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# Physics Difficulty and Problem-solving: Exploring the Role of Mathematics and Mathematical Symbols

**Olalekan Taofeek Badmus**[1*]
**Loyiso C. Jita**[2]

AFFILIATIONS
[1,2]Faculty of Education, University of Free State, Bloemfontein, South Africa.

CORRESPONDENCE
Email: Badmus.OT@ufs.ac.za*

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**Abstract:** Reports of difficulty in physics has been documented over the years, especially at the senior secondary level. The application of mathematics as a tool for understanding physical phenomena and problem-solving is well-established. The use of symbols and mathematical rigour is essential for effective problem-solving in physics. However, the teaching and learning of physics have encountered barriers, as highlighted in the literature on competencies in this field. This study focuses on exploring the interlink, context, and associated barriers in the teaching and learning of physics by reviewing existing literature on the application of mathematics and mathematical symbols. Through a theory synthesis design, the study examined the current state of literature on mathematical problem-solving in physics, as well as the differences between mathematising and the application of mathematics in physics. The competencies required of teachers and students were also highlighted in order to better equip physical sciences teachers to address the challenges faced by students in learning physics. The literature suggests that a well-sequenced approach to topics by both mathematics and physics teachers can facilitate knowledge transfer among students. Teachers are encouraged to provide step-by-step guidance to address students' mathematical deficiencies, particularly in the physics aspect of the physical sciences curriculum at the further education and training (FET) phase. It is recommended that topics between physics and mathematics be aligned and mathematical concepts be pre-teach to enhance students' contextual knowledge transfer.

***Keywords:*** Physics difficulty, problem-solving, mathematics, mathematical symbols.

## 1. Introduction

Students' source of difficulty in physics often lies in mathematical rigour. When students struggle with the mathematical aspect of physics lessons, teachers might suggest that they study more mathematics. However, the use of mathematics in science, particularly in physics, is not merely about doing mathematics. It serves a different purpose, which is to represent meaning about physical systems rather than expressing abstract relationships or symbolic representations from pure mathematics. The application of mathematics in physics is not the same as mathematising (Tuminaro & Redish, 2004).

Galileo Galilei's famous quote "Measure what is measurable and make measurable what is not so" highlights the importance of mathematics in giving meaning to the known. He also described the world as a perfect machine whose workings could only be understood in mathematical terms (Galilei, 1623). As described over the years, mathematics has far-reaching implications in the development of all aspects of human life, both within and outside of science. Observation and experimentation are two major processes in science that rely on mathematics and its applications (Dirac, 1940). Sense-making in science requires theorising physical phenomena in the form of equations, which allow for standards and universality in empiricism (Dirac, 1940). While science can take credit for the progress

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made in engineering and technology, mathematics existed before science as a field of study and can take even more credit for nurturing science into what it is today. The field of science has grown to be recognised as essential for the world's ease and sustainability. However, the increasing demand for science and its applications in various fields underscores the need for science education.

The demand for experts in the fields of Science, Technology, Robotics, Engineering, Aesthetics, and Mathematics (STRAEM) is growing (Badmus & Omosewo, 2020; Begg & Pierce, 2021; Burkholder et al., 2020; Guangming et al., 2019; Jardim et al., 2021; Park & Liu, 2021). Expertise in these fields requires certain competencies that translate into knowledge and practical skills. When considering the enrollment and performance of learners in sciences at both secondary and tertiary levels, empirical positions suggest a lack of interest due to the difficulties experienced in teaching and learning (Arcavi, 2005; Begg & Pierce, 2021). We believe that insufficient research has been conducted to explore the fundamentals of physics difficulty in relation to mathematics and mathematical symbols. Understanding the interaction among different forms of matter in relation to motion and energy is key to defining physics in its literary form. Do learners develop interest or not? This question has a place in the literature, given that interest and attitude are shaped by experiences (Mao et al., 2021). Positions derived from the literature suggest that physics, as one of the core science subjects, is perceived as more difficult due to several factors (Liu & Sun, 2021; Pingxia, 2018). The mathematical requirements and technical competence necessary for problem-solving in physics necessitate specific knowledge and skills that can be acquired through training in knowledge formation, operational guidance in experiments, understanding the nature of physics, and developing associative thinking patterns, among others (Dębowska & Greczyło, 2017; Guangming et al., 2019; Shaung, 2016). Additionally, learners are required to refine their skills, concepts, knowledge, and the scientific and humanistic spirit of inquiry to succeed (Hongjum, 2018; Liu & Sun, 2021).

Physics teaching and learning are guided by historical ideologies and conceptions, which influence the actions of both the teacher and the learner. The History of Science (HOS) and Nature of Science (NOS) provide educators with valuable guidance on how to integrate these concepts into the teaching and learning of physics (McComas, 2020; McComas & Clough, 2020). The teaching and learning of physics are not separate from pedagogical strategies that facilitate a comprehensive understanding of the science of matter, energy, and their interactions (Badmus & Jita, 2022a; Jardim et al., 2021; Viennot & Décamp, 2020). Additionally, there are established competencies for effective and efficient physics instruction that consider both NOS and HOS, which are particularly important within the physical sciences curriculum (Nouri & McComas, 2021; Sweeney & McComas, 2022). Physics educators hold diverse opinions when it comes to the challenges associated with teaching and learning of physics. Teaching physics effectively requires a strong foundation in both content and pedagogical knowledge, as well as mathematical competency and more. Similarly, learning physics requires a range of abilities in addition to the usual prerequisites in order to achieve satisfactory performance in the subject and embrace it as a discipline.

Physics educators have identified students' mathematical deficiencies as a primary cause for their difficulties (Arcavi, 2005; Redish, 2004; Tuminaro, 2004). Recently, experts have supported the idea that mathematics and physics are intimately connected (Bardini & Pierce, 2015; Begg & Pierce, 2021). The knowledge of mathematics is applicable to physics and vice versa. It is evident from the literature that there are associations and differences between these two disciplines (Dębowska & Greczyło, 2017; Liu & Sun, 2021; Tuminaro & Redish, 2004). These differences often confuse learners due to their interconnectedness (Begg & Pierce, 2021; Redish, 2004; Tuminaro, 2004). When studying physics, the application of mathematical knowledge is crucial, especially when comparing real-world phenomena to theoretical models. Additionally, mathematical manipulations and conceptual reasoning of physical phenomena using equations enable physics to have real-life applications. Therefore, a deep understanding of mathematics is necessary to cope with the rigour of physics

(Bardini & Pierce, 2015; Redish & Kuo, 2015; Uhden et al., 2012).In mathematics, symbols are often used with minimal substitutions and constants (Arcavi, 2005; Şahin & Yağbasan, 2012), whereas in physics, symbols carry significant meaning and typically represent values, constants, or derivatives that can be substituted in applications (Begg & Pierce, 2021).

Although mathematical limitations are a significant factor contributing to the difficulty students face in learning physics, scholars have also pointed out other legitimate concerns. These include the lack of emphasis on physics processes and insufficient attention to laboratory activities that aid learners in imagining and conceptualising, as well as the standards and ordering of parameters (Moelter & Jackson, 2012). Therefore, this study aims to aggregate knowledge from existing literature to address the challenges faced in physics classrooms through the application of mathematics and mathematical symbols. In this manuscript, we provide the rationale for using mathematics and mathematical symbols in physics teaching and learning, the competencies required for physics teaching and learning; an overview of the literature on the application of mathematics in physics; and an examination of the use of mathematical symbols in physics and other mathematical sciences in South Africa's Further Education and Training [FET] phase of the physical sciences curriculum.

### 1.1 Methodological design

Theory synthesis was used in this study to address the research objectives. In conceptual research, theory synthesis provides a basis for integrating literature and/or theories (Jaakkola, 2020). This paper aggregates relevant literature to argue for a new understanding of physics difficulty by connecting previously separate empirical studies. The argument is based on combining and integrating existing knowledge about the concept or phenomenon that requires intervention, allowing the researcher to develop a fresh perspective (MacInnis, 2011). We implemented theory synthesis by reviewing and analysing previous literature on the nature of physics, its teaching, learning, and challenges. We took a multidisciplinary approach to physics and mathematics and then established a theoretical foundation for synthesising the literature on mathematics and mathematical symbols in physics. We conducted a literature review on the global and local context related to the study and used it to gain a novel understanding of physics teaching and learning, highlighting the needs of both teachers and students. Additionally, we discussed various positions in the literature to form the positions in this study. The study concludes with recommendations based on the findings.

Ethical issues are of paramount importance in research studies. However, the authors assert that no ethical issues were violated in this manuscript. Since there were no direct human participants in this study, no primary data were collected from respondents that would have required consent. All intellectual works related to this study are properly cited.

## 2. Conceptual and Theoretical Appendage

This manuscript is centred around the theory of social constructivism and knowledge transfer. It asserts that knowing and knowledge creation are influenced by social experiences, where language plays a critical role in the formation and transfer of knowledge. In the field of physics, mathematics serves as a language of expression, with most physical phenomena having mathematical representations. The creation and regulation of knowledge are outcomes of experiences formed through social interaction (Bruner, 1961; Trevor et al., 2016; Vygotsky, 1962).

Constructivism, as a theory of learning, views learning as an active and constructive process that occurs within the learner's mind. While this theory provides insights into how learning takes place, it also addresses the essential element of knowledge transfer, particularly in the context of this manuscript: the transfer of mathematical knowledge for problem-solving in physics and physical sciences. Once knowledge is formed, the application of that knowledge occurs at a higher cognitive level. Importantly, applicable knowledge is transferable. In order for knowledge to be transferred (in this case, from mathematics and mathematical symbols), the learner must identify and articulate the problem to the brain (Gick & Holyoak, 1987). The brain then analyses the context in terms of the knowledge available to it and subsequently forms a contextual disposition of knowledge relevant to the problem. Transfer activity is then employed to address the problem, utilising the contextualised knowledge to solve it [physics problems] (Gick & Holyoak, 1987). This framework facilitates this type of knowledge transfer, allowing for linear, cyclical, and multi-directional [dynamic] transfer of knowledge (Turşucu et al., 2017).

## 2.1 Situated literature

The national interest report by Chubb et al. (2012) highlighted the declining enrollment in mathematical sciences in Australia. Taylor (2005) and Arcavi (2005) have also suggested that students prefer studying mathematics over mathematical sciences. In South Africa, the Department of Basic Education (DBE) has reported low enrollment and achievement in mathematics and physical sciences (DBE, 2019, 2020, 2021, 2022). Research has shown that the use of mathematical symbols, like letters, figures, compound templates, and symbolic expressions, is complex in mathematical science (Petrus, 2018; Redish, 2004; Tuminaro, 2004). Students often struggle with applying mathematical concepts and symbols in physics and other mathematical sciences (Badmus et al., 2020; Kuo et al., 2013). Hoban et al. (2012) found that students find it easier to read and write mathematics than to apply it, leading to a discouragement towards the field of mathematical sciences. De Lozano and Cardenas (2002) studied the application of mathematical knowledge in mathematics versus mathematical science [physical sciences]. They discovered inconsistencies in the usage of symbols, as mathematical science involves real-world applications and contexts that are not always explicitly stated, causing confusion among learners. For example, the equal sign [=] is used to indicate symmetry, transitivity, and reflexivity in mathematics but may be used differently in physics or physical sciences.

A critical and complementary approach to barriers in mathematical science was researched by Bardini and Pierce (2015). The study identified three areas of interest regarding the difficulties experienced by students: discontinuity, heightened complexity, and uncharted extension in symbol usage in mathematical sciences. These areas are specific to the transition from secondary school to university, where the knowledge of symbols is often discontinued. Additionally, the application of mathematics in this context is more complex and occasionally ambiguous compared to students' previous experiences. Bardini and Pierce also found that students have to deal with an increase in symbol density, symbol familiarity, new symbols, and known symbols in physics and mathematical sciences.

Another study by Niss (2012) investigated a framework for the identification of students' difficulties in solving real-world physics problems. Through theoretical analysis, three critical steps in problem-solving in physics were identified: analysing the problem situation, choosing the relevant physical theory, and mathematising. Niss noted that mathematisation in physics lacks a pre-existing model for operation, but learners face this limitation over the years, often due to the contextual nature and type of mathematics involved. These difficulties account for barriers that students lack the competencies to navigate.

Physics problem-solving difficulties and their implications were examined among Bachelor of Education students studying to become physics teachers in India by Reddy and Panacharoensawad (2017). The study utilised a 5-point Likert scale questionnaire to assess aspects of difficulty in physics. Areas identified as limiting the students included difficulty remembering equations and applying necessary mathematical problem-solving techniques. The study recommended that teachers/instructors should provide more assistance to help students overcome mathematical inadequacies and improve their problem-solving abilities, ultimately easing the difficulties they experience. Similarly, Retnawati et al. (2018) employed a phenomenological approach to qualitative research in order to investigate teachers' difficulties in teaching physics, with a focus on mathematical applications in Indonesia. Data was gathered through focus group discussions involving 15 high school physics teachers. The study revealed a lack of synchronisation between mathematical topics and physics topics, specifically in terms of the content being covered. This implies that teachers must teach prerequisite mathematical concepts alongside their physics instruction. Consequently, physics class time is allocated to teaching these concepts that are expected to be taught in mathematics classes. As a result, physics class periods are insufficient to cover all the material in the syllabus. The researchers recommended that synchronising the mathematics and physics syllabi would help alleviate the difficulties students experience in physics classrooms. Moreover, if this practice were implemented, physics teachers would have fewer explanations to provide regarding the mathematical aspects of their instruction.

Within South Africa, Basson (2002) conducted a study investigating the relationship between physics and mathematics. The study aimed to address the struggles students faced in physics, which were attributed to their understanding of mathematics. The study proposed utilising spatial operational capacity to explain the interconnection between these subjects. It concluded that hierarchical development in both areas is necessary for students to comprehend physics phenomena. In a similar vein, Makgato (2007) conducted a study examining the factors contributing to poor performance in mathematics and physical sciences in South Africa. The study utilised exploratory, descriptive research methods and found that teachers lacked sufficient content knowledge, negatively impacting student achievement.

More recently, Basson (2021) explored the alignment of content among physics, mathematics, and computer science in South African secondary schools. The study revealed that the content of mathematics and physics largely coincided, with physics topics being incorporated into the mathematics curriculum. However, despite evidence supporting an interdisciplinary approach, the study found no alignment or systematic integration of physics and mathematics in teaching and learning practices. It is worth noting that there have been few studies conducted in this context to propose policy directions for a multidisciplinary approach that facilitates topical alignment.

## 3. Context of the Study

The Department of Basic Education oversees education regulation from grades R-12 in the Republic of South Africa. After completing grade 12, students must take the National Senior Certificate (NSC) examinations to be admitted into higher education institutions, which are managed by the Council for Higher Education [CHE]. NSC examination reports from recent years have shown improvement in both enrollment and achievement (30% achieved) in mathematics and physical sciences (Department of Basic Education [DBE] examination reports, 2018; 2019; 2020; 2021; 2022). The NSC serves several purposes: equipping candidates with knowledge, skills, values, and attitudes for self-fulfilment; providing access to higher education; facilitating the transition from the educational institution to the workplace; and providing competent and skilled employees. Notably, there was an improvement in mathematics from 53.8% in 2020 to 57.6% in 2021 and in physical sciences from 65.8% to 69% during the same period. However, these quoted figures do not accurately represent the situation, considering the declining pass rate of 50%. Admission into STREAM fields in higher education institutions, particularly universities, requires scores above 50% in both mathematics and physical sciences. Therefore, a critical evaluation of Table 1 is necessary.

***Table 1:*** *Students' achievement in mathematics and physical sciences*

| | Total achieved at 50% and above | | | |
|---|---|---|---|---|
| **Subject/Year of Examination** | 2019 | 2020 | 2021 | 2022 |
| **Mathematics** | 20.3% | 22.3% | 23.0% | 22.0% |
| **Physical Sciences** | 33.1% | 26.3% | 27.3% | 30.4% |

From Table 1, the achievement of students scoring above 50% in both mathematics and physical science was not above 34% in the years under review. This means that courses requiring aboveaverage achievement in these subjects will suffer from low patronage. The economic implication of the aforementioned is the country's inability to train enough experts in STREAM disciplines. Hence, areas with a limited number of experts will be outsourced to promote economic growth. Table 1 was intended to cover a span of 5 years; however, the statistic for previous results (2018) was set as a benchmark at over 40%, and the result for 2023 is currently unavailable at the time of writing.

***Table 2:*** *NSC physical sciences diagnostic report from 2018-2022*

| Year | Challenging topics in physical science. Paper 1- physics | Report on students' challenges | Solutions from the report (summary) |
|---|---|---|---|
| **2018** | • Electrostatics and current electricity.<br>• Work energy and power.<br>• Doppler effect.<br>• Momentum. | -Drawing and Interpretation of graphs.<br>-Identifying variables in relation to equations describing graphs and graphical representations.<br>-Problem-solving using scientific formulae and substitutions.<br>-Application of mathematical principles.<br>-Use of correct unit of measurement. | Students should be availed of problem-solving activities which involve mathematical knowledge, especially in quadratic equations, simultaneous equations, binomials, factorisation, trigonometry and graphs in-class work, test and exams. [National Senior Certificate 2022 Diagnostic Report pg. 153-162] |
| **2019** | • Equations of motion.<br>• Photoelectric effect.<br>• Momentum.<br>• Work, energy and power.<br>• Electrostatics and current electricity. | -Problem-solving skills, mathematical skills.<br>-Drawing and labelling on forces acting on objects.<br>-Graph interpretation.<br>-Incorrect working with formulae.<br>-Application of mathematical principles. | Integrated problem-solving should become an integral part of teaching and learning. Problem-solving should be done on a variety of topics. Teachers include problem-solving activities in class activities. [National Senior Certificate 2019 Diagnostic Report pg. 203-213] |
| **2020** | • Doppler effect.<br>• Electrostatics.<br>• Electrodynamics.<br>• Photoelectric. effect. | -Drawing and Interpretation of graphs.<br>-Problem-solving skills involve the action of forces on objects.<br>-Identification of variables relating to equations.<br>-Incorrect substitutions and manipulation with formulae. | Students should be availed of problem-solving activities which involve mathematical knowledge, especially in quadratic equations, simultaneous equations, binomials, trigonometry, factorisation, trigonometry and graphs in-class work, test and exams. [National Senior Certificate 2021 Diagnostic Report pg. 206-2016] |
| **2021** | • Newton's laws of motion.<br>• Projectile motion<br>• Momentum and impulse<br>• Work, energy and power.<br>• Electrostatics and current electricity. | -Interpretation of graphs was a challenge.<br>-Problem-solving skill<br>-Identification of variables relating to equations.<br>-Application of mathematical concepts and principles | Students should be availed of problem-solving activities involving mathematical knowledge, especially in quadratic equations, simultaneous equations, binomials, factorisation, trigonometry and graphs in-class work, tests, and exams. |

| | | | |
|---|---|---|---|
| | | | [National Senior Certificate 2022 Diagnostic Report pg. 210-222] |
| **2022** | • Photo electric effect.<br>• Static and current electricity.<br>• Interpretation of graphs and drawings. | -Application of mathematical principles is a challenge to students.<br>-Drawing and interpretation of graphs.<br>-Problem-solving with equations and graphs is a challenge. | Students should be availed of problem-solving activities involving mathematical knowledge, especially in quadratic equations, simultaneous equations, binomials, factorisation, trigonometry and graphs in-class work, tests, and exams. [National Senior Certificate 2022 Diagnostic Report pg. 219-233] |

From Table 2, it is clear that students face challenges when it comes to applying mathematical concepts in physics. The report highlights a consistent lack of understanding and proficiency in using mathematical principles and problem-solving skills throughout the years under review. Although the report suggests that students should be taught various topics such as simultaneous equations, quadratic equations, geometry, trigonometry, binomials, and factorisation, it does not assign this responsibility to mathematics teachers, despite the fact that these topics fall within the realm of mathematics. Moreover, these topics are included in the mathematics curriculum and should have been taught to students over the course of their three years in school at this level. The crucial question here is: When were these topics taught? Were they taught at an appropriate time for an easy transfer of knowledge? However, it is important to note that the application of mathematical knowledge in physics differs from its use in mathematics itself. Therefore, the responsibility for incorporating mathematical knowledge into physics problem-solving lies with physics teachers. Explicitly integrating mathematical concepts into physics instruction can greatly benefit student learning. However, it is worth considering that revisiting mathematical topics within the physics classroom may be time-consuming and could detract from covering essential physics topics due to limited class time. To facilitate the transfer of knowledge, it is recommended to carefully sequence and align the teaching of mathematical topics with their corresponding physics concepts.

## 4. Rationale

Variations exist in the usage of mathematics across various disciplines. While mathematics is a broad field, its application in the field of science has been documented since the beginning of science itself. The underachievement of students in mathematics and physical sciences is a recurring issue in South Africa, requiring intervention (DBE, 2019, 2020, 2021, 2022; Prince, 2017). Reports indicate that students are increasingly avoiding core science disciplines, leading to a shortage of critical skills due to limited enrollment (Kirby & Dempster, 2018). There have been instances of students boycotting the physics aspect of physical sciences examinations (Kirby & Dempster, 2018). Students who take physical sciences in the FET phase often ignore physics-related questions (Badmus & Jita, 2023; Petrus, 2018). The teaching and learning of physics need to be revisited in order to address barriers to students' interest and learning. Previous studies have suggested a correlation between high performance in mathematics and physics, indicating that strong mathematical skills may contribute to success in the field of physical sciences (Makgato, 2007). Alternative perspectives in the literature also highlight the importance of mathematics in physics, particularly in specific areas where the knowledge of mathematics can be transferred and applied (Badmus et al., 2020; Badmus & Jita, 2023; Kuo et al., 2013). Mathematics and mathematical symbols have extensive applications in physics (Arcavi, 2005; Badmus & Jita, 2023; Begg & Pierce, 2021; Taylor, 2005). However, the term "mathematising" is often used to emphasise that the application of mathematical knowledge in physics (and other physical sciences) is not fundamentally different. Furthermore, problem-solving in physics requires the manipulation and rigour of mathematics, which is context-dependent. Effective teaching of science requires a deep understanding of content, pedagogy, and curriculum in order to provide learners with clear explanations and facilitate conceptual understanding (Mtsi & Maphosa, 2016; Taylor, 2019). This review aims to guide teachers on areas that require particular attention in addressing difficulties faced by students in physics during the FET phase.

## 5. Competencies for Physics Teaching

The goal of every curriculum is to develop key competencies among learners. Guiding learners and carrying out teaching activities are the roles of teachers, especially in the formal teaching and learning environment of a classroom. In order to effectively deliver knowledge, skills, and attitudes to benefit learners, certain competencies are required. In this section, we will explore the literature to gather the opinions of scholars regarding the competencies needed by physics teachers and in physics teaching.

According to Takayama (2013), there are three broad areas of competencies with sub-areas: the ability to act autonomously, the capacity to make use of interactive tools, and the ability to function in a socially heterogeneous environment. The Australian Standards for Teachers (APST) also requires certain professional knowledge, practice, and engagement specifically for physics teachers. As discussed by Redish and Kuo (2015), it is the responsibility of the teacher to reteach mathematical concepts and methods in physics topics or courses where students are struggling. Therefore, teachers are expected to provide the necessary competencies for learners to connect mathematical equations to their physical meanings, making the processing of mathematical tools and concepts easier for students in the context of physics.

Competency in physics is comprised of physics knowledge, methods, thinking skills, concepts, science, and humanistic spirits (Hongjun, 2018). Physics knowledge refers to understanding concepts and laws, while skills and thinking involve knowing students' perspectives, knowledge structures, and the best methods for imparting physics knowledge based on individual capabilities and capacities. For physics teachers, understanding their learners and their thought processes should be the foundation for effective teaching and presentation of acceptable knowledge. Physics is not an isolated subject; rather, it integrates with societal norms and values, contributing to the culture of science (Badmus & Jita, 2022b). The scientific spirit is characterised by inquiry, a scientific attitude, scientific ideals, and independent thinking. Additionally, a physics teacher should prioritise concerns for human values, dignity, universal self-care, spiritual experiences, and cultural phenomena, embodying a humanistic spirit. Key competencies for physics teachers include physics literacy, education and teaching literacy, scientific literacy, humanistic literacy, information literacy, and the ability for lifelong learning (Liu & Sun, 2020).

Moelter and Jackson (2012) emphasised the importance of symbol ordering in physics formulas, particularly in terms of standard form and how formulas are understood and applied in problem-solving by students. This study highlights the responsibility of teachers to explain and differentiate the reading and writing of parameters, constants, and variables in order to prevent confusion among learners. It is crucial for instructors to explicitly communicate this distinction to their students (Turşucu et al., 2017). Physics teachers should effectively clarify these boundaries in a contextual and explicit manner. For example, while equations are commonly presented in standard form, instructors must explicitly demonstrate how to change the subject of the equation for each parameter. Any challenges in this regard should be addressed before introducing students to the fundamentals of applying formulas in the given topic.

Alongside the focus on symbols, Retnawati et al. (2018) revealed a lack of complementarity between the mathematics and physics syllabi. Typically, mathematics topics are taught independently of their relevance to other subjects, such as physics. To enhance students' understanding of physics, it is recommended that topics and aspects of mathematics be organised in a systematic and sequenced manner. Failure to implement this approach could result in redundancy between the two syllabi. Ultimately, it is the responsibility of physics and mathematics teachers to negotiate and coordinate this sequencing rather than relying on a multidisciplinary curriculum approach.

## 6. Competencies for Physics Learning

The confusion learners face regarding why mathematics, mathematical concepts, and symbols have different applications in physics is fundamental to their learning (Kuo et al., 2013; Redish, 2014). Often, insufficient explanations are provided in the classroom due to teachers' limited time for critical discussion and sometimes a lack of explanation for the status quo. While teachers may find it convenient to understand and accept the application of mathematics, mathematical symbols, and concepts due to their training, learners often require more explanations, which are not always provided, leaving them with doubts about their abilities (Hsu, 2013).

It is important to note that a good knowledge of mathematics does not directly translate to better performance in physics, despite research on problem-solving in physics (Bardini & Pierce, 2015; Hsu et al., 2004; Redish, 2014). According to Redish and Kuo (2015), the struggle for students lies in the translation of mathematical knowledge and symbols rather than the mathematics itself, as is often perceived. They further argue that the mathematical equations learned in mathematics class may not look or feel the same as those in physics classes. This raises the question of what competencies are required of learners to perform as expected in physics classes.

Problem-solving and critical thinking abilities are essential for learners in physics (Burkholder et al., 2020). Physics problems require cognitive processes of sense-making, which involve the interpretation and evaluation of strategies (Adam & Wieman, 2015). Experts find these cognitive sense-making processes easier due to their experience (Arcavi, 2005). However, physics learners are considered novices in their reasoning patterns, including critical thinking and problem-solving capabilities (Burkholder et al., 2020; Larkin et al., 1980). In order to succeed, learners need the ability to break down physics problems into manageable pieces (Passow & Passow, 2017; Sabella & Redish, 2007). Cognitive adaptations such as unit conversion, equation manipulation, arithmetic rigour, symbol representations, and substitutions give experts an advantage in seeing complex problems as regular or everyday problems. On the other hand, learners require critical thinking and problem-solving capabilities to navigate through these processes, which experts consider routine. Interpretation and evaluation of physics problems and solutions also require these capabilities, which are often lacking in learners. Coaching, modelling, and scaffolding are suggested in the literature as approaches to help learners develop these necessary competencies (Arcavi, 2005; Bardini & Pierce, 2015; Redish, 2014).

Learners are encouraged to take mathematics seriously if they want to succeed in physics and other mathematical sciences. Chiu's (2015) study supports an alternative viewpoint to Redish and Kuo's (2015) study by stating that proficiency in mathematics provides learners with the logical reasoning skills needed for problem-solving in physics. The researchers also argue that learners with poor mathematical proficiency may lack the analytical mindset required to cope with the expectations of problem-solving in physics and other mathematical sciences. Similarly, symbols in physics are associated with constants, parameters, and variables, which are often conditioned and applicable under specific conditions. Unlike in mathematics, where symbols can be replaced without conditions, physics students must pay close attention to the changes in application and the complexity of the associated conditions. While teachers and textbooks are expected to clarify these conditions in both subjects, it is ultimately the students' responsibility to become familiar with them.

## 7. Discussion

The requisite mathematical foundation is essential for a proper understanding of physics concepts. However, the application of mathematics in physics differs slightly from its use in mathematics classes

(Redish & Kuo, 2015; Tuminaro, 2004). Nonetheless, the transfer of knowledge from mathematics to physics remains necessary (Reddy & Panacharoensawad, 2017; Redish, 2014; Turşucu et al., 2017). The role of teachers in explicitly teaching symbols and mathematics in physics classes involves clarifying how these concepts apply specifically to physics (Badmus & Jita, 2023; Begg & Pierce, 2021). The responsibility of teaching mathematics does not lie solely with mathematics teachers; instead, a well-sequenced syllabus created collaboratively by both mathematics and physics teachers can facilitate the transfer of knowledge from mathematics to physics. As a result, physics teachers are left with the task of explaining how mathematical knowledge applies to problem-solving in physics rather than teaching mathematics alongside its application, which often confuses students due to the differences in usage (Chiu, 2015; Retnawati et al., 2018; Retnawati et al., 2017). Teaching and learning the interrelations between these two subject areas without overwhelming students or causing confusion remains a challenge.

Formalising physical theory is incomplete without mathematical representation, even at ontological, epistemological, and social levels. It is crucial for students to understand this (Begg & Pierce, 2021). The mathematical symbols encountered by students in physics during the Further Education and Training (FET) phase are limited to that level. According to the literature reviewed, symbols can have different meanings and contexts at the tertiary level. While learners tend to hold on to previous knowledge regarding the application of mathematics and its symbols in physics, it is important to adapt and learn in the new context (Turşucu et al., 2017; Uhden et al., 2011). Students have been reported to engage in rote manipulation of formulas when solving physics problems (Arcavi, 2005; Uhden et al., 2012). Rather than seeing mathematics as a tool to enhance conceptual understanding in physics, students are generally more focused on obtaining the correct answer without considering the meaning behind it (Chiu, 2015). To succeed, students need to be comfortable with new parameters and embrace new knowledge of symbols. The role of the teacher is to facilitate this transition and familiarise students with new symbols and their context of usage. Discrepancies in teachers' content, pedagogy, and curriculum knowledge in South Africa have been documented (Taylor, 2019). However, physical science teachers who are willing to assist students in the physics aspect of the curriculum must be prepared to provide adequate and explicit guidance in these areas.

## 8. Conclusion and Recommendations

This study aimed to explore the difficulty of physics and problem-solving by considering the perspectives of mathematics and mathematical symbols. Mathematics and mathematical symbols play important roles in addressing difficulties encountered in both the teaching and learning of physics, as physics theories rely on mathematical equations to make sense. It is evident that the use of mathematics and mathematical symbols is dependent on the context and varies in different areas of physics. Based on the literature reviewed, aligning the curriculum may facilitate the transfer of knowledge from mathematics to physics through a multidisciplinary approach. According to the NSC physical sciences diagnostic reports, physics teachers have the responsibility of accommodating the mathematical needs of students in physics classrooms, as further explanation is necessary for students to conceptualise and understand the application of knowledge. It is recommended that professional development opportunities for physics teachers focus on equipping them to assist students in addressing their mathematical needs.

## 9. Declarations

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