeoGebra software for instructional purposes, placement at schools equipped with either a whiteboard or a computer laboratory for teaching practice, and strong academic performance in mathematics during their second year, along with evident commitment throughout the training period.

The data were collected in two phases: the first during the teaching practice and the second through stimulus-recall interviews conducted after the teaching practice. The data collection instruments included video recordings of the preservice teachers' GeoGebra-based lessons and semi-structured stimulus-recall interviews. Lesson observations focused on how GeoGebra applets were used to make mathematical content visual and dynamic. The stimulus-recall interviews enabled the preservice teachers to reflect on their teaching experiences and the influence of GeoGebra on their content knowledge and teaching proficiency.

3.1 Ethical considerations

All ethical requirements for conducting the study were meticulously adhered to in accordance with institutional research guidelines. Prior to participation, the preservice teachers were comprehensively informed about the nature, purpose, and procedures of the research. They were assured of the confidentiality of their responses, their right to withdraw from the study at any stage without penalty, and that their participation was entirely voluntary. Written informed consent was obtained from all participants. The study received ethical clearance from the Faculty of Education's Ethics Committee under clearance number FEDSECC026-06-23.

4. Findings and Analysis

Data were analysed using thematic analysis, following the six-phase procedure outlined by Braun and Clarke (2006). Thematic categories were deductively derived from an analytical framework that integrates two domains of the TPACK framework: Technological Content Knowledge (TCK) and Technological Pedagogical Knowledge (TPK), along with the five strands of mathematical proficiency proposed by Kilpatrick et al. (2001): conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. These frameworks served both as theoretical lenses and as a priori coding structures that guided the identification and interpretation of patterns across the data. A coding matrix (see Table 1) was developed to systematically analyse evidence from lesson observation videos and stimulus recall interviews, ensuring alignment between theoretical constructs and observed teaching practices. This hybrid approach enabled consistent categorisation of data and supported the emergence of meaningful findings. The first section of this report presents the findings organised according to the intersections of TCK and TPK with each strand of proficiency. The second section draws on interview data and is presented under two themes, enhanced content knowledge and improved teaching proficiency, to capture the preservice teachers' perspectives on how using GeoGebra influenced their professional growth. Table 1 presents the specific indicators used to code the data, reflecting how preservice teachers integrated GeoGebra into their lessons and demonstrated proficiency in teaching Grade 10 parabola functions.

Table 1: *Coding indicators for GeoGebra integration in parabola lessons*

Definition of TPACK Domains	
Technological Content Knowledge (TCK): Interactions between GeoGebra applets and parabola functions.	**Technological Pedagogical Knowledge (TPK)**: Interactions between GeoGebra applets and the teaching process.
1. Conceptual Understanding (CU): The understanding of core knowledge that influences teaching practice and student learning.	
TCK and CU: Evidence of using GeoGebra to: • Select appropriate applets to help learners develop concepts related to parabola functions. • Demonstrate dynamic visualisations of parabola functions using GeoGebra. • Create meaningful sliders to manipulate parabola functions effectively.	**TPK and CU**: Evidence of using GeoGebra to: • Visualise and explain mathematical concepts dynamically. • Demonstrate the effects of different parameters in algebraic functions through the manipulation of sliders.
2. Fluency (F): Fluency in performing instructional routines and classroom management, including addressing learners' barriers to learning.	
TCK and F: Evidence of using GeoGebra to: • Create applets suited for learners at various cognitive levels, allowing different methods of engagement (individual, pairs, groups). • Align GeoGebra applets with lesson objectives.	**TPK and F**: Evidence of using GeoGebra to: • Scaffold learners' conceptual development through sliders, enabling them to solve problems independently. • Ensure applets enhance learner understanding of key lesson objectives.
3. Strategic Competence (SC): The ability to solve teaching-related problems and effectively plan and implement instruction.	
TCK and SC: Evidence of using GeoGebra to: • Decide on content and implement applets that help learners solve problems related to parabola functions. • Offer problem-solving opportunities through worksheets or digital resources after engaging with GeoGebra	**TPK and SC**: Evidence of using GeoGebra to: • Facilitate lessons where dynamic visualisations are key to understanding, surpassing traditional teaching methods
4. Adaptive Reasoning (AR): The ability to reflect on teaching practices and student challenges	
TCK and AR: Evidence of using GeoGebra to:	**TPK and AR**: Evidence of using GeoGebra to:

• Reflect on the effectiveness of each GeoGebra applet and adjust strategies to address student challenges.	• Reflect on the relationship between the lesson content and the applets used to teach that content.
5. Productive Disposition (PD): A teacher's passion for teaching and engagement with student feedback to improve their teaching.	
TCK and PD: Evidence of using GeoGebra to: • Encourage learners to provide feedback on their learning experience with GeoGebra. • Value student input in shaping future lessons.	**TPK and PD**: Evidence of using GeoGebra to: • Facilitate learner reflection through assessments or discussions and adapt teaching practices based on their performance. • Encourage learners to share their mathematical thinking and reasoning.

4.1 Section 1: Intersections of TCK and TPK with each strand of proficiency

This section examines how preservice teachers used their Technological Content Knowledge (TCK) and Technological Pedagogical Knowledge (TPK) to support learners across key strands of mathematical proficiency. Through the integration of GeoGebra, the teachers enhanced conceptual understanding, fluency, strategic competence, adaptive reasoning, and productive disposition in their lessons.

4.1.1 TCK and conceptual understanding

Preservice teachers demonstrated TCK by effectively integrating GeoGebra to support learners' conceptual understanding of parabola functions. A consistent pattern across all four PSTs was their use of dynamic visualisations to illustrate how variations in the parameters a and q influenced the shape and position of the parabola. These visual representations helped bridge the gap between abstract algebraic expressions and their corresponding graphical interpretations. PST1, for example, began his lesson by sketching a parabola on the board before transitioning to GeoGebra. He created sliders for parameters a and q, allowing learners to observe how increasing the value of a made the parabola narrower and how negative values inverted its concavity. He explained, "Let's see what happens when we now increase the value of a… make a = 2 instead of a = 1." Learners responded actively, noting how the graph changed in real time, which indicated a solid grasp of the concept.

Figure I below show PST1 performing visual demonstrations of how changes in the parameter result in changes in either the narrowness or width of the graph.

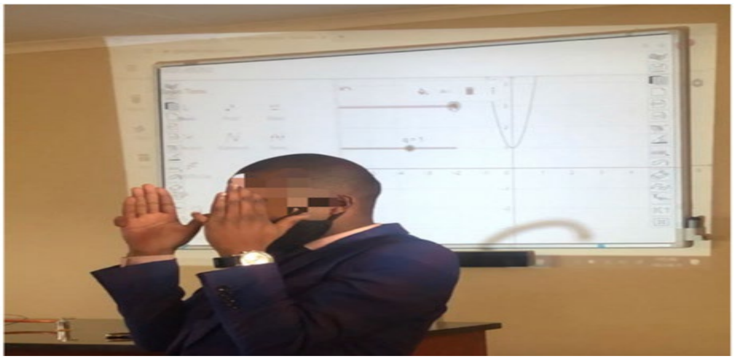

Figure I: *PST1 showing the effect of the parameter* a.

Similarly, PST3 used GeoGebra to compare a hand-drawn parabola with a dynamically generated graph of $f(x) = x^2$, showing learners how the value of $q = 0$ placed the turning point at the origin. PST2 reinforced the conceptual link between algebra and geometry by prompting learners to explain why the graph "disappears" when $a = 0$, clarifying that the equation $f(x) = q$ produces a horizontal line.

PST4 took this further by using two parabola functions, one fixed and one dynamic, to demonstrate transformations. Her learners were able to articulate how the graph shifted based on parameter values. One learner remarked, "The graph shifted up because q is bigger in this one," demonstrating engagement with the mathematical reasoning behind the visual changes.

These examples illustrate how the preservice teachers' TCK enabled them to make strategic use of GeoGebra's content-specific features to deepen conceptual understanding. This aligns with Kilpatrick et al.'s (2001) strand of Conceptual Understanding, which refers to students' ability to grasp mathematical ideas and relationships. By using GeoGebra to connect algebraic and graphical representations of the parabola, PSTs not only clarified content but also enhanced learners' long-term retention of mathematical structure.

4.1.2 TPK and conceptual understanding

Preservice teachers also demonstrated strong TPK by using GeoGebra not just to convey mathematical content, but to design pedagogical experiences that made abstract concepts more accessible. Their teaching practices reflected an awareness of how technology could mediate learning by supporting visualisation, exploration, and learner participation.

A key pedagogical strategy employed was real-time manipulation of sliders during classroom discussions. PST2 used this technique to help learners distinguish between positive and negative values of parameter a, stating, "If a is less than zero, the graph will concave down… if a is greater than zero, it will concave up," while slowly adjusting the slider. Learners visibly reacted with nods and verbal affirmations, indicating that they could connect the symbolic explanation with the visual changes. PST3 extended this by asking learners to predict how the graph of $f(x) = x^2$ would behave, and then confirming their responses using GeoGebra. Her method of alternating between student conjecture and visual verification encouraged active conceptual engagement.

Furthermore, PST4's pedagogical approach included comparative visualisations. She placed two functions on the same axes, one static and one dynamic, guiding learners to observe and discuss the differences in concavity and vertical shifts. This enabled learners to articulate relationships such as "This graph is higher because q is larger," showing an emerging ability to generalise mathematical patterns from dynamic representations. By selecting visual tools appropriate to the learning goal, sequencing tasks to build understanding, and prompting learners to articulate mathematical reasoning, PSTs used GeoGebra to scaffold conceptual development. These practices reflect the pedagogical use of technology to promote deep learning and align with the TPK domain of the TPACK framework. They also resonate with Kilpatrick et al.'s (2001) emphasis on Conceptual Understanding as foundational to mathematical proficiency.

4.1.3 TCK and fluency

Preservice teachers demonstrated TCK not only by selecting content-relevant applets but also by sequencing their use of GeoGebra to support learners' fluency in engaging with parabola functions. Fluency, as defined by Kilpatrick et al. (2001), involves carrying out procedures flexibly, accurately, and efficiently. PSTs used GeoGebra to streamline the delivery of content and reinforce an accurate, efficient understanding of graph transformations.

PST1 exemplified this fluency by structuring his lesson to begin with manual sketches before transitioning to GeoGebra. This allowed learners to first develop a foundational understanding, which was then enhanced by dynamic visualisations. He used GeoGebra to reinforce his explanation of turning points and axis symmetry, showing learners how changes in a and q affected the width and position of the graph. Integrating both modes of delivery, chalkboard followed by GeoGebra's software, supported learners at different levels of readiness, ensuring procedural coherence and fluency. Furthermore, PST4 also demonstrated TCK by using increasingly complex applets that gradually introduced more detailed representations of parabola transformations. Her lesson began with a basic parabola and built up to a comparison of multiple graphs on the same axes. This progression allowed learners to make connections across function types while retaining focus on key parameters. Learners could engage in structured exploration without being overwhelmed, thus supporting fluent interaction with new mathematical ideas.

The accuracy and responsiveness of GeoGebra significantly contributed to mathematical fluency by minimising errors commonly associated with hand-drawn graphs. Learners were afforded the opportunity to test, revise, and validate their understanding in real-time, a process facilitated by the preservice teachers' (PSTs) judicious use of technology and timing. Overall, the fluency of preservice teachers in content delivery was enhanced by their capability to select and adapt GeoGebra tools for optimal instructional pacing, conceptual precision, and visual clarity. These practices exemplify a high level of Technological Content Knowledge (TCK) integration, empowering learners to execute and interpret mathematical procedures with both confidence and clarity.

4.1.4 TPK and Fluency

Preservice teachers demonstrated TPK by using GeoGebra to structure lessons that supported learners' procedural fluency and engagement. They scaffolded instruction with the software to guide learners from teacher-led demonstrations to learner-driven exploration. Their pedagogical fluency was evident in how they integrated GeoGebra into the pacing and flow of their lessons.

PST2's lesson offered a clear example of this. She began by modelling how to manipulate the a and q sliders to show the effects on the parabola graph. After explaining and demonstrating each transformation, she then gave learners time to complete an activity involving $f(x) = -2x^2 + 8$. During the activity, she moved around the classroom, prompting learners to identify and interpret key features such as the vertex and intercepts. After the discussion, she returned to GeoGebra to show the correct graph and reinforce learners' visual intuition. In the following Figure II below, PST2 is displaying the correct graph of $f(x) = -2x^2 + 8$.

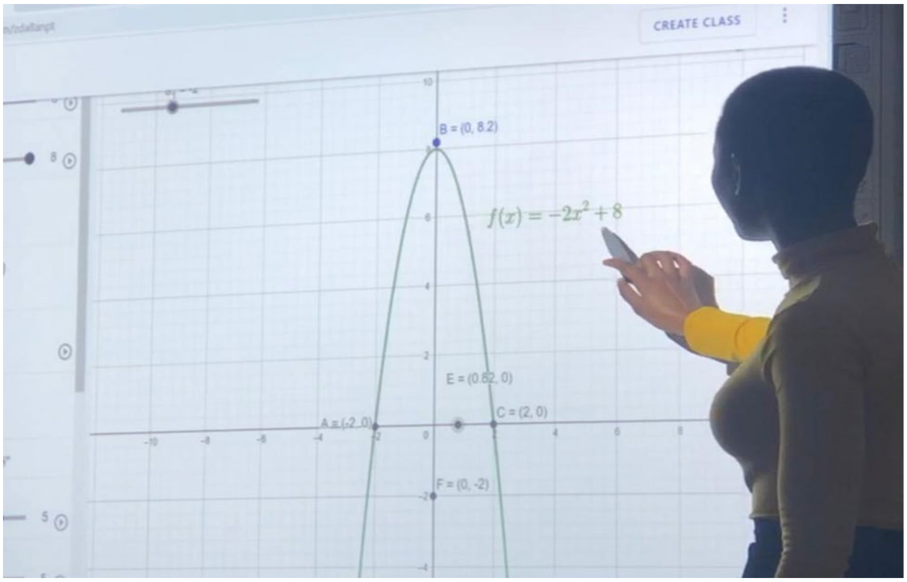

Figure II: PST2's GeoGebra screenshot showing solutions of the activity

Similarly, PST3 structured her lesson to shift responsibility from teacher to learner. After guiding learners through a substitution table for $y = x^2$, she transitioned to GeoGebra and asked learners to explain what happened as she manipulated the sliders. This approach allowed learners to first engage with the underlying logic of the function and then verify their understanding visually, reinforcing both procedural fluency and confidence.

Fluency was also fostered through GeoGebra's capacity for repetition and immediate feedback. PSTs allowed learners to explore "what-if" scenarios, such as changing the sign or magnitude of a, enabling them to predict, test, and revise their thinking, qualities of procedural mastery. Through these strategies, PSTs used GeoGebra not simply as a demonstration tool but as a scaffold for active learning. This aligns with Kilpatrick et al.'s (2001) definition of **Fluency**, as well as the **TPK** domain of the TPACK framework, where pedagogy and technology converge to support students in performing procedures accurately and meaningfully.

4.1.5 TCK and strategic competence

Preservice teachers demonstrated TCK by strategically selecting and using GeoGebra applets to support learners in solving problems and reasoning about mathematical structures. **Strategic competence**, as defined by Kilpatrick et al. (2001), refers to the ability to formulate, represent, and solve mathematical problems, a process the PSTs facilitated through technology-driven instruction. PST3 and PST4 both used GeoGebra not just for visual demonstration but to structure activities that required learners to apply their understanding of parabola functions. For instance, PST3 asked learners to describe and compare three different quadratic graphs, $p(x) = x^2$, $h(x) = -x^2$, and a dynamic $f(x) = ax^2 + q$, and then interpret the differences based on the parameters. Figure III below shows parabola functions that PST3 used to verify the graphs that were first drawn freehand.

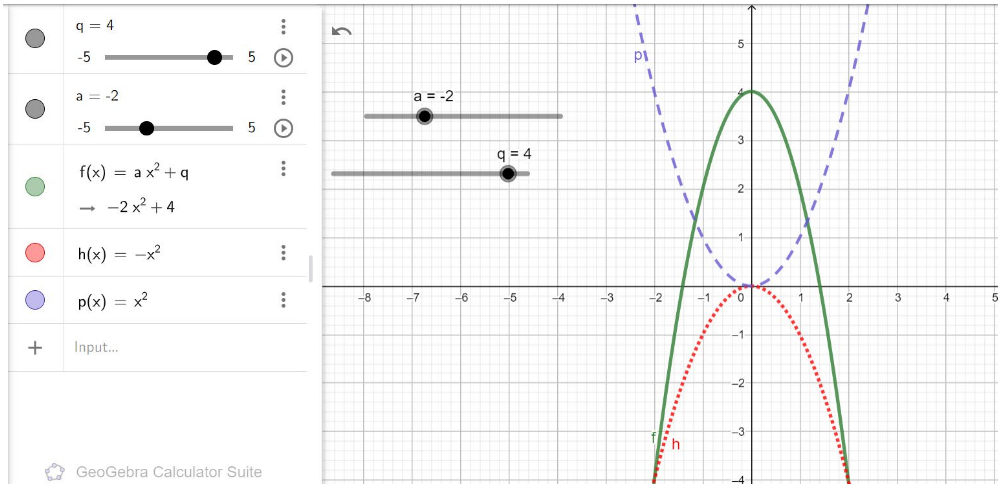

Figure III: *PST3's GeoGebra applet showing three parabola functions*

This prompted learners to formulate relationships between equations and graphs, developing their reasoning through strategic analysis.

Furthermore, PST1 enhanced strategic competence by giving learners the task of plotting $f(x) = x^2 - 9$ and $f(x) = -x^2 + 9$ on the same set of axes. After learners completed the activity manually, he used GeoGebra to verify their responses and facilitated a discussion on how parameter changes influenced graph behaviour. This structure allowed learners to hypothesise, test, and revise the central features of strategic mathematical thinking. Moreover, PST4's lesson included problem-solving opportunities that required learners to identify key graph features such as turning points and points of intersection, and to reason about the implications of missing x-intercepts. By constructing these scenarios using GeoGebra, she offered learners a content-rich environment for deepening their strategic understanding. These examples reflect how PSTs used their content knowledge of quadratic functions in tandem with GeoGebra's visual capabilities to promote strategic competence. Their ability to pose purposeful problems and guide learners in visual verification illustrates the effective enactment of TCK in supporting learners' mathematical reasoning.

4.1.6 TPK and strategic competence

Preservice teachers demonstrated TPK by designing lessons that used GeoGebra not only for illustration but also as a tool for engaging learners in mathematical problem-solving. Their pedagogy supported **strategic competence** by encouraging learners to make conjectures, justify their reasoning, and revise their thinking based on visual feedback. PST2's approach exemplified this integration. After demonstrating how parameters a and q affect the parabola, she posed questions that challenged learners to interpret transformations and construct equations from given graphs. For example, she asked learners to determine the equation of a parabola shown on a GeoGebra applet with labelled points. Rather than telling them the answer, she prompted them to explore and explain how the values of a and q could be deduced from the graph's features. This encouraged learners to strategically apply their knowledge in a problem-solving context.

PST3 used a similar strategy by presenting learners with a dynamic function $f(x) = ax^2 + q$ alongside fixed functions $p(x) = x^2$ and $h(x) = -x^2$, then inviting them to compare and contrast the graphs. Learners were asked to describe the transformations and infer parameter values based on visual differences. This dialogic use of GeoGebra facilitated exploration and mathematical reasoning, moving beyond rote learning to strategic interpretation. The use of GeoGebra enabled pre-service teachers (PSTs) to pose open-ended and visually rich problems, supporting learners' capacity to formulate solutions and reason mathematically. Such practices align with Kilpatrick et al.'s (2001) notion of Strategic Competence and reflect a pedagogical use of technology in which learners are active participants in the meaning-making process.

4.1.7 TCK and adaptive reasoning

Preservice teachers demonstrated TCK by adapting their instructional use of GeoGebra based on learners' conceptual challenges, thereby supporting adaptive reasoning—the ability to reflect, justify, and revise one's mathematical thinking (Kilpatrick et al., 2001). This was particularly evident when PSTs recognised gaps in learners' understanding and adjusted their content delivery accordingly, utilising GeoGebra's dynamic features.

PST1 exhibited strong adaptive reasoning when he observed that learners were struggling with the concavity of the graph. In response, he paused his GeoGebra demonstration, reverted to a hand-drawn explanation on the board, and then returned to the software for visual verification. This sequence helped consolidate understanding through both visual and symbolic representations, demonstrating his ability to modify content delivery to meet learners' needs.

Similarly, PST4 addressed confusion around parameter shifts by introducing an additional applet that compared two parabolic functions on the same axes, one static and one dynamic. This enabled learners to see the direct impact of changing a and q values, reinforcing the concept through side-by-side comparison. Her decision to adapt the lesson content in real time demonstrated an awareness of how to reframe mathematical ideas using technology to clarify misconceptions. These examples illustrate how preservice teachers used GeoGebra not only to present content but also to re-present it in ways that addressed emerging learner difficulties. Their ability to rethink and restructure their technological content choices reflects both adaptive reasoning and a deepening of TCK.

4.1.8 TPK and adaptive reasoning

Preservice teachers demonstrated **TPK** through their ability to adapt instructional strategies during lessons in response to learner feedback and confusion, using GeoGebra to mediate clarification. This flexibility reflects **adaptive reasoning**, which involves not only understanding mathematical relationships but also revising instruction to support learners' evolving needs (Kilpatrick et al., 2001). One striking example of pedagogical adaptability occurred when PST2 was asked by a learner why the parabola "disappears" when $a = 0$. Instead of simply stating the answer, she used GeoGebra to show how the quadratic function transforms into a horizontal line ($y = q$) when $a = 0$, making an abstract explanation tangible. Her calm and interactive explanation helped dispel the learners' confusion and reinforced their understanding through visual reasoning. PST3 also demonstrated pedagogical flexibility during a lesson on $y = x^2$. When learners were unsure how to describe the graph's shape based on the equation, she introduced a substitution table activity, enabling them to calculate and plot values manually. After this, she returned to GeoGebra to confirm the correct graph and invited learners to explain how their hand-drawn graph compared to the digital version. This shift in approach, from digital to manual and back, allowed learners to reconstruct their understanding and verify their reasoning visually. By using GeoGebra not just for delivery but for **real-time adjustment**, PSTs positioned themselves as responsive facilitators of learning. Their use of technology was not fixed but fluid, allowing for on-the-spot pedagogical shifts that supported individual and collective learner understanding. This aligns with the **TPK** dimension of the TPACK framework and the **adaptive reasoning** strand of mathematical proficiency.

4.1.9 TCK and productive disposition

Preservice teachers demonstrated TCK by using GeoGebra not only to teach mathematical content effectively but also to foster learners' confidence, interest, and positive attitudes toward mathematics, which are core elements of a productive disposition (Kilpatrick et al., 2001). Through intentional selection and use of content-specific digital tools, PSTs created opportunities for learners to engage with mathematics in ways that felt accessible and rewarding.

PST1 recounted how, after his GeoGebra-based lesson, learners shared that they had previously encountered parabolas but had never truly understood them until they saw the dynamic visualisations. One learner told him, "We so wish you had come earlier to teach us with GeoGebra." This feedback revealed a shift in learners' attitudes, highlighting how the visual interactivity of the tool helped demystify abstract mathematical concepts. Similarly, PST2 noted that learners appeared more motivated and participatory when engaging with GeoGebra-based activities. She encouraged students to manipulate the sliders themselves and reflect on what they observed, allowing them to feel ownership of their learning. PST4 also shared that learners responded with enthusiasm during her lesson, especially when comparing two parabolas and identifying how different values of a and q affected the graph. Thus, these examples show that the PSTs' use of GeoGebra to present accurate, content-rich, and interactive visualisations helped create a positive learning environment. Learners expressed enjoyment, showed increased willingness to participate, and demonstrated curiosity, all of which contribute to the development of a productive disposition toward mathematics. The preservice teachers' ability to achieve this through content-driven technology use reflects a robust application of TCK.

4.1.10 TPK and productive disposition

Preservice teachers also demonstrated TPK by using GeoGebra to foster a supportive, reflective, and learner-centred classroom environment, conditions that promote a productive disposition as defined by Kilpatrick et al. (2001). They engaged learners not only in understanding mathematical content but in developing positive attitudes toward learning mathematics through interactive pedagogical strategies.

PST4, for example, created space for learners to share their mathematical thinking during and after GeoGebra demonstrations. By encouraging class discussions around transformations of the parabola and asking open-ended questions, she allowed learners to reflect on what they were observing and articulate their understanding. Learners were visibly excited to participate, and many offered accurate observations and justifications, such as identifying how a negative *a* results in a concave-down graph. PST4 later shared that learners' misconceptions about non-real x-intercepts were resolved more effectively through these discussions than in her earlier chalkboard-based lessons.

PST3 similarly used GeoGebra for reflective learning as she regularly prompted learners to explain their reasoning during slider manipulations. She described how this process boosted her confidence as a teacher, stating, "GeoGebra made me feel more knowledgeable… I would own the fact that I am a teacher." This enhanced self-efficacy translated into a more engaging and responsive teaching style, which in turn encouraged learners to participate more openly. Thus, by using technology to enable learner voice, reflection, and positive reinforcement, PSTs modelled a teaching approach that values learner agency and collaboration. Such practices show how digital tools can support not just content learning but also the emotional and attitudinal aspects of mathematics education, aligning with both TPK and the development of productive disposition.

4.2 Section 2: Stimulus recall interviews

This section presents insights from preservice teachers' reflections during stimulus recall interviews, highlighting how their use of GeoGebra enhanced both their content knowledge and teaching proficiency. The interviews reveal that dynamic visualisations deepened their understanding of quadratic functions and strengthened their confidence and effectiveness in classroom practice.

4.2.1 Enhanced content knowledge

Across the interviews, preservice teachers consistently reported that their understanding of quadratic functions, particularly parabolas, was significantly deepened through their use of GeoGebra. The visual and dynamic nature of the software enabled them to internalise mathematical concepts that had previously been abstract or difficult to grasp through traditional methods.

PST1 reflected on how the tool improved his ability to visualise parameter effects: *"I was able to show the learners how parameter a and q affect the shape of a graph… with GeoGebra I managed to show the learners the instant changes that each parameter causes."* He also emphasised how the software reduced the margin for error: *"With GeoGebra you get the exact shape of the graph. When you draw it by hand, you can make mistakes and learners might think it's correct."* Similarly, PST2 shared that GeoGebra brought clarity to subtle but important mathematical ideas: *"GeoGebra did help me to be more knowledgeable… I experienced visually all that I was taught… I would see some changes in the graph when a moves between 1 and 4… such details are not easy to explain to students when using a chalkboard and chalk."* This visual learning allowed her to re-engage with previously learned content in more meaningful ways.

PST3 acknowledged a shift in her conceptual understanding: *"GeoGebra made me realise that I did not really understand the effects of a and q the way GeoGebra has demonstrated it… my content knowledge has improved a lot since I used GeoGebra."* Continuous manipulation of the sliders helped her build confidence in interpreting how parameter changes influenced the graph. PST4 similarly credited GeoGebra with resolving prior misunderstandings: *"Before using GeoGebra, I struggled to make sense of different gradient values and their impact on the shape of the graph… I didn't know that a decreasing graph meant the gradient is negative."* The software enabled her to connect symbolic and graphical representations of functions more effectively.

4.2.2 Improved teaching proficiency

In addition to strengthening their mathematical content knowledge, preservice teachers also described how GeoGebra enhanced their overall teaching proficiency. They noted improvements in lesson delivery, classroom confidence, error reduction, and learner engagement.

PST1 explained that the ability to demonstrate real-time graph changes boosted his confidence in the classroom: *"With GeoGebra, I managed to show the learners the changes in the graph right away… and I knew I wasn't making mistakes like I might on the board."* He added that learner feedback was overwhelmingly positive: *"After class, my learners said to me, 'Sir, we did learn these things before, but we never understood a single thing!'"* For PST2, GeoGebra transformed her teaching experience: *"It made my teaching easier and helped my students understand the parabola's transformations. I will definitely continue using it in the future."* She emphasised how the tool made it possible to design lessons that were interactive and conceptually rich.

PST3 also noted a significant shift in her teaching identity and confidence: *"GeoGebra made me more confident when I teach… I would own the fact that I am a teacher, and I am more knowledgeable than the learners."* She highlighted the practicality of the tool: *"It saved me time – I didn't have to make photocopies or draw everything manually."* Finally, PST4 reflected on how GeoGebra helped her address learner misconceptions more effectively: *"My learners didn't understand why the graph without x-intercepts behaves that way… I explained, but they didn't get it until I showed them with GeoGebra… then it made sense to them."* She emphasised the tool's ability to visualise concepts that are otherwise difficult to convey.

5. Discussion

This section presents a discussion of the findings, highlighting how preservice teachers applied TCK and TPK through GeoGebra to support learners across key strands of mathematical proficiency. It also reflects on how this process enhanced the teachers' own understanding and teaching skills.

5.1 Developing conceptual understanding through TCK and TPK

The findings of this study show that preservice teachers effectively used GeoGebra to support learners' conceptual understanding of parabola functions by connecting symbolic representations with dynamic visualisations. Through TCK, PSTs selected and manipulated applets that illustrated how changes in parameters a and q influence the direction, shape, and position of the graph. These real-time demonstrations enabled learners to visualise the immediate effects of algebraic modifications, which helped bridge the gap between abstract mathematical expressions and their graphical forms, an essential aspect of conceptual understanding (Kilpatrick et al., 2001). As PST3 reflected, *"GeoGebra made it easy to understand the effects of a and q; the more I used the sliders, the clearer it became."*

Furthermore, TPK was equally evident in how PSTs sequenced their lessons, used probing questions, and facilitated exploratory discussions. PST2's lesson, for example, included purposeful manipulation of sliders to contrast cases where $a > 0$ and $a < 0$, prompting learners to predict and explain the changes. These interactive approaches not only engaged learners but also supported the construction of mental models. This finding aligns with Sun (2023) and Hidayat et al. (2024), who emphasise that GeoGebra's dynamic feedback enhances learners' conceptual reasoning. It also resonates with Öçal (2017), who found that GeoGebra enables preservice teachers to convey complex ideas with clarity. In the context of rural, under-resourced classrooms, such as those in the Eastern Cape, GeoGebra served as a powerful conceptual bridge, benefiting both learners and preservice teachers in understanding the behaviour of quadratic functions.

5.2 Building procedural fluency with dynamic visual tools

The study found that GeoGebra supported preservice teachers in fostering procedural fluency by enabling learners to engage with mathematical processes in a clear, accurate, and efficient manner. Through TCK, PSTs used the tool to demonstrate multiple representations of the parabola function, allowing learners to practise interpreting and manipulating key parameters with precision. By transitioning from manual graphing to dynamic visualisation, PSTs scaffolded the learning process and reduced the risk of procedural errors. For instance, PST1 first drew a parabola manually and then confirmed its properties using GeoGebra, reinforcing both traditional methods and digital verification. PST4 gradually increased the complexity of the applets used, guiding learners through structured practice.

These practices are supported by Hidayat et al. (2024), who assert that dynamic tools like GeoGebra promote fluency by offering learners immediate feedback and opportunities to revise their understanding. The software's responsive environment allowed PSTs to address misconceptions in real time, helping learners refine their procedural skills with confidence. In general, the combination of visual accuracy, interactive pacing, and real-time responsiveness enabled PSTs to support procedural fluency effectively, in line with Kilpatrick et al.'s (2001) emphasis on flexibility and efficiency in mathematical practice.

5.3 Enhancing strategic competence through GeoGebra integration

The findings revealed that preservice teachers used GeoGebra to design and facilitate lessons that promoted strategic competence, defined as the ability to plan, solve instructional problems, and support learners in doing the same (Kilpatrick et al., 2001). Through both TCK and TPK, PSTs created opportunities for learners to engage in reasoning tasks, make predictions, and interpret mathematical relationships. For instance, PST2 challenged learners to determine equations from graphs using GeoGebra's visual cues and coordinates, prompting them to reverse-engineer their reasoning. PST3 used multiple graphs to facilitate comparison, encouraging learners to think critically about transformations. These strategies reflect deliberate instructional planning that goes beyond explanation to cultivate active problem-solving skills. This aligns with the findings of Muslim (2023) and Mensah (2023), who argue that GeoGebra enables deeper mathematical reasoning and exploration.

Furthermore, the tool's flexibility enabled PSTs to construct tasks that mirrored real-world mathematical thinking, supporting both learner autonomy and strategic engagement. Sudarsana et al. (2022) similarly emphasise the role of GeoGebra in enabling learners to work through complex problems visually and analytically. Moreover, in under-resourced rural settings like those in the Eastern Cape, such dynamic problem-solving approaches are critical. The capacity of GeoGebra to simulate varied scenarios and support experimentation compensates for limited access to manipulatives or printed resources (Manganyana et al., 2020; Mokotjo, 2023). In essence, the study illustrates that when PSTs are trained to design GeoGebra-enhanced lessons, they can successfully foster strategic competence, an essential component of teaching proficiency that empowers learners to become active problem solvers.

5.4 Fostering adaptive reasoning through responsive teaching

The study found that preservice teachers demonstrated **adaptive reasoning** by responding to learners' real-time challenges and adjusting their instructional approaches using GeoGebra. Adaptive reasoning, as outlined by Kilpatrick et al. (2001), involves the ability to reflect, explain, and adapt teaching to better support learner understanding. For instance, when a learner questioned why the graph "disappeared" at $a = 0$, PST2 used GeoGebra to show that the quadratic function becomes a horizontal line ($y = q$), turning a moment of confusion into a valuable learning opportunity. Similarly, PST3 modified her strategy when learners struggled with the function $y = x^2$ by introducing a table of values, then confirming the resulting graph using GeoGebra. These adjustments supported learners in making sense of concepts through both symbolic reasoning and visual feedback.

These actions illustrate a growing level of TPK among the PSTs, who used technology not only for demonstration but also as a tool for responsive teaching. This is supported by Weinhandl et al. (2021), who argue that adaptive content delivery is key to effective technology integration. Morales-López et al. (2021) further highlight that flexibility in using digital tools, rather than mere technical skill, distinguishes effective teaching. In the context of under-resourced rural classrooms, such flexibility is essential. GeoGebra enabled preservice teachers to both diagnose and address misconceptions on the spot while simultaneously deepening their own reflective practice. This process is echoed in the findings of Ngware et al. (2015), who advocate for technology as a means of enhancing pedagogical responsiveness and professional growth.

5.5 Nurturing productive disposition through engagement and confidence

The use of GeoGebra helped preservice teachers cultivate a productive disposition, both in themselves and their learners, by fostering enthusiasm, confidence, and a positive attitude towards mathematics teaching and learning (Kilpatrick et al., 2001). PST1 shared that learners became visibly excited and eager to engage when GeoGebra sliders were used: *"They said they had learned this before but never understood it until I used GeoGebra."* Similarly, PST4 observed that learners who initially held misconceptions about x-intercepts gained clarity through visual demonstrations, leading to increased participation. The preservice teachers also reported an increase in teaching confidence. PST3 noted: *"GeoGebra made me feel more confident when I teach… I felt like I owned the lesson."* This aligns with findings from Marange and Tatira (2023) and Rajagopal et al. (2015), who emphasise that interactive tools like GeoGebra can boost teacher self-efficacy and learner motivation.

By promoting learner reflection, encouraging mathematical talk, and providing accurate visual feedback, GeoGebra supported a learning environment where both teacher and learner engagement improved. In resource-constrained rural classrooms, this positive shift is critical for sustaining interest and participation in mathematics (Khalo, 2020; Manganyana et al., 2020).

5.6 Preservice teachers' growth in content knowledge and teaching proficiency

The integration of GeoGebra significantly contributed to preservice teachers' development of both mathematical content knowledge and teaching proficiency. Through frequent use of the tool, PSTs reported a clearer understanding of the effects of parameters in quadratic functions, particularly the roles of a and q in shaping and shifting the parabola. PST2 reflected: *"With GeoGebra, I experienced visually all that I was taught… I could finally understand how small changes in a affect the width of the graph."* Similarly, PST4 shared how her understanding of gradient and graphical shifts improved after interacting with GeoGebra applets. In terms of teaching proficiency, PSTs developed confidence in lesson planning, delivery, and learner engagement. GeoGebra allowed them to facilitate more interactive, accurate, and visually supported lessons. PST1 emphasised: *"GeoGebra helped me teach without worrying about making mistakes on the board… it gave me confidence."*

These findings support Zambak (2015) and Sedega et al. (2018), who highlight the role of technology in strengthening both pedagogical and content expertise. Moreover, the ability to design, implement, and adapt digital tools for instruction reflects growth in TPACK, specifically in the blending of technology with pedagogy and content (Koehler & Mishra, 2006). Importantly, this growth occurred in a rural, under-resourced context, demonstrating that, with appropriate training, preservice teachers can harness digital tools to overcome traditional instructional barriers and deliver high-quality mathematics lessons (Morales-López et al., 2021; Mokotjo, 2023).

6. Conclusion And Recommendations

This study explored how the integration of GeoGebra enhanced preservice teachers' proficiency in teaching Grade 10 parabola functions within a rural South African context. Drawing on the TPACK framework and Kilpatrick et al.'s strands of mathematical proficiency, the findings revealed that GeoGebra supported the development of conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and a productive disposition. The tool not only deepened content knowledge but also improved instructional confidence, adaptability, and learner engagement. Based on these findings, it is recommended that initial teacher education programmes incorporate structured training on dynamic visual tools like GeoGebra. This training should go beyond technical skills and focus on pedagogical integration aligned with curriculum content. Furthermore, providing access to digital tools in rural teaching practice schools could bridge resource gaps and support meaningful mathematics learning. Future research may explore the long-term impact of GeoGebra use across different topics and its influence on learner achievement.

7. Declarations

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